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THE PHYSICAL REVIEW.

ANNOUNCEMENT OF THE TRANSFER OF THE REVIEW TO THE AMERICAN PHYSICAL SOCIETY.[1]

WITH the present number the AMERICAN PHYSICAL SOCIETY takes over the PHYSICAL REVIEW and assumes the entire responsibility for its conduct. In so doing the society wishes to give expression to its deep appreciation of the great service done to physics and physicists in America by the editors who, in July-August, 1893, put forth the first number of a new journal, and to Cornell University, which assumed the financial risk. There was at that time no journal in this country entirely devoted to physics, and there was no national society. During nearly twenty years the original editors have carried on the arduous task of maintaining this journal on a high standard, and it is difficult to estimate the value of their efforts in furthering the cause of physics in America. In this manner the way for the foundation of the American Physical Society was prepared, and early in its history the society and the REVIEW entered into relations which have continually become closer. The former editors have now thought best to complete their task by transferring their control to the American Physical Society, and the PHYSICAL REVIEW now becomes the journal of that society, national in scope, and looks for the coöperation of all American physicists. A. G. W.

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[1] For a more detailed statement of the action of the Society in taking over the REVIEW and of the arrangements made for its future management, see the Minutes of the Sixty-fifth meeting, pages 61–66, in this number.

DIFFRACTION AND SECONDARY RADIATION WITH ELECTRIC WAVES OF SHORT WAVE-LENGTH.

BY A. D. COLE.

SEVERAL years ago the writer presented to the American Physical Society the results of a study of diffraction effects obtained with electric waves of short wave-length.[1] In that work, as has been the usual practice among experimenters with electromagnetic radiation of wave-length less than a meter, the radiating source was placed at the focus of a parabolic cylindrical mirror. With this arrangement the experiments find their optical analogy in diffraction phenomena of the type commonly called the "Fraunhofer class," where the light used has a plane wave-front. In the experiments about to be described, however, the exciter of electric waves was provided with no converging arrangement, either mirror or lens. Thus a "point source" (or more strictly a line source) was approximated, so that the results have their optical analogue in the "Fresnel class" of optical experiments. This present study differs from the earlier one also in this point; that the receiver also was unprovided with any arrangement for intensifying the effect of the radiation upon it. The usual parabolic mirror would be inadmissible in any study which attempts to compare intensities for points only a few centimeters apart, but in the earlier work a narrow reflecting strip about 1.5 cm. wide was placed behind the exciter at about one quarter wave-length distance, as suggested by Righi.[2] In the present experiments however, as it was desired to keep the conditions as simple as possible, both receiver and exciter were used alone.

The exciter used was of the modified Righi type described in earlier papers.[3] The oscillating system consisted of two cylinders of aluminium, each 2.4 cm. long and .32 cm. in diameter. They were placed with axis vertical and separated by a minute spark gap, whose length could be adjusted by a slow-motion screw pressing upon one limb of the glass frame, which served as a support for the two cylinders and the oil reservoir surrounding the spark gap. The two auxiliary spark gaps in air were from 2 to 3 mm. long. The receiver was of the Klemencic thermal type.

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[1] Science, April 5, 1907 (Report of Meeting of Section B, A. A. A. S.).

[2] Righi, Die Optik der elektrischen Schwingungen, p. 36 (1898).

[3] PHYS. REV., 33, p. 241, 1906.

A thermal receiver was preferred because its indications are proportional to the energy received and it was desired to study the distribution of the energy and not simply locate maxima and minima. The receiver consisted of two brass cylinders, having in place of the spark gap a thermo-junction of fine iron and constantin wire of .02 mm. diameter. These cylinders were smaller and shorter than those of the exciter, but could be lengthened by fitting over them a pair of larger cylinders. These larger cylinders were of the same diameter at those of the exciter and had a hole bored axially in one end, of diameter equal to that of the small cylinders (.21 cm.). This end was also provided with a diametral slit to give springiness and range of adjustment. Special care was taken to insure a close fit and firm pressure of the sliding cylinders upon the smaller fixed cylinders. To adapt the same receiver for use with exciters of different period and to be able to tune to resonance conditions in each case, eleven pairs of sliding cylinders were prepared, varying in length from 5 to 44 mm.[1] In the present work however only the one exciter described was used; its length was 4.8 cm. and the receiver was found by a series of careful experiments to be in resonance with it—as will be shown presently—when its length was 3.9 cm.

Diffraction Experiments.

Exciter and receiver having been brought to the same vibration period by adjustment of the length of the receiver, some experiments were tried with such conditions as would produce diffraction if light radiation were used. Naturally it was found impracticable to imitate optical experiments in the relation existing between wave-length and distances travelled. Satisfactory intensity was obtained only when the total distance of travel was less than a meter, *i. e.*, only a few wave-lengths. Even at such distances the required intensity is not easy to get when no mirror or lens is used with either exciter or receiver unless one uses an interrupter of high frequency. In the present case a Wehnelt interrupter was used, in connection with an induction coil capable of giving a 12 cm. spark.

Fig. 1 shows the result of an experiment to measure the diffraction due to a thin edge of conducting material. Exciter and receiver were placed 60 cm. apart, with the axis of each vertical. Halfway between them was a flat sheet of zinc 47 cm. high by 37 wide, the center of one vertical edge being on the line *ER*. (The metal screens are not shown full size in the figures.) The receiver was mounted at *R* on a revolving arm which could swing horizontally about a center immediately below the refracting edge. The arm was swung about, step by step, through an angle of

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[1] Phys. Rev., 20, p. 269, 1905.

55 degrees, from 15 degrees behind the screen edge to 40 degrees on the other side of the axial line. Readings were taken at each 5°, four in each position. Five times at approximately equal intervals during the readings the intensity was read with screen removed and receiver in the central position. There was a constant and gradual diminution of intensity during the series due to the gradual lengthening of the spark gap. The rate of this diminution is shown by the slope of the dotted line in the figure. To compare the readings for the several positions more fairly, the mean for each position was corrected to the value corresponding to the mean intensity for the series. From the figure we observe that along the line of the edge the intensity is reduced to one fifth that obtained when the screen is removed. It falls off gradually behind the edge, reaching value one tenth at about 8° and thereafter changing but little. On the other side of the axis it rises, more rapidly than it fell away on the protected side, and with increasing rapidity until at 27° from the center the same intensity is reached as with the screen away. Beyond this point the increase is less rapid, but the effect is everywhere greater than with the plate away (so that the word screen becomes a misnomer). A similar experiment was tried with the distance *SR* increased to 50 cm. (making *ER* 80 cm.), but no significant difference appeared with the made to changed distance. If with the greater distance the receiver had been travel along the tangent at *R* instead of along the arc, it might have been possible to get diffraction bands as in the corresponding optical experiment, for then two half period elements of the wave front would become exposed when *R* was removed about 50 cm. to one side of the axial line and the conditions for a minimum reached.

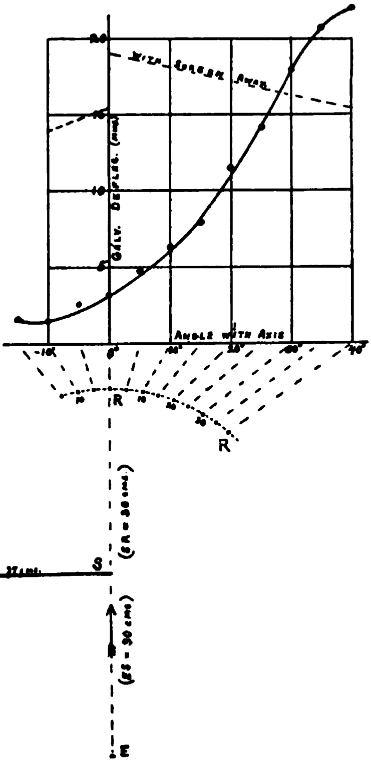


Fig. 1.

In Fig. 2 the results of an experiment are shown where two such plates were used, with their inner edges 20 cm. apart. In this case the radiation passed through an opening of 20 cm., or about 1.5 wave-lengths in width. The total distance of the receiver from the exciter was 80 cm., and from the plane of the plates was 50 cm. Readings were taken for positions 5° apart through an angle of 50° from the axis. The curve shows the energy distribution found. It is seen that the effect of the plates was to increase the energy on the axial line by more than 50 per cent. above that shown when the plates were removed. As R is swung out to one side

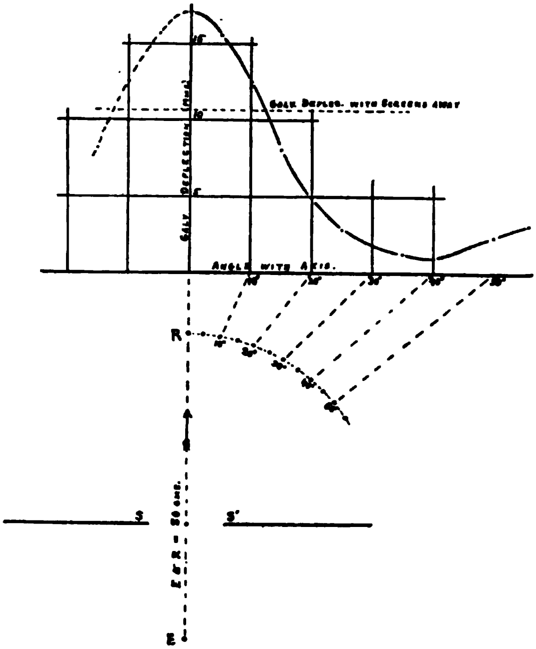
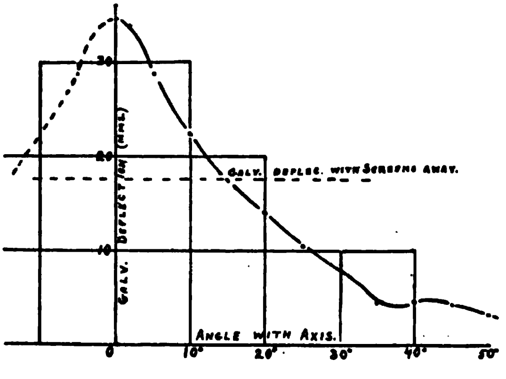


Fig. 2.



Opening between Screens 25 cms.

Fig. 3.

the energy received decreases, slowly at first and then more rapidly. At 12° the same intensity is indicated as with the plates removed (shown on the curve by the horizontal dotted line). At 30° the geometrical shadow of the nearer plate is entered and the intensity is now about one sixth of that amount. At about 40° a minimum appears with intensity about 5 per cent. of that along the axis. (The readings here however were small and somewhat irregular and another comparison of the energies at 40° and 55° showed them nearly equal.)

A similar series was taken with a 25 cm. opening between the plates. The result is shown in Fig. 3. In this case the energy along the axis is twice that found when the plates are removed. The rate of falling off is somewhat more rapid than with the 20 cm. opening.

A clearer indication of maxima and minima ("fringes") is shown in the experiment which furnished the data for Fig. 4. This however was an experiment of the Fraunhofer class, since the exciter was placed at the focal point of a concave parabolic mirror (of 7.5 cm. focal length and 35 cm. width of aperture). A narrow reflecting strip of sheet metal was placed behind the receiver. Thus larger readings were secured. The distance between plates was also slightly different (17.6 cm. instead of 20). A minimum appears in the curve at about 37° and a second weak maximum shows beyond.

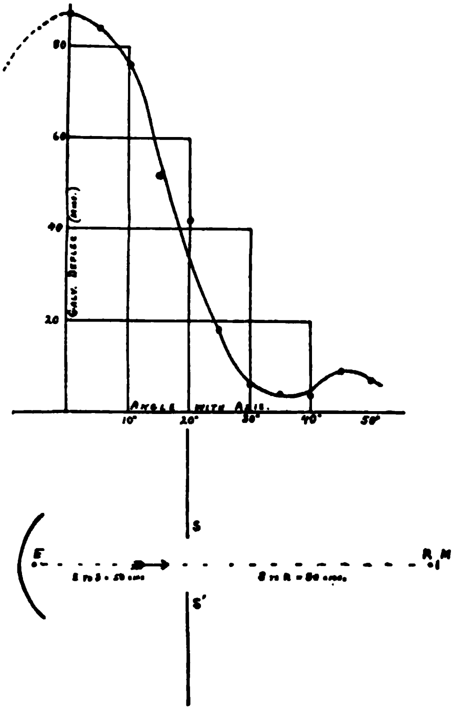


Fig. 4.

In Fig. 5 is shown the effect when the plates are located with reference to the exciter as shown in Fig. 2, but the receiver is placed successively at seven different points on the axis. Curve *A* shows the change in intensity as the receiver is moved from a position 40 cm. from the exciter by 10 cm. steps until it was 100 cm. away. Curve *B* shows the result of the same changes, when the plates were taken away. In every position of the receiver it is seen that the presence of the plates increases the amount of energy received. This increase is most marked when the receiver is 50 cm. distant where the ratio of ordinates of the two curves is greater than 2 : 1. The difference is not so great at 60 and 70 cm.

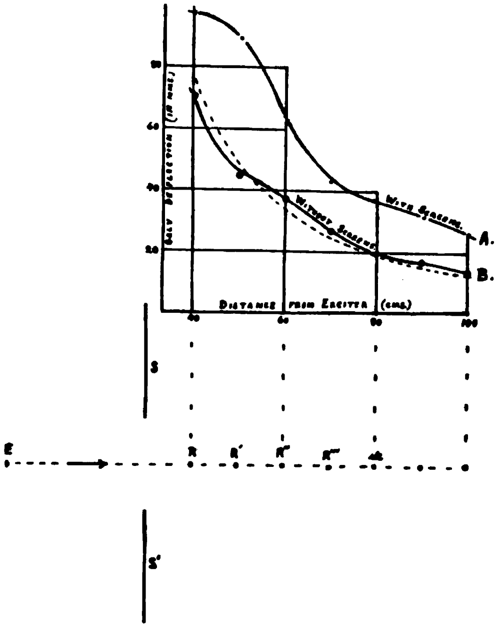


Fig. 5.

distance, but increases again beyond. Curve *B* shows that without the screens the decrease of energy with increase of distance approximately follows the inverse square law. The dotted curve is drawn so that its ordinates are inversely as the squares of the distances between exciter and receiver and this curve approximately coincides with curve *B*.

In another experiment the distance between exciter and receiver was kept fixed at 80 cm. but the plates were shifted along by steps of 20 cm., taking in succession the seven positions indicated in the lower part of Fig. 6. They were in each case 25 cm. apart, with the radiation axis half way between them. As before the effect of the plates is everywhere to increase the energy received at *R*, as shown in the upper part of Fig. 6, where the ordinates show the relative amounts of energy received for the several positions of the screens. The increase is most marked when the plates are 20 cm. from the exciter and is almost zero when the exciter is in the plane of the plates. For other positions the increase lies between 30 and 60 per cent.

Fig. 7. shows the results obtained when the positions of exciter and receiver and the plane of the plates are kept unchanged throughout the series, but the width of the opening between the plates is changed by short steps from 0 to 24 cm. The data for the full-line curve were obtained with the exciter and receiver 60 cm. apart and with the plates half way between them as shown in the lower part of the figure. The data for the dotted curve were obtained with *R* 20 cm. more distant, *i. e.*, 50 cm. beyond the plane of the plates. The form of the curve is about the same in each case. In both we notice that the radiation received at *R* is very small when the distance between the plates is less than one fourth the wave-length (3 cm.). Then it rises rapidly and nearly in proportion to the width of the opening up to about 1.5 λ (18 cm.). Then the increase is slower up to 24 cm. In the experiment where the total distance is 60 cm., an effect equal to that which we have with the plates removed is secured when the opening is about one wave-length in width. The largest effect is about 60 per cent. greater than this.

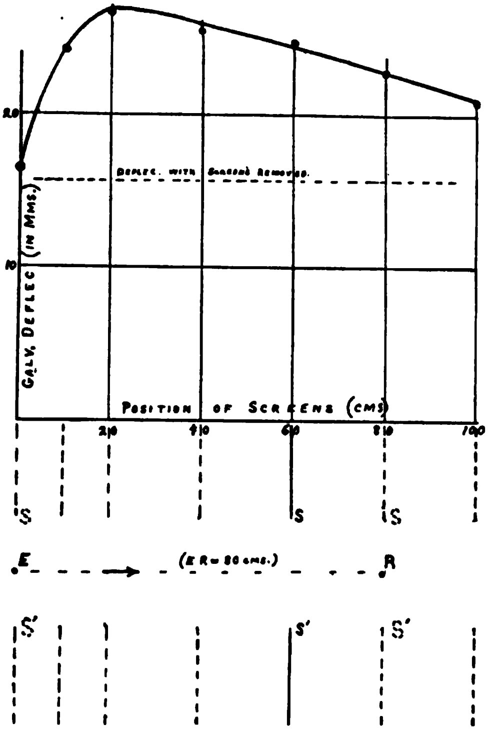


Fig. 6.

A few words about the relative dimensions of exciter and receiver and the wave-length used in these experiments. Early in the course of this work a paper was published by Dr. J. E. Ives[1] in which a different ratio of length of exciter to receiver was found for resonance than that which the author had named in a paper in Phys. Rev., 20, p. 268. In the earlier experiments the adjustment of length of the receiver was secured by sliding slitted tubes of thin sheet copper over the copper cylinders of the receiver. Dr. Ives's results renewed a suspicion that these thin tubes did not fit as tightly as they ought and perhaps were too light in metal also. Therefore a new receiver was made provided with the set of heavier and better-fitting cylinders described early in this paper. These were adjusted as described in the earlier paper. The results are shown in the six curves of Fig. 8. These all show a decided and rather sharp maximum at about 40 mm.—perhaps the best mean value is 39 mm. The exciter length was 48 mm., and the ratio of lengths is therefore 1.23, which lies between the value previously found and that of Ives, although it is considerably nearer to Ives's value (1.43 for cylinders of 2.5 mm. diam.).

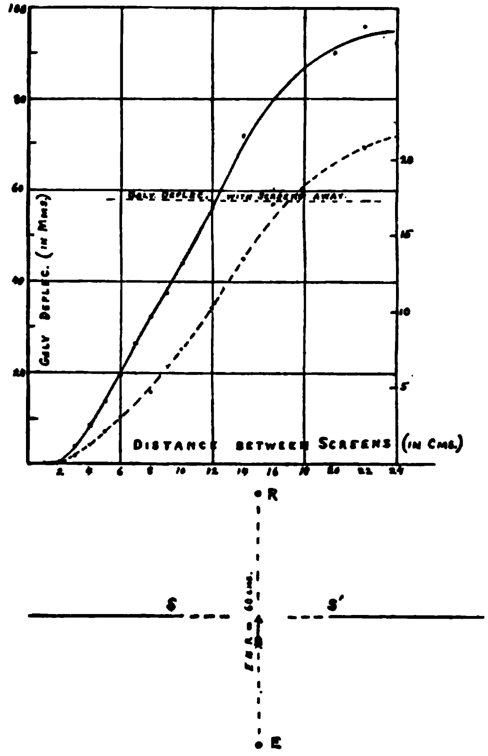


Fig. 7.

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[1] Phys. Rev., 30, pp. 199–221, 1910.

I have not certainly located the cause of error in the early experiment, but it is very likely in the imperfect fit of the sliding cylinders. In the present work the good agreement of the several experiments was obtained in spite of the fact that the conditions were purposely changed in the series shown by the different curves and one other not shown. Thus in one case both exciter and receiver were in concave mirrors, in another neither of them, in the others the exciter alone had a mirror; the distance between exciter and receiver was in one case 30 cm., in another 50 and in others 75; the length of the spark gap was also varied in different experiments.

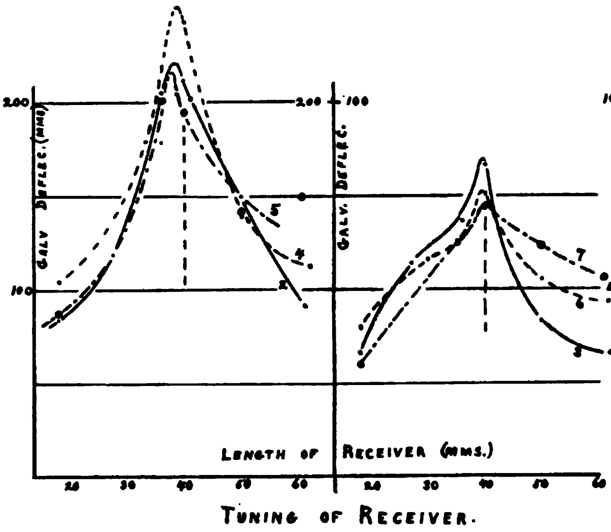

Fig. 8.

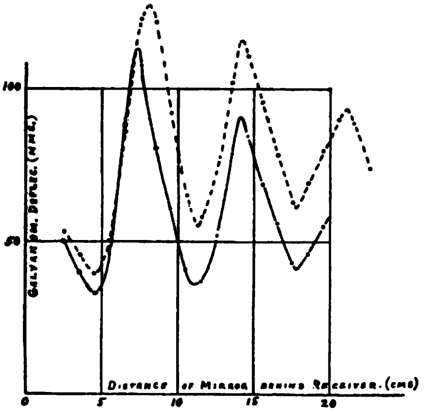

Fig. 9.

Exciter and receiver having been brought into resonance, the wave-length was determined from the interference curve secured by shifting the distance of a large plane sheet of zinc mounted behind the receiver. The data for two such series are shown in Fig. 9. In general each point on one of these curves is the mean of five readings. Thus the dotted curve shown involves over a hundred galvanometer deflections. The two curves shown were taken more than a week apart. The wave-length figured from the dotted curve is 13.1 cm. The curves seem satisfactory so far as regularity is concerned, also in the ratio of maxima to minima ordinates and in their agreement with each other. Many other curves were taken indicating the same wave-length. This wave-length is however larger than has been found for apparatus of about these dimensions by Ives and others using the interferometer method. If there is anything radically wrong in the method here used it would be desirable to know it, as it is essentially that used by Hertz in his classical interference-by-reflection experiments. It has been shown by the author[1] also that results by this method agree with those given by the Boltzmann two-mirror method and by the method of reflection from a single mirror behind the exciter.

EXPERIMENTS WITH INTERPOSED RODS.

Some measurements were made to find the magnitude of the screening effect of various conductors placed between exciter and receiver. First a thin brass rod was used, 40 cm. long and .32 cm. in diameter. It was placed in a vertical position on the line of exciter and receiver, which were 80 cm. apart. For each position along the line a series of readings was taken as follows: First three or four readings with no rod, then an equal number with the rod in position, again several readings with the rod removed, and so on until ten or twelve readings of each kind were obtained. The screening effect found was surprisingly large for any position, but differed considerably for different distances from the exciter as the following tabulation shows. The first column shows the distance of the rod from the receiver and the second the per cent. of radiation intercepted.

Rod from *R*, cm.	Intercepted by Rod, Per Cent.
2.4	77
5.	60
7.5	44
10.	37
15.	26
20.	27
25.	24
40.	32

One typical set of readings is shown, each of the eight positions tabulated having a similar set.

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[1] Phys. Rev., 20, p. 271, 1905, and 23, p. 244, 1906.

	Rod Away.		Rod 10 cm. from *R*.
			14.
			14.2
	22.2	←	14.5
	22.0		
	25.0	→	15.0
			15.1
	22.8	←	14.2
	22.9		
	23.7	→	14.2
			15.6
			14.0
	22.0	←	14.4
	21.5		
	23.0		
	25.0		
Means....	23.0		14.5

$\frac{14.5}{23} =$ 63 per cent. passing,

∴ 37 per cent. intercepted.

In the above experiments great care was taken to have the thin rod exactly on the line connecting exciter and receiver. But inasmuch as the diameter of the rod was only about one fortieth of a wave-length, it was difficult to think of so large a screening as a shadow or as a diffraction effect. The effect of moving the rod to one side was therefore tried. It was placed on the central line and half-way between *E* and *R*. Then it was moved 2 cm. to one side and a series of readings taken. So also at 4 and 6 cm to one side. In all three positions a large screening effect was found and nearly the same for each, about 25 per cent. Plainly this is no diffraction effect. Similar experiments with a thin wire are later described.

EFFECTS OF LATERAL SHIFTING.

The thin rod was now replaced by a narrow strip of sheet metal, 3 cm. wide (and 34 cm. high). Its screening effect was measured on the central line, at the quarter, half and three quarter points (reckoning from *R* to *E*). It proved to be 49 per cent., 52 per cent. and 54 per cent. at the three points. The effect of moving this strip also to one side of the central position was tried, and with a surprising result. When 5 cm. from the central position, it cut down the effect at *R* more than when directly in line. The sidewise motion was continued, a series of readings being taken for each 5 cm. shift. The result is shown graphically in curve *a* of Fig. 10. At 15 cm. to one side it actually exerted a helpful influence, for the radiation received at *R* was greater than when the strip was removed.

Next a wider strip was tried in the same way. It was 7.5 cm. wide. The result is shown in curve *b* of Fig. 10. The effect is evidently of the same kind but greater. The maximum and minimum occur at the same points but they are more pronounced.

But the greatest surprise came when the last experiment was repeated with the thin edge turned toward the exciter. Almost the same curve as before was obtained, except that in the central position the obstructing power of the piece of metal was distinctly *greater* when it stood edgewise than when its broad side was presented. The results are shown graphically by the points surrounded by small squares in Fig. 10. It is seen that they all lie very close to the curve *b* except the one representing the deflection in the central position, whose height is scarcely two thirds of the ordinate of the curve at that point. The same experiment was then tried with the .32 cm. rod (used in the first test along the axis) The result appears in curve *c* of Fig. 10. It is of the same general character as the others, but the maximum and minimum are less pronounced.

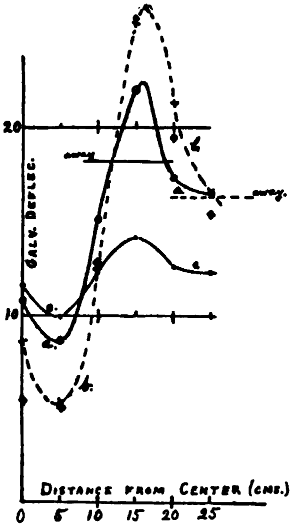

Fig. 10.

These phenomena seem hard to explain from the standpoint of the optical analogy, or if regarded as diffraction effects. A leading physicist suggested that the obstructing effect was proportional to the amount of metal present in the obstruction. This seems unlikely however, as any absorbed energy would doubtless take the form of alternating currents of a frequency so enormous that they would appear as "ultra-skin-currents" (as they have been aptly termed). Experimental evidence that it is not a question of mass of metal was obtained by some measurements on the obstructing effect of a vertical wire having a diameter of .72 mm., much smaller than the thin rod before used. It was found to absorb 19 per cent. of the energy when placed at the half way point and more than 40 per cent. when 5 cm. in front of the receiver. The absorbing power of the 3.2 mm. rod was 32 per cent. and 60 per cent. in the same positions, less than twice as much, while its mass was about twenty times that of the thin wire. That the result is not determined by the mass of metal is also seen by comparing curves *c* and *a*. The characteristic effect is much more marked in curve *a*, due to the thin strip, than it is in curve *c* due to the rod; yet the mass of the strip is smaller than that of an equal length of the rod in the ratio of 14.5 to 23.

The obstructing effect of the thin wire was so great when placed parallel to the electric force that it was tried in the horizontal position (perpendicular to the electric force), but no absorbing power could then be detected. A series of ten readings with wire horizontal at center—readings taken in groups as before described—gave a mean value of 14.8, while the companion series with the wire removed showed 14.7 mm. galvanometer deflection.

To explain such strong absorption effects optical analogies fail us, and it seems reasonable to refer them to the effects of radiation coming from oscillations induced in the obstructing conductors differing in phase something like a half period from that of the direct radiation. This has been called by Righi secondary radiation.[1] In the reference given he describes an experiment where another resonator similar in dimensions to the one used as receiver was placed at various distances to one side of the receiver and its influence upon the latter noted. This influence was found to vary with the distance, helping in some positions and opposing in others. At distance $= \lambda/2$ a maximum was obtained. This was tested in our apparatus by using a vertical strip of sheet metal whose length and width were equal to the length and diameter of the receiver. When this was placed 6.5 cm. ($= \lambda/2$) to one side of R on a line perpendicular to the axis, it was found to increase the energy received about 12 per cent. Trying non-resonant vertical conductors the long rod of .32 cm. diameter had no measurable influence, but a flat strip 7.5 cm. wide (with its width parallel to the radiation axis) gave a well-defined set of maxima and minima, but with distances between adjacent maxima and minima somewhat less than a half wave-length.

These later experiments and certain quantitative deviations of the diffraction phenomena from what is to be expected from optical analogy seem to find a possible explanation in the influence of secondary radiation from conductors in the vicinity. This suggested a more thorough and systematic study of the influence of neighboring conductors, which has since been made. A description of these experiments, a preliminary account of which has been presented to the Physical Society, will be given in a later paper.

SUMMARY.

1. A study was made of the distribution of radiant energy with several arrangements of electric wave apparatus likely to furnish diffraction effects. Among them were the following cases: radiation passing a thin edge of opaque material, radiation passing through openings of several widths, tested (*a*) for lateral distribution and (*b*) for variation along the

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[1] Righi, Die Optik der elektrischen Schwingungen, Chap. II., 1898.

axis. The effect of the screens was to increase the energy received at certain points and some cases analogous to diffraction bands are described.

2. The effect of shifting a pair of "screens" along from one position to another along the radiation axis is described.

3. The effect of a gradual change in the width of an opening through which radiation passed was studied for two points on the axis. Very little energy passed when the opening was less than a quarter wave-length; with openings greater than one and a half wave-lengths more energy appeared beyond than when no screens at all were used.

4. Some resonance and interference results are shown by curves.

5. Large absorption or obstruction effects were found when either a thick or a thin wire was placed at any point along the radiation axis, if its length were parallel to the electric force. A flat strip reduced the intensity even more when placed edgewise than when broadside to the radiation.

6. When such a wire or strip was moved laterally by short steps from a central position strong maxima and minima were shown at the receiver.

7. One of Righi's "secondary radiation" experiments was repeated with the somewhat different apparatus used in this study and his result verified. The suggestion is made that many of the effects observed in these experiments can be explained by the interference of such secondary radiation with that directly received.

Ohio State University,
August, 1912.

THE VELOCITY OF ELECTRONS IN THE PHOTO-ELECTRIC EFFECT, AS A FUNCTION OF THE WAVE LENGTHS OF THE LIGHT.

By David W. Cornelius.

THE object of this investigation is a determination of the potential acquired by alkali metals when illuminated by light of different wave-lengths, and the determination of the velocity of the electrons as a function of the frequency of the incident light. Besides the direct interest of these measurements, the importance of this investigation lies in its connection with the general theory of radiation of the black body as developed by Planck by means of the calculus of probability and thermodynamics. In Planck's theory, which has been well confirmed by the experiments on the radiation of the black body, which also gives good values for the elementary quantities of nature, the elementary oscillator emits the radiant energy not continuously, but intermittently in definite units such that the elementary unit of radiation energy is proportional to the frequency of the light. The equation is

$$E = hn,$$

where E is the energy, n is the frequency and h is a constant of proportionality. On this theory we should expect the positive potential of the photo-electric effect to increase proportionally to n. Some of the first determinations of the velocities of electrons emitted from alkali metals under the action of light of different wave-lengths have been made by Jakob Kunz,[1] who found that the potentials are nearly proportional to the square of the frequencies of the incident light, and that the velocities are almost independent of the temperature and of the intensity of the light. Two other recent papers make further measurements important. J. R. Wright[2] finds that there is a decided maximum in the curve representing potential and frequency in the case of zinc; and R. Pohl and P. Pringsheim[3] find that there are two different photo-electric effects in the alkali metals, the ordinary and the selective effect. They measured, however, the photo-electric currents, and not the equilibrium potential acquired by the metals. It is therefore important to ascertain whether there are two different effects to be found in this equilibrium potential, whether there is a maximum, or whether the potential increases continuously throughout the range of the visible light. Finally the influence of the temperature on the equilibrium potential has to be determined.

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[1] J. Kunz, Phys. Rev., Vol. 29, 3, 1909; Vol. 31, 5, 1910.

[2] J. R. Wright, Phys. Rev., Vol. 33, 1, 1911.

[3] R. Pohl and P. Pringsheim, Verhandlungen, d. D. Phys. Gesellschaft, 12, p. 215, 1910; and R. Pohl, Verhandlungen, d. D. Phys. Gesellschaft, 11, 715, 1909; 13, 961, 1911.

DESCRIPTION OF APPARATUS AND METHOD.

The arrangement of the essential parts of the apparatus used in this research is shown in Figs. 1 and 2. The source of light is a carbon arc *L*.

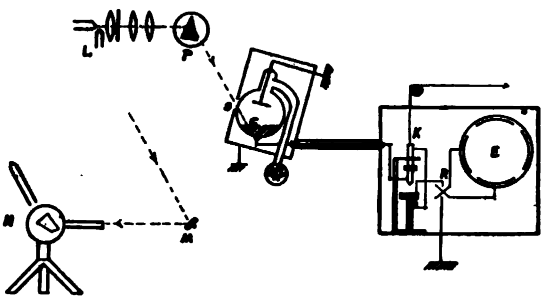

Fig. 1.

The light passes through a slit and system of lenses which gives a beam of parallel light upon the prism *P*, which is capable of rotation, thus providing for an intense source of light, which after passing through the prism, falls upon the slit *S* of a light-tight box containing the photo-electric cell. By rotating the prism *P* any desired wave-length can be made to pass through the slit *S* and fall upon the photo-electric metal *C*. The illuminated electrode *C* of the cell is connected through a key *K* and commutator *R* to a pair of quadrants of a Dolazalek electrometer *E*. Static charges were avoided by having the cell, connections, keys and measuring instrument, which were manipulated from a distance, all inside of earthed conductors. The insulators were made of sulphur and amber plugs. The light which passes through the slit *S* falls upon the metal in the photo-electric cell. The mirror *m* is mounted so that it can be rotated about the axis *ab*. The light of the same wave-length as is incident upon the metal is therefore incident upon the mirror. The light is reflected from the mirror into a direct reading spectrometer *H*, made by Hilger, which was calibrated by means of the sodium lines. This proved to be a very satisfactory arrangement, since the readings of wave-length could be made as the rotating prism was rotated into a

Fig. 2.

desired position. The suspension of the electrometer was a fine quartz fiber which was made conducting by $CaCl_2$ for a portion of the work, and a fine phosphor bronze wire for the remainder. The deflection of the electrometer was read by the image of an incandescent light focused on a millimeter scale at a distance of about five meters from the mirror of the electrometer needle. The electrometer was calibrated by means of a standard Weston cell, resistances and storage battery in the usual way. The calibration of the electrometer is practically a straight line. That is, the deflections were proportional to voltage. The sensibility was .0011 volt per mm. deflection. The electrometer was calibrated several times with practically the same results. The maximum potential acquired by the metal, for incident light of any given wave-length, was determined by the deflection of the electrometer. Various voltages were used on the needle ranging from 40 to 140, depending upon the sensibility desired.

A number of photo-electric cells were constructed, containing different metals. The designs of the cells varied considerably. The determination of the velocity of electrons emitted from the surface of a photo-electric metal in a vacuum tube, when acted upon by light, was attempted by different methods. The stream of negatively charged electrons will be deflected if it passes through a magnetic field. The velocity of the electrons can be calculated by the ratio e/m, where e is the charge and m the mass of the electron, along with H, the magnetic field and the deflection of the stream of electrons. The magnetic deflection method was tried in several tubes. The advantage of this method is that it gives an independent determination of the velocity of the electrons, so that it would be desirable to be able to use it. After repeated trials the magnetic deflection method was given up, since it was found impossible to measure the photo-electric current. The method of measuring the equilibrium potential by means of an electrometer is subject to certain objections which could not be made against the deflection method. Any conductivity along the glass wall or through the vapor of the alkali metal diminishes the final value of the equilibrium potential. Moreover the potential measured by the electrometer is due to the equilibrium potential and partly to contact electro-motive forces or potential differences of any other nature. All those influences could at once be avoided by the magnetic deflection method.

Several cells, which are not shown in a diagram, were unsuccessful. Two rubidium cells showed some interesting features. Exposed to light they showed in the beginning considerable sensitiveness, which decayed rapidly until they did not respond even to intense light. After being left in the dark for a short time, they would sometimes recover. The photo-electric metal was melted several times, but apparently was not effective in changing the behavior of the cells. The same behavior was observed in a potassium cell. It was put in sunlight for several hours, but this did not result in the cell becoming consistently active. This phenomenon might be called a fatigue, the cause and character of it are still uncertain. Thus we see that this work is attended with many difficulties. There are a number of sources of error. A small leak in the tube, a slight impurity of the photo-electric metal, and its surface conditions, gas developed in the tube due to wax joints or occluded gas in the metals in the tube, static charges on the glass of the tube, imperfect insulation between the electrodes of the cell, enter as factors, rendering this experimental work difficult of execution. In several experiments it has been observed that the photo-electric metal in the dark chamber acquired spontaneously a negative potential, which developed very slowly and arose to quite considerable values. This effect has since been studied by J. W. Woodrow. In those cells which gave the best and most consistent results this negative effect has not been observed. If it existed it must have been so slow that it could not effect the readings to any measurable amount.

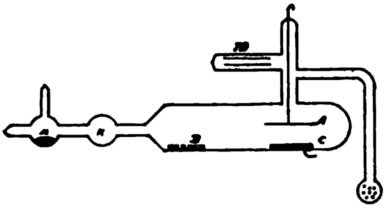

Fig. 3.

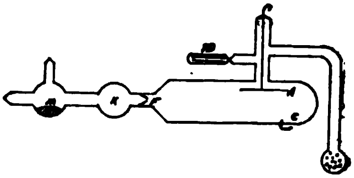

Fig. 4.

The general type of cell which has proved to be the most satisfactory in this investigation is shown in Figs. 3 and 4. The metals used were potassium, potassium "fixed" with hydrogen, cæsium, and cæsium "fixed" with hydrogen. There were only slight variations of the design of the cell to suit the introduction of the different metals, or conditions desired for the cell.

Cæsium was prepared for introduction into the cell, by first drying cæsium chloride by melting it in contact with dry hydrochloric acid gas. Fourteen grams of dry cæsium chloride were mixed with 2.5 grams calcium, placed in an iron boat in a combustion tube of Bohemian glass. When the temperature rises the reaction beween the Ca and the CsCl, or RbCl, becomes quite violent and the calcium and the salts spread out and condense together with the rubidium, or cæsium, in the cooler parts of the combustion tube. To prevent this mixture, a plug of asbestos and iron wire keep the iron boat in position and allow the rubidium, or cæsium, vapor alone to pass through the plug. The combustion tube is connected by means of sealing wax and glass tubes to the photo-electric cell. After the tube was exhausted the combustion tube was heated gradually until the metal distilled and ran through the connecting tubes into the cell. Rubidium and cæsium were both prepared in the same manner. Potassium was introduced into the bulb *M*, of Figs. 3 and 4, distilled into *K* and poured into *C*. The cells were all exhausted by a Gaede pump. The vacuum in each case was tested by the characteristics of the discharge, from an induction coil between two electrodes in the system. A flame was kept under the charcoal bulb for several hours until the charcoal ceased to develop gas and the pump was able to exhaust the system to a stage of hard Roentgen rays. If the metal introduced was to be left in the pure state without hydrogen, the tube was sealed off from the pump as soon as the metal was distilled and transferred into the final position. However, if the metal was to be "fixed" with the hydrogen, the cell was not sealed off from the pump until later. The hydrogen was introduced by means of palladium, Pd, of Figs. 3 and 4. This metal was used as a cathode in a solution of three parts water. and one part H_2SO_4 When the electric current passes through this cell the Pd absorbs a large amount of hydrogen which is given off again by gently heating the dry metal with a bunsen flame, in Pd of Figs. 3 and 4. When the cell contains a small amount of hydrogen and a discharge passes from the alkali metal to the anode the surface of the alkali metal assumes very intense colors, due to the combination of the metal with hydrogen or else to a colloidal transformation of the metal. This process we may call fixing or forming. In this forming the potassium assumes intense blue or purple colors, while the cæsium exhibits a greenish gray or bronze surface.

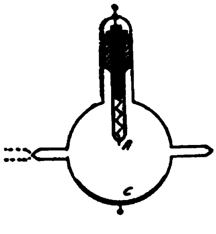

Fig. 5.

A cæsium cell No. 10 without charcoal bulb is shown in Fig. 5. Cæsium is distilled into the bulb at *C* when the electrode *A* is raised, by means of a magnet, in order that no cæsium vapor condense upon the electrode. The diameter of the bulb is about 5 cm., and the distance between *A* and *C* about 2 cm. The data for this cell are given in Table I.

The curve of Table I. is quite similar to that obtained by Jakob Kunz[1] under similar circumstances. The cells which he used did not have any charcoal bulbs attached. Thus it will be noted that this cell which does not have charcoal bulb attached verifies his work. The variations of this curve from a smooth curve is in a measure due to residual pressure in the cell. This error is reduced almost entirely where charcoal and liquid air are used.

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[1] J. Kunz, Phys. Rev., Vol. 29, 3, 1909.

Table I.

Cæsium Cell No. 10. *t* = 23° *C.*

Wave-length. λ $\mu\mu$	Frequency². $\frac{N^2}{10^{30}}$	Volts. v	Wave-length. λ $\mu\mu$	Frequency². $\frac{N^2}{10^{30}}$	Volts. v.
420	.510	1.168	570	.278	0.606
450	.444	1.090	600	.250	0.517
480	.391	1.010	630	.227	0.387
510	.346	0.920	660	.207	0.343
540	.309	0.748	690	.189	0.325

Volts on needle of electrometer 126. K_{126} = .001202.

Cæsium cell No. 12 was similar to Fig. 3. The metal was distilled upon an iron plate at *B* of Fig. 3 and moved by means of a magnet to a final position *C*. *A* is an aluminium plate 1 × 3 cm. The distance between electrodes was about 4 mm. The tube was 2 cm. in diameter and 18 cm. long. The cæsium was not "fixed" with hydrogen before the cell was sealed from the pump. The cell was first tried with the metal pure, but was found not to be photo-electric. After three days' effort it was decided to "fix" the metal. It was done in the usual manner. However, since the cell had been previously sealed from the pump, the residual hydrogen in the tube was not pumped out after fixing, but left for the charcoal to absorb when immersed in liquid air. The results of this cell are given in Fig. 6. Liquid air was kept on the charcoal bulb continuously during the series of observations. The time elapsing between the readings for curves 1–4 was about 43 hours.

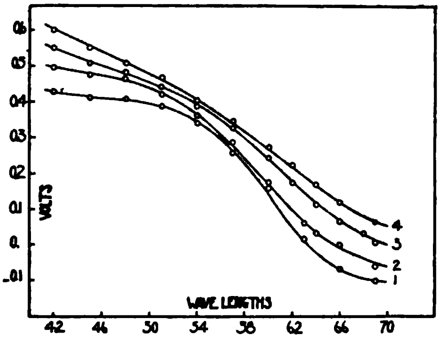

Fig. 6.

The characteristics of the curves change gradually with time. This is due probably to the absorption of the residual hydrogen, by the charcoal. The change seems to indicate that the curves are approaching a form such as would finally make the maximum potential proportional to the square of the frequency. It was impossible to keep liquid air on the cell continuously, and make observations until such a steady state was reached. It is probable, moreover, that the surface of the metal undergoes slight changes so that the sensitiveness either increases or decreases in the course of time.

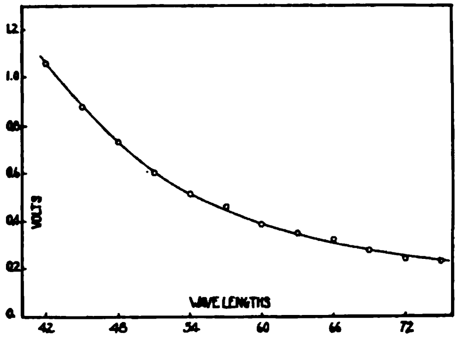

Fig. 7.

Potassium was placed in bulb *M*, Fig. 4, distilled into *K* and poured through funnel *F* into final position *C* in cell No. 4. The surface of the potassium became darkened to a reddish purple color, as the cell was left exposed to the light of the room for six days before the observations, shown in Table II. and curves of Figs. 7 and 8 were taken.

TABLE II.

Cæsium Cell No. 4. t = 23° C.

Wave-length. $\mu\mu$	Frequency^2. $\frac{N^2}{10^{30}}$	Volts. v	Wave-length. $\mu\mu$	Frequency^2. $\frac{N^2}{10^{30}}$	Volts. v
420	.510	1.070	600	.250	0.394
450	.444	0.890	630	.227	0.356
480	.391	0.730	660	.207	0.335
510	.346	0.609	690	.189	0.279
540	.309	0.523	720	.173	0.249
570	.278	0.472	750	.160	0.243

Volts on needle of electrometer 127. $K_{127} = .00150$.

These curves are very significant. The curve of Fig. 7 certainly is not a straight line as Planck's law would require, but Fig. 8 shows a straight line within a reasonable error of observation. There is a slight variation in the points near the red end of the spectrum, but that is probably due to the great difficulty of determining those points accurately. It was found that the maximum equilibrium potential was independent of the intensity of the incident light, within the limits of the

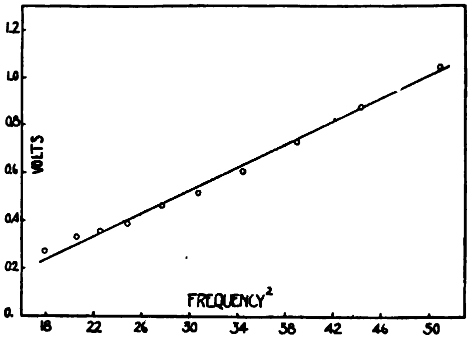

Fig. 8.

full intensity due to the arc, and the intensity of the light due to an incandescent lamp directed upon the cell.

Cell No. 8 was a cell in which the active metal was potassium fixed with hydrogen. It was similar to Fig. 3. The potassium was distilled upon a platinum plate (which had iron attached) at position *B*, Fig. 3, and moved into position *C* by means of a magnet. The electrode distance was about 2 mm. The potassium was fixed in the usual manner, as described before, but was not photo-electric. The movable electrode was moved back to position *B* and the potassium melted. When the electrode was removed back to position *C*, it was photo-electric. Observations were made upon the cell for three consecutive days, which indicated a positive potential of the alkali metal when exposed to light. The potential was erratic however. Nine days later consistent observations were made upon the cell as shown in data of Table III., and Figs. 9 and 10.

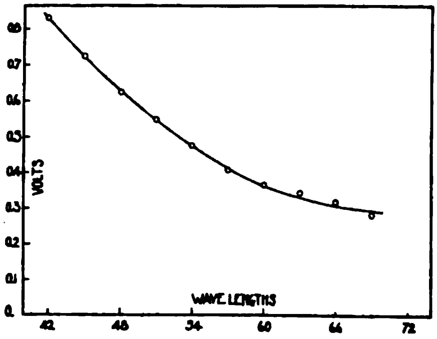

Fig. 9.

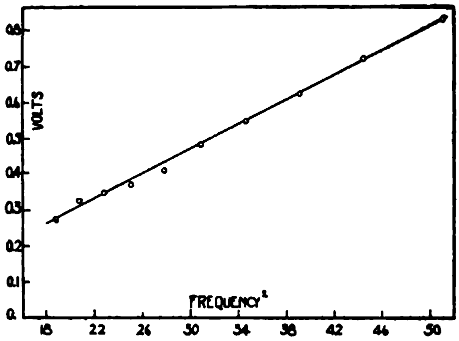

Fig. 10.

Table III.

Cæsium Cell No. 8. $t = 23°$ *C.*

Wave-length. μμ	**Frequency².** $\frac{N^2}{10^{30}}$	**Volts.** v	**Wave-length.** μμ	**Frequency².** N^2	**Volts.** v
420	.510	.835	570	.278	.407
450	.444	.722	600	.250	.361
480	.391	.622	630	.227	.348
510	.346	.537	660	.207	.326
540	.309	.488	690	.189	.266

Volts on needle of electrometer 106. $K_{106} = .00141$.

These plates show the same characteristic properties of the phenomenon. The equilibrium potential is proportional to the square of the frequency. This is always the case when the metal is in the permanent state of activity.

Cæsium was poured into position *C* in cell No. 13, which was of the form shown in Fig. 4. The surface was "fixed" in the usual manner. The color of the surface was a greenish gray and partly bronze. When liquid air was applied to the charcoal bulb, the metal was not active in the beginning. Violet light was incident upon the cell for an hour, or more, before the electrometer indicated a positive deflection of 510 mm. which is equivalent to .062 vol A set of observations was taken, as recorded in curve 1 of Fig. . he maximum potential was reached very quickly after the metal was connected to the electrometer, but was very difficult to read accurately. Liquid air was kept continuously on the bulb and a series of observations was taken about twelve hours later, as recorded in curve 3, Fig. 11. About nine hours later another series was taken as shown in curve 3 of the same figure. The cell was left undisturbed for four days (due to a lack of liquid air). After this time another series of observations was taken upon the cell as given in Table IV. and curve 1 of Figs. 12 and 13. (The time required for the completion of a series is about two hours.) Then following this series another was immediately taken, Table IV. and curve 2, Figs. 12 and 13. The cell was placed in an ice bath and another set of observations for 0° C. was taken which is recorded in Table IV. and curve 3 of Figs. 12 and 13. Two

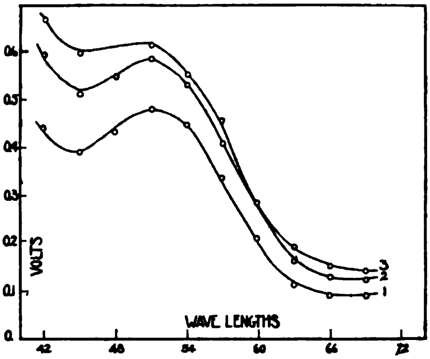

Fig. 11.

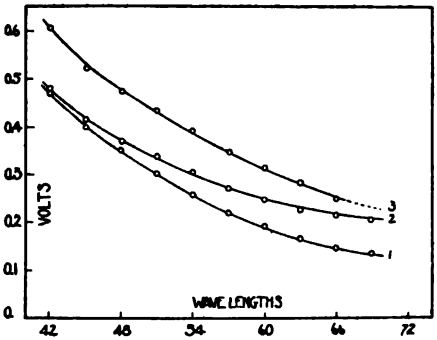

Fig. 12.

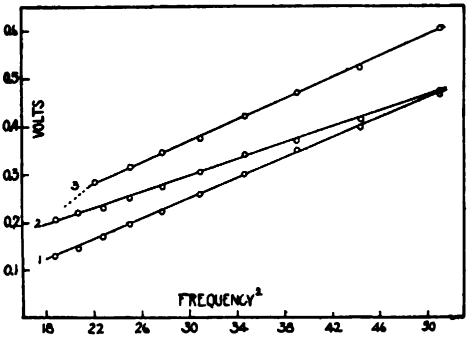

Fig. 13.

days later sets of data were taken for variation in temperature as shown in Table V.

Table IV.

Cæsium Cell 13. 4 *days later than Fig.* 11.
λ = wave-length. N^2 = frequency². v = volts.

λ	N^2	Time, 3 P. M. Tem., 23° C. v	Time, 6 P. M. Tem., 23° C. v	Time, 10 P. M. Tem., 0° C. v
420	.510	.468	.414	.604
450	.444	.392	.373	.520
480	.391	.355	.368	.470
510	.346	.301	.342	.420
540	.309	.259	.305	.382
570	.278	.220	.272	.345
600	.250	.197	.249	.306
630	.227	.168	.228	.288
660	.207	.145	.220	.248
690	.189	.132	.204	.204

Volts on needle of electrometer 127. K_{127} = .00120.

Table V.

Cæsium Cell 13. 2 *days later than Figs.* 12 *and* 13.
λ = wave-length. N^2 = frequency². v = volts.

λ	N^2	Tem., 23° C. v	Tem., 40° C. (?) v	Tem., 0° C. v
420	.510	.475	.600	[1]
450	.444	.405	.528	[1]
480	.391	.378	.486	.483
510	.346	.348	.450	.455
540	.309	.318	.414	.420
570	.278	.288	.367	.374
600	.250	.258	.342	.348
630	.227	.242	.294	.298
660	.207	.228	.248	.256
690	.189	.205	.212	.214

Volts on needle of electrometer 127. K_{127} = .00120.

The curves of Fig. 11 indicate a change with time similar to Fig. 6. There seems to be a growth in the sensibility of the cell as well as a smoothing of the curves. The surface of the photo-electric metal seems to undergo a change until it finally reaches a steady state, in which the maximum potential varies as the square of the frequency of the incident light.

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[1] Reading of the electrometer uncertain. Possibly rise of the temperature of the cell, poor insulation, and negative effect are the causes of the difficulty. The cell, in this case, became negative in a very few minutes when in the dark.

The set of curves 1 and 2, in Figs. 12 and 13, shows the cell to have reached a steady state. It is interesting to note the development of the cell. There is some suspicion to believe that the sensibility increases within the length of time (two hours) required to make a set of observations. Curve 2 is read from the red to the violet end of the spectrum. Curve 3 for ice temperature is a little uncertain at the end points, the electrometer being difficult to read in that region. All the errors, imperfect insulation, rise of temperature of the cell, etc., tend to lower the points. There is a little uncertainty, therefore, as to the character of curve 3, since the shifting of a few of the end points may be critical in determining the form of the curve.

GENERAL RESULTS.

Let us consider the significance of this experimental work. We see that Planck's law, which states that the maximum potential acquired by the alkali metal should be proportional to the frequency of the incident light, is not corroborated. But, on the other hand, the maximum equilibrium potential of the metal P varies as the square of the frequency n of the light. This relation is expressed by the equation,

$$P = kn^2 + P_0,$$

where k is a factor of proportionality and P_0 a constant. The theory, as developed by J. Kunz,[1] is expressed in the form $E = kn^2$, where E is the energy of the vibrating Faraday tube in the beam of light, k is a factor of proportionality and n the frequency of the light. If light falls upon a photo-electric metal a part of the energy is reflected and a part absorbed, both being proportional to the square of the frequency. In order that an electron may escape from the metal, its kinetic energy must be sufficient to overcome the attraction of the positive charge left behind. An electron, in passing through the metal, may lose some energy by collision with molecules of the metal before it reaches the surface. Let us call this loss of kinetic energy w. Then the electron leaves the surface of the metal with a kinetic energy $E = kn^2 - w$. The metal acquires a positive charge and a maximum potential of P, when the electrons are escaping, which is in equilibrium with the energy of the escaping electrons. If the charge of an electron is e, then $Pe = akn^2 - w$. We are able to calculate the values for the constant w, from the experimental observations. The straight lines of Figs. 8, 10 and 13, which represent the relation of maximum potential and square of frequency, may be expressed in the form

$$P = sn^2 + w.$$

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[1] J. Kunz, PHYS. REV., Vol. 29, 3, 1909.

If $P = 0$, then $sn^2 = w$. Thus graphically w is the point where the curve crosses the y axis, if $P = y$ and $x = n^2$. Several values have been calculated from the data which are given in the table below. The method of calculation may be made clear by a sample calculation taken from Fig. 13,

$$s = \frac{P_1 - P_2}{n_1^2 - n_2^2},$$

where P_1 is potential in absolute units for a frequency n_1, and P_2 is the potential in absolute units for a frequency n_2. This gives

$$s = \frac{P_1 - P_2}{n_1^2 - n_2^2} \times 300,$$

where P_1 and P_2 are expressed in volts.

$$s = \frac{(.468 - .132)}{300 \times .51 \times 10^{30} - .18 \times 10^{30}} = .34 \times 10^{-32}$$

for the slope of curve 1, Fig. 13. Applying this value in the formula for the case where $\lambda = 420\mu\mu$

$$w = P_1 - sn_1^2 = .468 - (.34 \times 10^{-32} \times .51 \times 10^{30} \times 300)$$
$$= - .052 \text{ volt} = .174 \times 10^{-4} \text{ abs.}$$
$$e \times w = 4.65 \times 10^{-10} \times 174 \times 10^{-4} = - .0807 \cdot 10^{-12} \text{ ergs.}$$

This represents the energy lost by the electron in reaching the surface or the amount of work necessary to drag an electron to the surface of the metal.

Value of Work to Bring Electron to Surface.

Table VI.

Metal in Cell.	Cæsium 23° C.	Cæsium 0° C.	Cæsium 23° C., 48 Hrs. Later.	Pot. Cell No. 1.	Pot Cell No. 8 with *H*.
Values of $S \times 10^{-32}$.	.340	.368	.273	.817	.574
	.344	.376	.273	.827	.575
	.338	.369	.272	.822	.563
	.346	.370	.277	.833	.573
	.340	.369	.270	.847	.575
Mean $S \times 10^{-32}$...	.342	.374	.273	.829	.572
Work in volts.....	−.052	+.037	+.057	− .160	−.041
Work in ergs $\times 10^{-13}$	−.807	+.572	+.888	−2.480	−.667

The velocity of an electron emitted from the surface is given by the equilibrium relation $Pe = \frac{1}{2}mv^2$, where P is the maximum potential, e

the charge of the electron, m its mass and v its initial velocity of emission from the surface. Hence

$$v = \sqrt{2\frac{e}{m}P}$$

gives the formula for the calculation of the velocity where e/m is 1.77×10^7 and P has values as determined by observations.

VALUES OF INITIAL VELOCITY OF ELECTRONS.

TABLE VII.

Cell.	λ in $\mu\mu$.	P Abs. Units.	$V = \sqrt{2\frac{e}{m}P}, \frac{\text{cm}}{\text{sec}}$.
Pot. No. 4..........	420	1.09×10^8	6.21×10^7
Pot. No. 4..........	690	$.266 \times 10^8$	0.97×10^7
Pot. No. 4..........	750	$.233 \times 10^8$	0.91×10^7
Pot. No. 8..........	420	$.835 \times 10^8$	5.45×10^7
Pot. No. 8..........	690	$.266 \times 10^8$	0.97×10^7
Cæsium No. 13 with H	420	$.472 \times 10^8$	4.09×10^7
Cæsium at 23° C.....	690	$.780 \times 10^8$	0.78×10^7
At ice..............	420	$.600 \times 10^8$	4.61×10^7
At 0° C............	690	$.177 \times 10^8$	0.79×10^7

Planck's law as expressed by the equation,

$$E = hn = \tfrac{1}{2}mv^2 + C, \tag{1}$$

which may be written

$$hn = Pe + C,$$

gives the right order of magnitude for the velocity of the electron although it does not express the relation of the velocity of the electrons as a function of the frequency of the incident light. h has a value of 6.548×10^{-27} ergs sec. The velocity is given by the equation (1) $v = \sqrt{2hn/m}$. For light of wave-length $420\mu\mu$, $n = 0.715 \cdot 10^{15}$. The mass of the electron is 8.7×10^{-28} grams. These values applied to the formula give

$$v = \sqrt{\frac{2hn}{m}} = \sqrt{\frac{2 \times 6.55 \times 10^{-27} \times .715}{8.7 \times 10^{-28}}} = 10.3 \times 10^7 \text{ cm. sec.}$$

This compares well with the experimental values. However, the experimental values for the maximum potential acquired by the metal, for light of different frequencies, are different from those calculated by means of the equation of Planck,

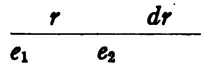
$$hn = Pe + C,$$

where h is 6.5×10^{-27}, e is 4.65×10^{-10} and C is 2.4×10^{-12}. This variation of the experimental results from Planck's law is made clear by a set of calculations for potassium cell No. 8, which is shown in Table VIII.

TABLE VIII.

$\frac{n}{10^{14}}$	Potential V_1 Calculated by Planck's Law, Volts.	Potential Observed V_2.	Difference $V_1 - V_2$ Volts.
.435	.266	.266	
.455	.354	.326	.028
.476	.438	.348	.090
.500	.541	.361	.180
.527	.654	.407	.247
.556	.781	.488	.293
.588	.909	.537	.372
.625	1.063	.622	.441
.666	1.242	.722	.520
.714	1.440	.835	.605

The work done in expelling an electron can be computed. The work done in moving an electron of charge e_1 (which must be equal to e_2 or a multiple of e_2) through a distance dr is



$$dw = \frac{e_1e_2}{r^2}\,dr.$$

The work done in moving the electron to infinity (*i. e.*, beyond the field of attraction of the atom) is:

$$w = e_1e_2\int_R^\infty \frac{dr}{r^2} = -\frac{e_1e_2}{r}\Big|_R^\infty = \frac{e_1e_2}{R},$$

where R is the radius of the atom. Now

$$w = \frac{e_1e_2}{R} = \frac{1}{2}m_2v^2.$$

Hence

$$v = \sqrt{\frac{2e_1e_2}{Rm_2}}.$$

This is the velocity necessary that the electron may escape from the metal.

If an electron revolves about the positive atom it has kinetic energy due to its rotation. The velocity of the electron is determined by the equilibrium of the centrifugal and centripetal forces. If the electron has a mass m_2, a charge e_2, linear velocity v and the positive atom has a mass m_1, charge e_1 (which must equal e_2 or a multiple of e_2) and a radius R; then

$$\frac{1}{2}\frac{m_2 v^2}{R} = \frac{1}{2}\frac{e_1 e_2}{R^2},$$

if the revolution is in a circle, which is the case when m_1 is large as compared to m_2. This gives

$$\text{K.E.} = \tfrac{1}{2} m_2 v^2 = \frac{1}{2}\frac{e_1 e_2}{R}.$$

F. A. Lindemann[1] has used this formula in his determination of the wave-length for which the selective photo-electric current reaches a maximum. Thus the work that has to be communicated to the electron by the incident light is the difference between the total work done in the expulsion of an electron and its energy due to rotation, viz.,

$$w - \text{K.E.} = \frac{1}{2}\frac{e_1 e_2}{R}.$$

The values of the velocity, computed by this deduction, are near the values found by experiment. Suppose that

$$e_1 = \text{charge of the atom} = e_2 = 4.65 \times 10^{-10}$$

$$\frac{e_2}{m_2} = 5.35 \times 10^{17}.$$

R_k = radius of potassium atom = 2.37×10^{-8} cm. used by Lindemann.[2]

$$\therefore v_k = \sqrt{\frac{2 \times 4.65 \times 10^{-10} \times 5.35 \times 10^{17}}{2.37 \times 10^{-8}}} = 4.58 \times 10^7 \frac{\text{cm.}}{\text{sec.}}$$

= velocity of the electron from potassium atom.

The atomic volumes vary as the cubes of the atomic radii, so we have

$$R_{cs}^3 = R_k^3 \frac{a_{cs}}{a_k},$$

which determines the value of R_{cs} when R_k is known. $R_{cs} = 2.75 \times 10^{-8}$ cm. Then

$$v_{cs} = \sqrt{\frac{2 \times 4.65 \times 10^{-10} \times 5.35 \times 10^{17}}{2.75 \times 10^{-8}}} = 4.25 \times 10^7 \frac{\text{cm.}}{\text{sec.}}$$

= velocity of electron from cæsium in order that it may escape.

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[1] F. A. Lindemann, Verh. der Deutschen Phys. Gesell., 13, No. 12, 1911.

[2] F. A. Lindemann, Verh. Deutschen Phys. Gesell., 13, No. 12, 1911.

By means of the formula,

$$w_L = \frac{e_1 e_2}{2R},$$

the work necessary to expel an electron from the metal can be computed. Using values as in the above calculation, we get for the work which needs to be supplied by the incident light to expel an electron, in case of potassium,

$$w - \text{K.E.} = \frac{1}{2}\frac{e_1 e_2}{R} = \frac{1}{2}\frac{(4.65 \times 10^{-10})^2}{2.47 \times 10^{-8}} = 4.57 \times 10^{-12} \text{ ergs.}$$

For cæsium,

$$w - \text{K.E.} = \frac{1}{2}\frac{(4.65 \times 10^{-10})^2}{2.75 \times 10^{-8}} = 3.82 \times 10^{-12} \text{ ergs.}$$

The electron theory gives a means of calculating the time required for an electron to be emitted photo-electrically. The differential equation for the condition of resonance of the electron, when acted upon by an electrical wave, due to the vibration of the incident light, is

$$m\frac{d^2x}{dt^2}\frac{dx}{dt} + a^2x\frac{dx}{dt} + \frac{2}{3}\frac{e^2}{C}\left(\frac{d^2x}{dt^2}\right)^2 = E_0 \cos \phi t \frac{dx}{dt}.$$

This may be written

$$\frac{dE}{dt} + \frac{2}{3}\frac{e^2}{C}\left(\frac{d^2x}{dt^2}\right)^2 = E_0 \phi t \frac{dx}{dt}. \tag{1}$$

E is the energy of the vibrating electron, e its charge, C the velocity of light, E_0 the amplitude of the electric force in the light vector, times e, ϕ the frequency $= 2\pi n$, and x the displacement from the position of equilibrium. A particular solution gives $x = h \cos(\phi t - \alpha)$.

$$\frac{dx}{dt} = -h\phi \sin(\phi t - \alpha), \quad \operatorname{tg} \alpha = \frac{2}{3}\frac{e^2}{C}\frac{\phi^3}{a^2 - m\phi^2},$$

$$h = \frac{E_0}{m\sqrt{(\phi_0^2 - \phi^2)^2 + \frac{2}{3}\left(\frac{e^2\phi^3}{Cm}\right)^2}},$$

where ϕ_0 is the frequency of the applied electrical force and ϕ is the frequency of the electron's vibrations. Introducing these values in the equation (1) gives

$$dE + \frac{2e^2}{3C}\left(\frac{d^2x}{dt^2}\right)^2 dt = -\frac{E_0^2\phi \cos \phi t \sin(\phi t - \alpha)dt}{m\sqrt{(\phi_0^2 - \phi^2)^2 + \left(\frac{2}{3}\frac{e^2}{C}\frac{\phi^3}{m}\right)^2}}.$$

If the electron is considered to have a unit charge then the work done in one revolution, in time T, is

$$E_T - E_0 + \frac{2}{3}\frac{e^2}{C}\int_0^T \left(\frac{d^2x}{dt^2}\right)^2 dt = -\frac{E_0^2\phi \int_0^T \cos\phi t \sin(\phi t - \alpha)dt}{m\sqrt{(\phi_0^2 - \phi^2)^2 + \left(\frac{2}{3}\frac{e^2}{mC}\phi^3\right)^2}}.$$

To integrate $\int_0^T \cos\phi t\,(\sin\phi t \cos\alpha - \cos\phi t \sin\alpha)\,dt$, put $\phi t = x$, $\phi dt = dx$, $dt = dx/\phi$. Then the integral becomes $-\sin\alpha\cdot T/2$.

Hence,

$$E_T - E_0 + \frac{2}{3}\frac{e^2}{C}\int_0^T \left(\frac{d^2x}{dt^2}\right)^2 dt = \frac{E_0^2\phi \sin\alpha T}{2m\sqrt{(\phi_0^2 - \phi^2)^2 + \left(\frac{2}{3}\frac{e^2}{mC}\phi^3\right)^2}}.$$

Since $\phi T = 2\pi$,

$$E_T = \frac{E_0^2 \sin\alpha\pi}{m\sqrt{(\phi_0^2 - \phi^2)^2 + \left(\frac{2}{3}\frac{e^2}{mC}\phi^3\right)^2}}$$

is the work done in one revolution or complete vibration of the light vector considering the moving charge as unity. If the electron is in resonance with the light vector, then $\phi_0 = \phi$ and $\alpha = \pi/2$. Hence,

$$E_T = \frac{E_0^2 \sin\alpha\pi}{\frac{2}{3}\frac{e^2}{C}\phi^3} \quad \text{becomes} \quad \frac{E_0^2\pi}{\frac{2}{3}\frac{e^2}{C}\phi^3}.$$

If the charge of the electron be taken as e instead of unity, as in the above consideration, the work dw done by the electric force E_0 is

$$dw = E_0 e dx.$$

Therefore,

$$E = \frac{(E_{0cs}e_{cs})^2\pi}{\frac{2}{3}\frac{e_{em}^2}{C}\phi^3} = \frac{3}{16}\frac{E_0^2\lambda^3}{\pi^2}.$$

The values for $E_0 = 0.003$ E.S.U. (estimated) and $\lambda = 420\mu\mu = 4.2 \times 10^{-5}$ cm., when substituted in the above equation, give

$$E = \frac{3}{16}\frac{(.003)^2(4.2 \times 10^{-5})^3}{(3.1416)^2} = 1.3 \times 10^{-20} \text{ ergs.}$$

This is the work supplied to the electron by one vibration of the light vector. It was found by means of the previous deduction that the work necessary to liberate an electron is 4.2×10^{-12} ergs. Thus we get the number of vibrations necessary to liberate an electron which is

$$\frac{4.2 \times 10^{-12}}{1.3 \times 10^{-20}} = 3 \times 10^{8}.$$

Therefore, the length of time to liberate an electron is that required for 3×10^{8} vibrations of the light vector. For $\lambda = 420\mu\mu$, the time is

$$t = \frac{\lambda}{C} = \frac{4.2 \times 10^{-5}}{3 \times 10^{10}} = 1.4 \times 10^{-15} \text{ sec.}$$

Hence the time for 3×10^{8} vibrations is $3.10^{8} \times 1.4 \times 10^{-15} = 4.2 \times 10^{-7}$ second.

Thus the time required for the liberation of an electron from a molecule by the incident light is very small. A conception of this period of time, however, can be obtained in terms of the distance, traversed by light in this interval, which is $3 \times 10^{10} \times 4 \times 10^{-7} = 120$ meters.

Summary.

The principal results of this investigation are as follows:

1. A small amount of residual gas in the photo-electric cell influences its behavior.

2. The surface conditions of the alkali metal has a very large influence upon the photo-electric effect; which may be constant from the beginning or increase or decrease in the course of time.

3. It takes, as a rule, some time for a cell to reach a steady state of sensibility.

4. The attempts to measure the velocities of the electrons by means of the magnetic deflection have been unsuccessful.

5. The results obtained indicate that the equilibrium potential depends to some extent upon the temperature of the metal, above zero degrees; below zero degrees the equilibrium potential seems to be independent of the temperature. Further experiments upon this point are necessary.

6. The relation of the velocity of the electrons and the frequency of the incident light is the same for: pure potassium, potassium "fixed" with hydrogen, cæsium and cæsium "fixed" with hydrogen. And the relation between the equilibrium potential and the frequency of the incident light is the more constant the nearer the metal approaches the permanent state of sensitiveness.

7. The theoretical and calculated values of the initial velocity of the electrons are both of the order of 10^{7} cm. per second.

8. The theoretical value of the time required for the expulsion of an

electron due to the resonance effect of the incident light is of the order of 10^{-7} second.

9. The equilibrium potential of the electrons in the photo-electric effect varies directly as the square of the frequency of the incident light. Planck's law, according to which the units of the electromagnetic energy are proportional to the frequency is not confirmed by the results of this investigation.

10. While the theory of resonance shows that a beam of light may supply a sufficient amount of energy for the electron to escape it cannot satisfactorily account for the essential fact that the velocity of the electrons escaping from the metal is proportional to the frequency.

The author takes great pleasure in acknowledging his indebtedness to Professor A. P. Carman for the facilities for this investigation and to Professor Jakob Kunz, both for his general supervision of the work and for many valuable suggestions.

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UNIVERSITY OF ILLINOIS.
May 6, 1912.

ON THE COMPARATIVE ABSORPTION OF γ AND X RAYS.

By S. J. Allen and E. J. Lorentz.

Introduction.

In the April, 1912, number of this magazine one of the authors published a number of results on the absorption of γ rays by matter. These results were confined mainly to the hard γ rays after they had passed through different thickness of matter. The results showed: that the γ rays are not homogeneous; that the absorption per unit of mass is a function of the atomic weight, increasing in a fairly uniform manner with the heavier atoms; that the hardening of the rays by passing through matter depended on the atomic weight, and was large for the heavy atoms. In the case of compound substances the absorption depends on the kind and relative number of atoms present, and is independent of the chemical grouping.

The present paper is a continuation, and deals with the soft γ rays, and X rays of various degrees of hardness. An attempt is made to get very hard X rays so as to bridge over the gap between them and the softest γ rays and to compare their absorptive characteristics side by side.

While not as successful as was hoped the authors obtained X rays whose coefficients of absorption were only about four times those for the softest γ rays. Aluminum was used as the standard of comparison.

The work of Benoist has shown that the absorption of X rays is some function of the atomic weight, increasing in general with heavy atoms, but showing many anomalous results. A number of atoms, notably that of silver, exhibit quite different characteristics for hard and soft rays.

The very important results of Barkla and others on the absorption of X rays have definitely proved the existence of true secondary X rays, which he has termed the fluorescent or characteristic X rays. These secondary rays are produced by the absorption of the primary rays only when the hardness of the primary is greater than that which the particular substance under consideration is capable of producing. These secondary rays are homogeneous, and their hardness increases with the atomic weight of the bombarded substance. They are not dependent upon the quantity, but only upon the quality, of the exciting primary radiation.

Besides this true secondary radiation from the bombarded substance there are a large number of corpuscles liberated, and also a certain amount of scattered primary rays.

No true secondary radiation of the same nature as this characteristic X radiation has been noticed for the γ rays, and the anomalous results for X rays is also lacking in any marked degree.

The evidence at present, to decide whether the ionization in a closed vessel exposed to X rays is caused in any part by the direct influence of the primary rays or wholly by the secondary effect of the corpuscles, does not seem conclusive.

EXPERIMENTAL ARRANGEMENTS.

The apparatus used in these experiments was quite simple and can be quickly described. That used for the γ rays is shown sketched in Fig. 1. It consists of a gold leaf electroscope A of brass sides and top, mounted above and directly over the pole pieces N and S of a powerful electro-magnet. The electroscope was surrounded on the sides by a thick cylinder of cast iron B. The base was closed by a thin sheet of iron. The electroscope was operated and read in the usual manner.

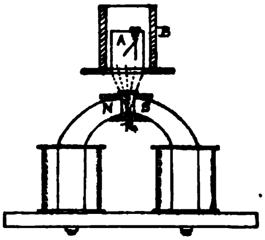

Fig. 1.

The radium *Ra* covered by only the thin glass walls of the containing tube was placed at the base of the pole pieces and halfway between them at the top of the pole pieces was placed the absorbing layer.

The stream of γ and β rays, which left the radium and passed upward between the pole pieces towards the electroscope, would by the time it reached it consist almost wholly of γ rays, since the β rays would be completely turned aside by the powerful magnetic field.

The iron shield around the electroscope protected to a considerable degree the interior from the magnetic field, so that the ionization would not be complicated by the curved trajectories of the electrons set free by the γ rays from the walls of the electrocsope.

The emergent electron radiation from the various absorbing layers would also be turned aside to a great extent by the magnetic field.

The sides of the pole pieces and all neighboring metallic masses were covered with cardboard to cut down as far as possible the secondary radiations.

By these arrangements the ionization inside the electroscope could be taken as a measure of the intensity of the γ rays.

The stream of γ rays reaching the electroscope would thus be quite pure and as soft as it was possible to obtain from radium in radioactive equilibrium.

Experiments were first made to determine how strong a magnetic field was necessary to completely eliminate the electron radiations. After a strength of field of 4,500 C.G.S. units was reached it was found that further increase of field did not materially reduce the ionization, and all experiments were made at this strength.

The distance between the radium and electroscope was about 15 cm. and the absorbing layer was placed about half way.

In the case of the X rays several arrangements were used. For the soft rays, where the tube could be run in a steady manner, the gold leaf electroscope, and also the radium balanced electrometer were used, and fairly consistent results obtained.

For the medium and very hard rays these methods were no longer of any use, since the variations in the running of the tubes were so abrupt and rapid as to prevent time or "rate of leak" readings being taken with any consistency.

In order to eliminate as far as possible the effect of this rapid variation in the tube a balance method was used.

The principle of this can be readily seen by an examination of Fig. 2. A rectangular brass box was divided into two compartments *A* and *B* by an insulated brass plate *a*. The chamber *A* contained an insulated fixed plate *b*, and the chamber *B* an insulated plate *c* which could be moved in and out, changing the volume of *B* at will. The plate *a* was connected to one pair of quadrants of the electrometer, the other pair being earthed. Plate *b* was connected to one pole of a battery and plate *c* to the other, the middle of the battery being earthed.

The top of the brass box was covered with a lead plate of sufficient thickness to absorb all X rays. Two openings, *d* and *e*, were cut in this plate, one opening into chamber *A*, and the other into *B*.

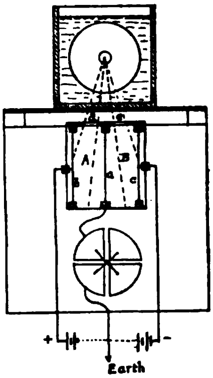

Fig. 2.

The box, electronometer, keys, and all connections, were completely enclosed in a tin box to shield them from all electrostatic influences. The top of this box was covered by a thick lead screen with an opening cut to allow the rays to pass through. Above this was placed a large wooden box, filled with oil, in which the X ray tube was immersed to prevent, punctures, and sparking between the terminals.

The electric potential was furnished by a large static influence machine, capable of giving in dry weather a spark of 30 inches or more. The wire leads from the machine to the tube had to be enclosed in glass tubes to prevent brush and spark discharges.

The X-ray tube was connected by a two-way cock to a Gaede mercury pump, and to an ordinary mercury hand pump. By means of the two pumps a low vacuum could be maintained.

The passage of current through the tube at very high degrees of hardness was found by the authors to be very irregular, and intermittent. In order to start the tube a very high potential was necessary, but after it had started the potential dropped to a much lower value. Sometimes the tube would flicker back and forth very rapidly, the walls instead of having a uniform fluorescence would be covered with a large number of brilliant green spots with dark spaces between. From the experience of the authors it would seem very difficult to get an X-ray tube to run much harder.

The method of making an absorption test was as follows: The tube being started, and the windows *d* and *e* opened, the plate *c* was moved back and forth until the electrometer on being separated from earth indicated a balance. The layer of material to be investigated was then placed over the window *e*, thus disturbing the balance. Various layers of aluminum were then placed over the window *d* until the balance was restored. In this case the rays absorbed by the given substance were equal to that absorbed by the aluminum, and the necessary thickness of aluminum was taken as the "equivalent" or standard. This of course gives only the comparative absorption in terms of the aluminum standard and not the absolute.

By closing the chamber *B* completely, and connecting the quadrants to the radium standard, an absolute measurement of the absorption by aluminum could be made, provided the tube would run steady enough. For the very hard rays this was not found to be practical, and so only the comparative absorptions could be obtained.

RESULTS FOR SOFT γ RAYS.

In Table I. are expressed the results for soft γ rays, column one giving the substance, column two the amount of rays transmitted, column three the value of the coefficient of absorption, calculated in the ordinary manner, divided by the density, and column four the relative absorption in terms of aluminum.

By an examination of this table it will be seen that the results are the same as already obtained for the hard rays, but very much more marked. Elements below aluminum in atomic weight have practically the same absorption per unit mass, but above, the divergence increases rapidly with the atomic weight.

The decrease of λ/d with the absorption for the light atoms is not very marked, but with increasing atomic weight it becomes quite large. It would seem from these results that the γ rays are not homogeneous, but it is possible that they exist in groups when we are dealing with radium in radioactive equilibrium.

Moseley and Makower in a paper in the Philosophical Magazine for February, 1912, show that there is a soft γ radiation from Ra *B*, for which they calculated in an approximate manner λ for lead to be from 4 to 6 cm. This would give $\lambda/d \times 100$ values from 35 to 53.

The largest value for lead obtained by the authors was about 21, which is of the same order of magnitude. This value of course contains some hard rays which would tend to make it smaller.

In the following table are also shown results for pure liquids and aqueous solutions.

Table I.

Substance.	Transmission in Per Cent.	$\frac{\lambda}{d} \times 100.$	Relative Absorption Al=1.
Carbon	90	5.46	.97
Carbon	85	5.56	.99
Carbon	80	5.61	1.01
Aluminum	90	5.61	1.0
Aluminum	85	5.61	1.0
Aluminum	80	5.55	1.0
	75	5.45	1.0
Sulphur	90	6.02	1.07
Sulphur	85	5.67	1.01
Sulphur	80	5.52	.99
Wrought iron	90	6.76	1.20
Wrought iron	85	6.30	1.12
Wrought iron	80	5.83	1.05
Copper	90	9.86	1.76
Copper	85	8.29	1.48
Copper	80	7.26	1.31
Zinc	90	10.53	1.88
Silver	90	11.1	1.98
Silver	75	8.42	1.54
Tin	90	12.0	2.14
Tin	80	9.71	1.73
Tin	75	8.96	1.64
Mercury	90	17.2	3.07
Mercury	80	14.3	2.58
Mercury	75	12.9	2.37
Lead	90	20.7	3.69
Lead	85	17.9	3.19
Lead	80	15.1	2.72
Bismuth	90	24.9	4.44
Bismuth	85	20.7	3.69
Bismuth	80	17.5	3.15

Pure Liquids.

Water, H_2O	90	5.85	1.04
	85	6.01	1.07
	80	6.19	1.11
Ether, $(C_2H_5)_2O$	90	5.8	1.04
	85	6.0	1.07
	80	6.2	1.11
Amyl alcohol, $C_5H_{11}OH$	90	6.0	1.07
	85	6.1	1.08
	80	6.3	1.14
Benzene, C_6H_6	90	5.8	1.04
	85	6.0	1.07
	80	6.2	1.11
Amyl bromide, $C_7H_{11}Br$	90	6.8	1.21
	85	6.5	1.16
	80	6.1	1.08

Aqueous Solutions.

Substance.	Transmission in Per Cent.	$\frac{\lambda}{d}\times 100$.	Relative Absorption Al=1.
Chloroform, $CHCl_3$	90	6.2	1.11
	85	6.1	1.08
	80	6.0	1.07
Potassium bromide, KBr	90	6.9	1.23
	85	6.8	1.21
	80	6.7	1.20
Potassium chloride, KCl	90	5.7	1.02
	85	5.9	1.05
	80	6.1	1.09
Potassium iodide, KI	90	11.3	2.01
	85	9.7	1.73
	80	8.9	1.60
Cadmium iodide, CdI_2	90	8.7	1.55
	85	8.4	1.49
	80	7.9	1.42
Copper chloride, $CuCl_2$	90	6.4	1.14
	85	6.3	1.12
	80	6.2	1.11

For the liquids containing only C, H, and O, the values of λ/d are the same, thus agreeing with the additive law. It will be noticed that there is an apparent increase in the values of λ/d with increasing absorption. The same is true for pure carbon but to a less extent. Whether this is a true increase, or due to the thick layers necessary in the case of these light liquids to get appreciable absorption, is not clear.

Whenever there is present an atom of greater atomic weight than that of aluminum, it can be distinguished by the increased absorption.

The results for the aqueous solutions agree closely with the additive law.

The results for the pure metals are shown plotted in Fig. 3, the ordinates being the absorption per unit mass, and the abscissæ atomic weights.

The resulting curves are fairly smooth, but not uniform, and indicate undue absorption for elements around Cu and Zn.

All these results for the soft γ rays can be considered as an extension of those for the hard γ rays of the previous paper, and are in general agreement with them.

Results for X Rays.

In Table II. are expressed the results obtained for X rays by the "rate of leak" method, column four giving the absolute absorption per unit of mass, and column five the relative absorption in terms of aluminum. These results extend from a very soft ray to a very hard one.

In order to bring out all the characteristics a very careful examination of the table is necessary, but it can be said in general that the absorption per unit mass increases with the atomic weight, being very marked in the case of soft rays, and less so for the hard ones.

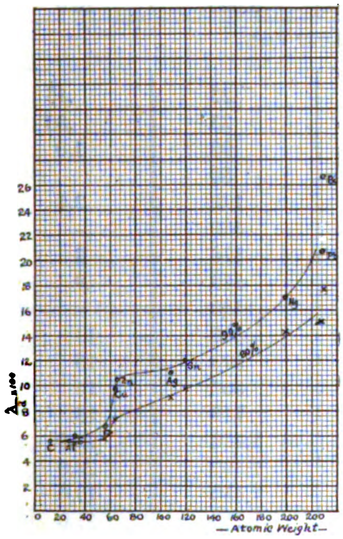

Fig. 3.

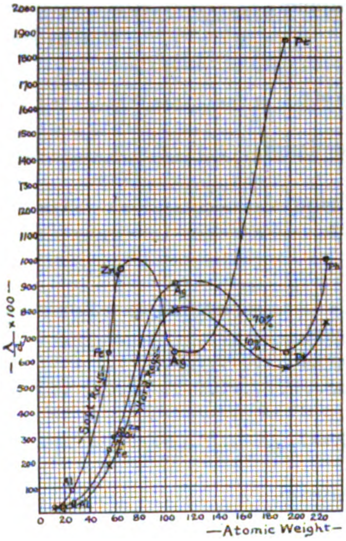

Fig. 4.

There are however exceptions to this, notably in the case of silver. For very soft rays, silver is a much poorer absorber than platinum, but for hard rays it is considerably better. Carbon also shows large variations, for soft rays it is very transparent compared with aluminum, but for hard rays it is much nearer to it.

TABLE II.

Very Soft Tube, about 4,000 Volts Potential Difference.

Substance.	Transmission in Per Cent.	$\frac{\lambda}{d} \times 100.$	Relative Absorption.
Carbon	30	250	.12
Aluminum	16	1,600	1.00
Silver	30	7,600	5.0
Soft Tube, about 13,000 Volts Potential Difference.			
Carbon	70	74	.11
	50	68	.12
	20	59	.14
	10	55	.16
Aluminum	70	670	
	50	560	1.0
	20	410	
	10	340	
Iron	6.7	2,020	6.5
Zinc	4.8	2,910	10.0
Silver	50	3,400	6.0
	20	2,400	6.0
	10	2,100	6.2
Platinum	5	5,600	20.0
Medium Hard Tube, about 35,000 Volts.			
Carbon	42	25	.35
	25	23	.33
Aluminum	59	90	1.0
	17	50	
Sulphur	18	120	2.4
Iron	70	580	–
	49	570	8
	23	540	9
	10	450	–
Nickel	9	520	13
Copper	11.5	590	15
Zinc	42	860	16
	24	710	15
Silver	26	1,430	23.0
	14	1,390	28.0
Platinum	46	1,450	19.0
	26	1,260	21.0
Hard Tube, about 100,000 Volts.			
Carbon	70	24	.60
	50	22	.58
	20	19	.63
	10	19	.68
Aluminum	70	40	
	50	38	1.0
	20	30	

TABLE II.—*Continued.*

Substance.	Transmission in Per Cent.	$\frac{\lambda}{d} \times 100$.	Relative Absorption.	
Aluminum	10	28		
Sulphur	40	67	1.7	
Iron	70	253	6.3	6.5
	50	240	6.3	
	20	203	6.7	
	10	190	6.8	
Nickel	70	300	7.5	8.1
	50	276	7.3	
	20	256	8.5	
	10	256	9.1	
Copper	70	315	8.0	8.2
	50	281	7.4	
	20	258	8.6	
	10	247	8.8	
Zinc	70	333	8.3	8.8
	50	319	8.4	
	20	278	9.2	
	10	264	9.4	
Silver	70	910	22.7	24.3
	50	830	21.8	
	20	800	26.6	
	10	740	26.4	
Platinum	70	630	15.6	17.7
	50	620	16.3	
	20	570	19.0	
	10	560	20.0	
Tinfoil	70	1,000	25.0	2.5
	50	890	23.4	
	20	750	25.0	
	10	740	26.4	

It can be seen clearly that as the rays get harder the difference in the absorption per unit mass for the different elements gets less and less. This fact would be in confirmation with the hypothesis of the identity of γ and X rays.

In Fig. 4 results are plotted for hard and soft rays, and serve to illustrate the very anomalous character of silver.

In Table III. are shown results obtained by the balanced method described above, column four giving the relative absorption per unit mass in terms of aluminum. This relative absorption is calculated from the formula, $t_a d_a / t_x d_x$, where t_x and d_x are the thickness of layer and density of any substance, while t_a is the equivalent thickness of aluminum to produce equal absorption, and d_a the density of aluminum.

It can be seen from the table that the results are practically the same as those by the "rate of leak" method, and wherever the hardness of the ray is the same they are in close agreement.

Table III.

Soft Tube, about 10,000 *Volts.*

Substance.	Thickness in Cms.	Equivalent of Al.	Relative Absorption.	
Carbon	.64	.052	.14	
	1.28	.103	.14	
	1.92	.159	.15	
	2.56	.240	.16	
	3.20	.302	.19	
Sulphur	.635	.920	1.95	
Iron	.0160	.437	9.4	9.3
	.325	.870	9.2	
Nickel	.0495	1.670	10.3	
Zinc	.0145	.436	11.1	12.4
	.0287	.933	12.4	
	.0472	1.730	13.9	
Silver	.0040	.0860	5.52	
	.0120	.3563	7.62	
	.0240	.8640	8.26	
	.0280	1.004	9.57	
Platinum	.0025	.380	18.6	19.6
	.0050	.761	19.1	
	.0075	1.230	21.0	
Tinfoil	.0049	.450	22.9	24.9
	.0098	1.000	25.5	
	.0147	1.550	26.3	
Tube about 25,000 Volts.				
Carbon	.64	.054	.15	
	1.28	.125	.17	
	1.92	.185	.17	
	2.56	.266	.18	
	3.20	.344	.19	
	4.48	.513	.20	
	5.76	.706	.22	
Sulphur	.635	.918	1.77	
Iron	.0160	.361	7.8	7.9
	.0325	.734	7.8	
	.0620	1.440	8.0	
Nickel	.0495	1.650	10.2	
Copper	.0420	1.420	10.3	
Zinc	.0185	.529	10.6	11.0
	.0327	.990	11.2	
	.0472	1.440	11.3	
Silver	.0040	.102	6.6	
	.0120	.412	8.8	
	.0160	.650	10.4	
	.0240	.986	10.5	
	.0280	1.257	11.5	
Platinum	.0025	.390	19.6	19.8
	.0050	.785	19.5	

TABLE III.—*Continued.*

Substance.	Thickness in Cms.	Equivalent of Al.	Relative Absorption.	
Platinum	.0075	1.200	20.0	19.8
	.0100	1.618	20.2	
Tinfoil	.0049	.514	26.1	26.5
	.0098	1.057	26.9	

Tube about 50,000–60,000 *Volts.*

Carbon	1.28	.1296	.18	
	2.56	.318	.22	
	3.84	.540	.25	
	5.12	.842	.29	
Sulphur	.635	1.010	2.1	
Iron	.0160	.412	8.9	8.7
	.0325	.827	8.8	
	.0620	1.555	8.7	
	.0945	2.330	8.5	
Nickel	.0495	1.865	11.6	
Copper	.0425	1.511	10.8	
Zinc	.0142	.558	14.5	13.9
	.0287	1.077	13.9	
	.0427	1.534	13.3	
Silver	.0040	.133	8.6	
	.0120	.663	14.2	
	.0160	1.004	16.1	
	.0240	1.860	19.9	
	.0280	2.220	20.4	
Platinum	.0025	.442	22.1	21.2
	.0050	.917	22.9	
	.0075	1.284	21.4	
	.0100	1.661	20.7	
	.0125	1.913	19.1	
Tinfoil	.0049	.560	28.5	28.1
	.0098	1.157	29.6	
	.0147	1.681	28.5	
	.0196	2.040	26.0	
Water	.47	.028	.16	
	1.41	.109	.21	
	2.82	.240	.23	
Alcohol	.94	.042	.15	
	2.82	.153	.18	
Copper chloride	.47	.589	2.35	
	.94	1.134	2.27	
	1.41	1.673	2.48	
Lead nitrate	.235	1.060	8.8	
	.470	1.839	7.7	
Potassium iodide	.141	.987	11.2	
	.376	3.113	13.2	

TABLE III.—*Continued.*

Very Hard Tube, over 100,000 *Volts.*

Substance.	Thickness in Cms.	Equivalent of Al.	Relative Absorption.	
Carbon	1.27	.208	.27	
	2.54	.482	.33	
	3.81	.733	.34	
	5.08	.945	.33	
Sulphur	.625	.668	1.4	
Iron	.0165	.316	6.6	5.2
	.033	.500	5.2	
	.0765	1.116	5.1	
	.1305	1.641	4.4	
	.2175	2.882	4.6	
Nickel	.0495	.896	5.5	5.5
	.0990	1.830	5.6	
	.1485	2.610	5.4	
Copper	.045	1.03	6.9	6.8
	.100	2.46	7.4	
	.145	2.89	6.0	
Zinc	.0142	.372	9.6	8.0
	.0287	.646	8.2	
	.0490	.954	7.1	
	.0777	1.760	8.3	
	.1270	2.420	7.0	
Silver	.0040	.317	20.2	21.1
	.0123	1.116	23.5	
	.0246	2.046	21.4	
	.0369	2.790	19.4	
Platinum	.0025	.415	20.7	18.5
	.0050	.750	18.7	
	.0060	.952	19.5	
	.0075	1.116	18.7	
	.0120	1.860	19.4	
	.0170	2.252	16.5	
	.0195	2.600	16.3	
Tinfoil	.0049	.460	23.4	22.3
	.0098	.930	23.7	
	.0196	1.674	21.3	
	.0248	2.050	20.7	
Lead	.0052	.480	23.2	

A number of the results are shown plotted in Fig. 5, where the ordinates represent the thickness of layer and the abscissæ the equivalent thickness of aluminum. The curves are practically straight over a considerable range, with the exception of silver, whose anomalous behavior is indicated by the curvature, resembling a parabola. In the case of the very hard tube the curve for silver is nearly straight, showing that the ratio of the absorption by silver to that of aluminum is constant. This would indicate that the rays examined were practically homogeneous. The ray before reaching the ionization chambers had to pass through about 6 sheets of ordinary tinfoil which would cut out any soft rays which might be present.

The tube ran in too irregular a manner to get any very accurate measurements by the rate of leak method, but a few approximate ones were obtained which gave for aluminum a value of λ/d = about .22. These hard X rays thus gave a value for aluminum only about four times greater than the γ rays did. For lead the value of λ/d for the X rays calculated from the relative absorption would be about 5.1, which is about twenty-five times the value of λ/d for the γ rays. The hardest X rays here obtained are still considerably softer than the softest γ rays, but it would seem that if the absorption by any material for the X and γ rays ever becomes equal it will be reached by the light atoms long before the heavy ones.

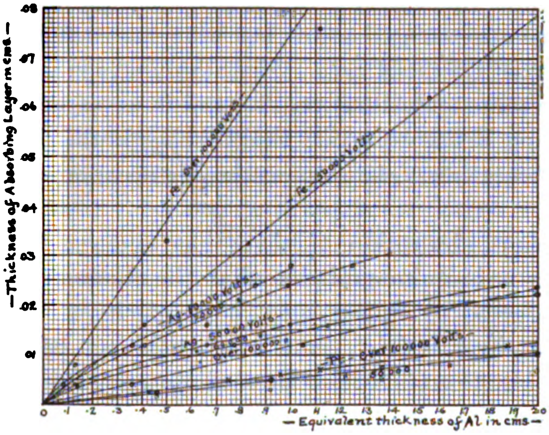

Fig. 5.

It can be seen that even for the hardest rays silver has a very high absorptive power.

Conclusion.

The chief points brought out in this paper can be conveniently summarized as follows:

1. For the soft γ rays λ/d follows the same general characteristics as already found for the hard rays, but increasing with the atomic weight in a more rapid manner.

2. For the X rays, λ/d, examined over a wide range and by different methods, gives results which are in general agreement with those of Benoist. λ/d is an increasing value of the atomic weight in general, but some substances, notably that of silver, show very anomalous results.

The value of λ/d for silver is much less for the soft rays, and much greater for the hard rays, than one would be led to expect.

3. Very hard X rays were obtained whose coefficient of absorption measured by aluminum was only four times greater than that of the γ rays. These rays are nearly homogeneous, and still show a very high absorbing power for silver.

4. None of the very high weight atoms, such as lead, platinum, etc., show any marked anomalous results. The attempt therefore to obtain X rays as hard as the softest γ rays has not been altogether successful, but it has been approached, and it has been shown that the anomalous results still persist for the X rays. Since the γ rays have not been shown to exhibit the same anomalous results, and if we assume that γ and X rays are the same in nature, a very rapid change in the absorptive powers of certain elements must take place as the X rays approach closely to the γ rays.

The anomalous absorption of silver has not yet been completely explained, but it is quite probable that the fluorescent secondary X radiation discovered by Barkla must have much to do with it. This radiation from silver is fairly hard and would not be excited by soft primary rays, but to some extent by medium hard rays, and largely by very hard rays. The fluorescent radiation could only have an effect when the primary beam was of sufficient hardness to excite strongly the radiation characteristic of the element under consideration. The heavy atoms should show no very great anomalous behavior until the very hard primary rays were used.

The latest results on the fluorescent radiation by Chapman[1] show that the high atomic weight elements give off a soft homogeneous radiation similar in character to that which Barkla calls series *L*. There is also, from these atoms, a considerable scattering of the primary, and some hard radiation of series *K*.

In a paper in the September number, 1911, of the Phil. Mag. Barkla gives a good summary of the present knowledge on the fluorescent radiation and its effect upon the absorption curves of any element for X rays. He ascribes all selective or anomalous absorption to the presence of the homogeneous radiation characteristic of the particular element under investigation. In the neighborhood of an absorption band, *i. e.*, at the point where the fluorescent radiation starts for any given element, the absorption, in terms of an element like aluminum having no absorption band, would increase rapidly at first and afterwards decrease. The characteristic form of an absorption curve would be the same for all elements. An absorption band would result from an abnormal absorption of energy from the primary beam due to the sudden transformation into the fluorescent radiation.

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[1] J. C. Chapman, Proc. Roy. Soc., May, 1912.

A confirmation of this view of absorption is seen from the author's results for silver. Barka gives for the value of λ/d for the homogeneous radiation from silver in terms of aluminum, 2.5. From Table II. above we see that for a tube of about 13,000 volts P.D. the value of λ/d for aluminum is from 6.7 to 3.4, and therefore below that necessary (2.5) to start the homogeneous radiation. Under these conditions the relative absorption of silver in terms of aluminum is practically constant, and of low value. In the next tube running at 35,000 volts P.D. λ/d for Al = (.9 − .5), above that required. The relative absorption of silver in terms of aluminum is now much greater. Between these two hardnesses of tube the absorption band for silver occurred.

The results for the other elements experimented upon also show confirmation of this theory, but not to so marked a degree.

The relative absorption for most of the elements increases to a maximum and then decreases with the hardness of the rays, except silver, which over the range experimented upon has not begun to decrease appreciably.

From the results of Benoist and others it would seem that the elements round-about silver exhibit the most marked anomalous characteristics.

If the X and γ rays are the same in nature but only differ in penetrating power, we might expect to find fluorescent radiations for γ rays. If this were true the absorption curves for γ rays would be affected in an analogous manner to X rays.

It is known that the secondary γ rays are of two kinds, high velocity β rays, and penetrating γ rays of the same nature as the primary. These latter rays according to Florance[1] are simply the scattered primary, of the same penetrating power, and not homogeneous.

There is no evidence yet of any homogeneous γ rays characteristic of the various elements.

In this respect then X rays and γ rays differ in character, and since it has been clearly shown that the hardest X rays excite fluorescent radiation, and that silver maintains its anomalous behavior, the γ rays if they are of the same nature as X rays must have lost this power rather suddenly.

The various theories which have been advanced to account for the production of X and γ rays do not seem complete enough at present to account for all the phenomena connected with γ and X rays.

University of Cincinnati,
September, 1912.

[1] Florance, Phil. Mag., 1910.

STUDY OF RESISTANCE OF CARBON CONTACTS.

By A. L. Clark.

WHILE engaged in some experimental work with telephone transmitters, it was found that the resistance of carbon contacts varied to a remarkable extent, not only with pressure and current, but also with the time and in a very regular manner.

The first two effects have been known for a long time, but the regular variation with time after eliminating the effect of change due to a rise of temperature, seems to have escaped notice—at least I have been unable to find any account of such work. To be sure it has been well known for a long time in telephone practice that oxidation and breaking down of contacts occur, but no attempts to show any regularity in such phenomena have been discovered.

The effects of pressure on contact resistance of carbon was investigated very thoroughly by Bidwell who also[1] investigated the effects of current strength on contact resistance. He found that the passage of a current through the contact caused the permanent change in resistance, sometimes decreasing it and sometimes increasing it, depending on the value of the current. In this connection, he found that if the current does not exceed a certain value, the resistance is decreased and more as the current is larger, provided the current does not rise above this value. On the other hand, if the current exceeds a certain value there is a permanent increase in the resistance.

At the time of the investigation described below, the writer did not have access to Bidwell's paper, consequently much of that work was repeated, but the regularity of the phenomena just described seems to be new and worth notice and such matters as are not mentioned in other researches are described below.

This paper is an account of an investigation of the time relations of changes in resistance in carbon contacts such as are commonly used in telephone transmitters. Experiments were made on carbon in the form of grains, balls and plates, always with the same results.

It will be convenient to divide carbon contacts into two classes, loose and tight. By a loose contact its mean one where the particles are held so that the contact is made under very slight pressure. In this case there is no change of resistance due to expansion, which usually causes increased pressure, but the contact may break down, as will be seen below. By tight contact is meant one where the particles are held with sufficient firmness to maintain electrical contact even with large currents. In the first case, that is, loose contact, there are noticeable permanent changes in resistance. In the latter, tight contact, these changes are very slight. Microphonic action is not impossible in the case of tight contact. Indeed, the microphone is usually an example of tight contact in the sense explained above.

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[1] Shelford Bidwell, "Elec. Res. of Carbon Contacts," Proc. Roy. Soc., XXXV., 1, 1883.

Loose contact was studied first by attaching a small carbon ball or a small grain of carbon to a thin phosphor bronze wire with a bit of a liquid cement, containing graphite and known as "clamp paste." This hardens in a few hours, is very strong and conducts well enough. In other experiments, the carbon balls and fragments were partially coated with copper electrolytically and then soldered to the phosphor bronze wire. Any other small wire possessing considerable elasticity would do equally well, but the phosphor bronze wire was at hand and served very nicely. The method of attaching the wire to the fragment seems to have no effect on the result. The wire was about 5 cm. long and was held by its end in a clamp so that the bit of carbon could be moved to touch another rigidly held piece of carbon. The circuit is shown in Fig. 1.

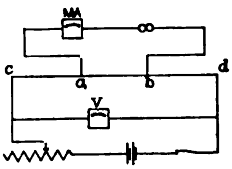

Fig. 1.

A 4-volt storage battery is arranged to send its current through a potentiometer wire, *cd*, and an adjustable resistance *R* in series. A sensitive voltmeter is applied at the ends of the wire, *cd*. The variable contacts *a* and *b* lead the current through a milammeter and the experimental carbon contact in series. The milammeter should be as near dead beat as possible. By moving *a* and *b* any required voltage may be applied and the current observed. It was found that when a small current is sent through the contact that there is a very regular decrease in resistance which if the contact is undisturbed seems to be permanent. The slightest jar, even that made by a wagon passing in the street outside, is sufficient to change the contact in such a way that the phenomenon repeats. Bidwell noticed that the resistance of such a contact is decreased by the current, but did not notice the regularity of the effect. Fig. 2 shows the manner in which the resistance changes. Here carbon balls ¼″ in diameter were used. The time was measured by a metronome beating 40 beats to the minute. If a stronger current is used, the decrease is more noticeable, and if a strong current is followed by a weak one, the weak one has no effect on the resistance. It is not absolutely certain that there is no recovery from this change of resistance, but there is no experimental evidence to show that such a recovery occurs. Since there was no room available which could be made sound and jar proof, this point has not been carefully investigated. The sensitiveness of a loose contact to change in pressure is most extraordinary. When standing near the table on which the apparatus was placed, it was found that shifting the weight from one foot to the other distorted the floor and table enough to change the pressure sufficiently to show a decided effect in the milammeter.

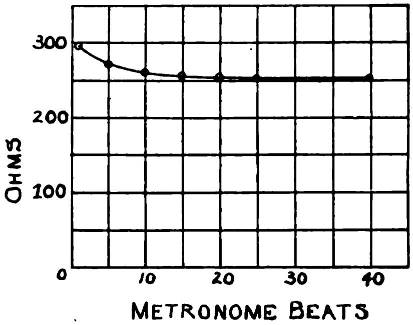

Fig. 2.

Table I. shows the result of several successive runs, beginning with a low E.M.F. and increasing to a value below the critical point. It will be seen that after the first run the resistance changes very little.

TABLE I.

E.M.F. in Volts.	I.	II.	III.	IV.
.4	445	308	296	296
.6	383	289	287	277
.8	254	222	236	236

If, as Bidwell points out, the E.M.F. passes a certain critical value, the phenomenon changes. There the resistance increases with the time and with remarkable regularity. As Bidwell noticed, the critical value depends somewhat on the pressure at contact. It also depends on contact area, so that the critical value probably depends on current density rather than on E.M.F. The higher the applied E.M.F., or better the higher the current density, the more rapid is the rise in resistance. Fig. 3 shows the phenomenon for two balls in loose contact when the applied E.M.F. is 1.6 volts.

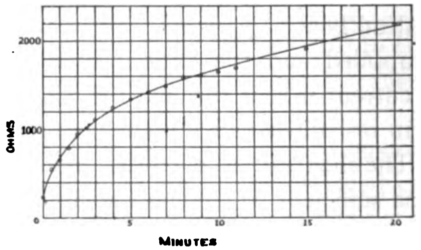

Fig. 3.

As will be seen, the resistance increases rapidly at first, then more slowly and after about two minutes has increased to five-fold the original value. There is no evidence of recovery in this case. If the current be interrupted at any time and the circuit be left open for an hour or two and then closed, the resistance begins with the same value as when the current was interrupted and the curve is absolutely continuous, unless the contact has been jarred. With large E.M.F.'s but still so small that no visible arc forms, the resistance rises rapidly to infinity.

With smaller E.M.F.'s the change is very slow. If the initial value of the E.M.F. is near the critical value, it may happen that when the current is turned on the resistance falls for a moment, then rises. Apparently the fall in resistance gives rise to a current density in excess of the critical value and the second case is exhibited. Occasionally the resistance falls very quickly to a very low value and the contact then shows signs of cohesion, requiring a slight but unmistakable pull to separate the particles. Of course the contact is no longer loose. On the other hand, when the phenomenon of increase of resistance is observed and the particles are observed under a high power microscope, traces of a feathery adhering substance are sometimes seen, suggesting ash. This is not always visible and where seen may be small fragments abraded by rubbing, but the existence of ash is suggested.

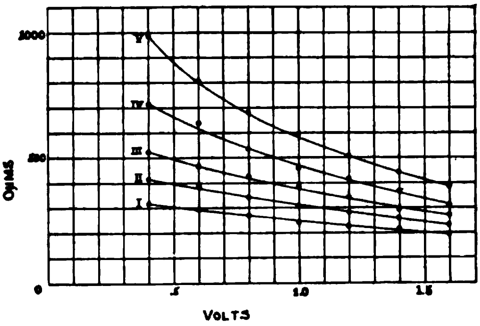

Fig. 4.

Table II. and Fig. 4 show the effect of successive passage of the E.M.F. beyond the critical value. Each such passage gives rise to an increase of resistance. In this experiment, the circuit was closed for as brief a period as possible. The longer the circuit is closed on the high value of the E.M.F. the higher the next curve of the family will rise.

Table II.

E.M.F.	I.	II.	III.	IV.	V.
.4	313	413	523	712	988
.6	290	382	462	635	808
.8	267	342	425	533	684
1.0	243	307	380	463	592
1.2	223	282	340	411	504
1.4	212	258	294	362	445
1.6	186	229	274	309	385

The phenomena observed are totally different when the pressure becomes great enough to break through the nonconducting layer, whatever it is. For example, a ball was held against a vertical disk of carbon with pressure produced by the elasticity of the phosphor bronze wire and the phenomena of loose contact were observed. It was then placed on the same disk held horizontally, the pressure now being caused by the weight of the ball and the contact was tight. When the contact is tight there is no breakdown, even when the current is large enough to cause a bright arc. In another experiment, two thick carbon disks were placed near together in the same vertical plane and a small fragment of carbon dropped between them. Such a contact may be loose or tight, behaving very erratically.

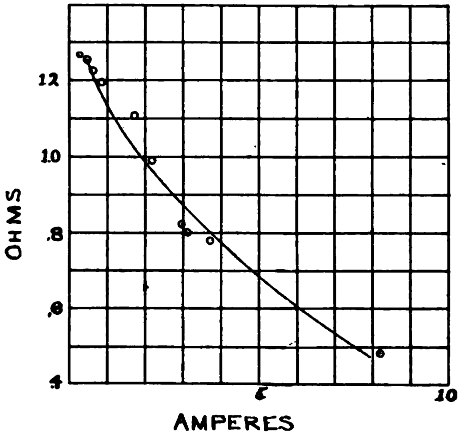

Fig. 5.

Fig. 5 shows the change in resistance due to different currents in the case of tight contact. In this experiment, a small ball was held against a carbon disk with moderate pressure. As will be seen from the readings in Table III., it is doubtful whether there is a decided change in resistance with time. The first column shows time in half minutes. The other columns show the resistances at these times for the applied electromotive forces given at the top of the columns.

TABLE III.

Time.	.06 Volt.	.21 Volt.	.4 Volt.	.56 Volt.	.72 Volt.	.83 Volt.	1.09 Volts.	1.60 Volts.	1.9 Volts.
0	4	3	2.9	3.4	3.2	2.82	2.66	1.54	1.29
½	4.8	3	2.9	3.38	3.18	2.77	2.68	1.55	1.23
1	4.4	3	3.2	3.3	3.25	2.99	2.6	1.54	1.23
1½	4.2	3	3.3	3.28	3.25	2.88	2.6	1.50	1.25
2	4.4	3	3.28	3.33	3.22	3.03	2.55		
2½			3.27	3.34	3.22	2.92			
3			3.26	3.34					

In some cases of tight contact, *e. g.*, a small ball against a disk under slight pressure, it has been found that the resistance decreases with time, but if the current be interrupted by opening the switch for a few minutes and closing it again, the resistance has a much smaller value than when the switch was opened. The following data will make this evident. With an E.M.F. of four volts the resistance fell in four minutes from 158 to 146 ohms. After a rest of two minutes the resistance was 135 ohms and the circuit being closed for three minutes, no further change in resistance was observed. But a rest of two minutes brought the resistance down to 99 ohms, where it remained practically constant for three minutes. Another two-minute rest brought it to 91 ohms. In experiments like this each period of rest brought about a marked decrease in resistance. This phenomenon has been observed a number of times, but no satisfactory explanation has been suggested.

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ANGLE IN VECTOR ALGEBRA; AND COMPOSITION OF ROTATIONS.

By Frederick Slate.

ALTHOUGH angular displacement, velocity and acceleration have their place as axial vectors recognized in vector algebra, the foundations of their acceptance into the scheme are seldom laid in satisfactory fashion, and their relation to the kinematics of a rigid body is not always stated accurately. The ideas here offered have for object to supplement the view of these points that is usually presented, and so render it systematically more complete.

The specification of plane angle as the quotient of circular arc by radius includes no orienting elements explicitly, and is consequently inadequate for the purposes of vector expression. But the orientation can be supplied easily and without affecting the ordinary measure of magnitude. Writing for differential angle

$$d\mathbf{a} = \frac{ds}{r}[\mathbf{r}_1\mathbf{t}_1] \tag{1}$$

the appended vector product determines the plane of the angle-element by means of unit vectors in the radius and the tangent for the element of circular arc (ds), while the tensor assigns proper magnitude (ds/r). For this primary form we can then substitute the convenient equivalent

$$d\mathbf{a} = \frac{1}{r^2}[\mathbf{r}d\mathbf{s}], \tag{2}$$

which clearly remains valid also for any radius vector ($\mathbf{r}$) and any curve-element ($d\mathbf{s}$) at its extremity. This original limitation to differentials becomes necessary in order that the unit vectors may be specified locally; but it disappears effectively when one plane contains the pole and a finite arc. In that simpler case

$$\mathbf{a} = \int_0^{s_1} \frac{1}{r^2}[\mathbf{r}d\mathbf{s}] \tag{3}$$

is evidently to be taken as an algebraic summation of coplanar areal elements representable on a common normal, sign in the individual contributions to the total being rendered consistent with the standard conventions by the sequence chosen for the factors in the vector product. Although the vector factors separately vary from one element to another, their product is in a correct sense common to all the terms summed when eq. (1) or eq. (2) is integrated. The more general conditions, however, are seen to involve the interpretation of the process in eq. (3) as a vector-summation.

Plane angle or angular displacement is thus fully established as an axial vector through this necessary connection with the "area described by the radius vector"; being represented as twice the area of the corresponding sector in a circle of unit radius. And without need of further discussion it is then apparent how plane angles, finite as well as infinitesimal, must conform without exception to the rules governing other axial vectors. And further this entails logically a similar conclusion regarding angular velocity and angular acceleration through their aspect as increments of angle, dimensional differences among vectors of the same type being reconciled, here and elsewhere, by familiar considerations.

Turning now to solid angle, begin by assuming a surface-element (*dS*) on a sphere of radius (*r*); next extend the scope of the symbols to the case where an element of any surface is taken at the extremity of its radius vector; and notice that all accepted details in the measurement of solid angle (*A*) fit naturally into the differential expressions

$$dA = \frac{dS}{r^2}(\mathbf{r}_1\mathbf{T}_1) = \frac{1}{r^3}(\mathbf{r}d\mathbf{S}). \qquad (4)$$

These have been so constructed as to introduce a scalar product; first of unit vectors orienting the radius vector ($\mathbf{r}_1$) and the surface-element ($\mathbf{T}_1$), and secondly of vectors parallel to these, but including also the tensor factors. The operations indicated leave the measure of magnitude undisturbed. And the result being scalar the need of discrimination that attaches to eq. (3) does not enter at the next stage; so that we have in general the scalar sum

$$A = \int_0^{S_1} \frac{1}{r^3}(\mathbf{r}d\mathbf{S}), \qquad (5)$$

which is interpretable immediately as a volume three times that of the spherical sector of unit radius which ordinarily appears in the discussion of solid angle. The correct signs among the elements constituting the algebraic sum follow automatically from the individual cosine-factor which is implied in each scalar product ($\mathbf{r}d\mathbf{S}$).

The application to the kinematics of a rigid solid turns upon the expression of linear displacement at any definite point in terms of the "angular displacement of the solid." There are two current plans of calculation, according as the origin is taken (1) at the center of mass (which may then itself be moving), or (2) at a fixed point in unchanging configuration with the solid. We shall make the development definite by choosing the latter alternative: there is no essential divergence of results, for the purpose here. From the origin (*O*) draw a fixed line (*OA*) about which angular displacement (a_1) is supposed. Then for any selected point (*Q*) the radius vector (r) must be of constant length, but is subject to change (in direction) as a vector; and for differential linear displacement

$$d\mathbf{r} = d\mathbf{s} = [d\mathbf{a}_1\mathbf{r}]. \tag{6}$$

Combination with eq. (2) shows the necessary identity, $d\mathbf{s} = d\mathbf{s}$. Proceeding to finite angular displacement, linear displacement is shown as a vector sum (though the elements of angle are additive algebraically), of obvious geometrical meaning,

$$\int_{r_0}^{r_1} d\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_0 = \int_0^{a_1} [d\mathbf{a}_1\mathbf{r}]. \tag{7}$$

After completing this displacement let further angular displacement occur about any other fixed line (*OB*). Then similarly

$$\mathbf{r}_2 - \mathbf{r}_1 = \int_0^{a_2} [d\mathbf{a}_2\mathbf{r}], \tag{8}$$

and by addition preserving the sequence of integrals,

$$\mathbf{r}_2 - \mathbf{r}_0 = \int_0^{a_1} [d\mathbf{a}_1\mathbf{r}] + \int_0^{a_2} [d\mathbf{a}_2\mathbf{r}]. \tag{9}$$

The second member does not coalesce into the form

$$\int_0^{a} [d\mathbf{a}_1 + d\mathbf{a}_2,\ \mathbf{r}]$$

for the plain reason that the radius vector is not a common factor (as a vector) in pairs of elements that can be segregated one from each integral; and moreover the series of its (vector) values in either integral would not be reproduced if the sequence were reversed. We can detect in these facts (1) the foundation of Rodrigues' Theorem, and (2) the harmony of that theorem with full exhibition of vector qualities in angle, as set forth in what precedes. For note that the necessary and sufficient condition rendering legitimate and equivalent the combinations

$$\mathbf{r}_2 - \mathbf{r}_0 = \int_0^{a} [d\mathbf{a}_1 + d\mathbf{a}_2,\ \mathbf{r}] = \int_0^{a} [d\mathbf{a}_2 + d\mathbf{a}_1,\ \mathbf{r}] = \int_0^{a} [d\mathbf{a}\mathbf{r}] \tag{10}$$

is that the vector denoted by the symbol ($\mathbf{r}$) should be made common (or unique); and this is met by supposing the processes of displacement simultaneous. The limitation to infinitesimal angle is secondary then, as an accompaniment of simultaneousness; and at most marks a progressive temporary (or instantaneous) adjustment within an interval of finite displacement.[1] The remaining steps in adapting the thought of eq. (10) to angular velocities and accelerations are so evident that they need not detain us, except to emphasize the simultaneousness again as dominant, for instance in the "Parallelogram of Angular Velocities," thus: Consider rotation ($\dot{\mathbf{a}}_x$) about the X-axis as occurring *alone*, and producing linear velocity ($\mathbf{v}_1$) at a point given by its distance ($\mathbf{r}_1$) from that axis; and similarly for rotation about Y-axis and Z-axis, each alone. Then

$$\mathbf{v}_1 = [\dot{\mathbf{a}}_x\mathbf{r}_1]; \quad \mathbf{v}_2 = [\dot{\mathbf{a}}_y\mathbf{r}_2]; \quad \mathbf{v}_3 = [\dot{\mathbf{a}}_z\mathbf{r}_3]. \tag{11}$$

It is next required to adjust the *simultaneous* occurrence of the velocities in eq. (11) to producing the actual motion. That is, introducing the radius vector, to satisfy the condition

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = [\dot{\mathbf{a}}_x, \mathbf{r} - \mathbf{x}] + [\dot{\mathbf{a}}_y, \mathbf{r} - \mathbf{y}] + [\dot{\mathbf{a}}_z, \mathbf{r} - \mathbf{z}]. \tag{12}$$

But $[\dot{\mathbf{a}}_x\mathbf{x}] = 0$, etc.; so we may write

$$\mathbf{v} = [\dot{\mathbf{a}}\mathbf{r}]; \quad \text{where } \dot{\mathbf{a}} = \dot{\mathbf{a}}_x + \dot{\mathbf{a}}_y + \dot{\mathbf{a}}_z; \tag{13}$$

and find the accurate interpretation of the scope and meaning of the result in equations (6) to (10).

University of California.

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[1] These details may be justified by Heaviside's remark that "This property is notoriously difficult to understand by Cartesian mathematics"; and we may entertain a doubt whether his vector proof is not over-simplified (El. Mag. Theory, Vol. 1, p. 181). The older books sometimes were content with showing that the resultant axis is a line of zero velocity. Denial of vector quality to angle is a modern confusion (Coffin, Vector Analysis, 2d ed., p. 241).

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SIXTY-FOURTH MEETING.

A REGULAR meeting of the Physical Society was held in the Physical Laboraory of Northwestern University, Evanston, Ills., on Saturday, November 30, 1912.

In the absence of the president and vice-president Mr. Crew was made chairman of the meeting.

The secretary made a report to the meeting regarding the mail ballot held in May, 1912, the report being in substance the same as that presented at the New York meeting on October 12, 1912, and given in the minutes of that meeting.[1]

At the close of the meeting the Society voted to express to the authorities of Northwestern University, to the members of the Department of Physics, and to the University Club, its high appreciation of the hospitality extended to the Society.

The following papers were presented:

Sound Wave Photography. ARTHUR L. FOLEY.

A Method of Producing Known Relative Sound Intensities and a Test of the Rayleigh Disc. G. W. STEWART and HAROLD STILES.

The Effects of Gas Currents on Sound, with Applications to Ventilating Systems of Auditoriums. F. R. WATSON.

The Value of e. R. A. MILLIKAN.

The Cooling Effect Produced by the Emission of Ions from Hot Bodies. H. L. COOKE and O. W. RICHARDSON.

The Evidence that Sodium belongs to a Radioactive Series of Elements. F. C. BROWN.

Ionization of Potassium Vapor by Ultraviolet Light. S. HERBERT ANDERSON.

Photographs of Retrograde Rays. CHAS. T. KNIPP.

Note on the Distribution of Velocities of Photo-electrons from Thin Cathode Films when Illuminated by the Mercury Arc. PAUL H. DIKE and F. C. BROWN.

New and Old Expressions for the Variation of the Mass of the Electron with Increasing Velocity. JAKOB KUNZ.

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[1] PHYSICAL REVIEW, XXXV., p. 397.

The Relation between Current and Time Rate of Change of Electric Force. Max Mason.

The Stability of Residual Magnetism. N. H. Williams.

Summary of a Study of the Electric Furnace Spectrum of Iron. (By title.) Arthur S. King.

The Infra-red Arc Spectrum of Iron in the Region 8000–30000 A.U. H. M. Randall.

Distribution of the Elements in the Solar Atmosphere. Charles E. St. John.

Some Direct Measurements of the Temperature Correction to the Refractive Index of Water around 20° C. F. A. Molby.

The Effect upon the Elastic Properties of Copper Wire Produced by Increase of Temperature and Passage of an Electric Current. H. L. Dodge.

A Method of Production of Light-negative Selenium. F. C. Brown.

A Substitute for a Bronson Resistance. (By title.) D. W. Cornelius and J. G. Kemp.

Air Resistance to Falling Bodies. (By title.) A. A. Somerville.

The Effect of Drawing on the Elasticity of Certain Wires. L. P. Sieg.

On the Cause of the Apparent Difference between Spark and Arc Sources in Imparting Initial Speeds to Photo-electrons. R. A. Millikan.

Note on a Possible Kinematic Explanation of Spectral Series with Constant Frequency Intervals. Ernest Merritt.

Adjourned at 5 P.M.

Ernest Merritt,
Secretary.

Minutes of the Sixty-fifth Meeting.

A JOINT meeting of the Physical Society with Section B of the American Association for the Advancement of Science was held in the physical laboratory of the Case School for Applied Science, Cleveland, Ohio, December 30, 1912, to January 1, 1913. The presiding officers were President W. F. Magie, of the Physical Society, and Vice-President A. G. Webster, of Section B.

The annual business meeting of the Physical Society was held Wednesday afternoon, January 1, 1913.

The chair appointed Messrs. F. R. Watson and F. A. Saunders to act as tellers for the annual election of officers and, the tellers having reported the result of their count of the ballots, the following officers were declared elected for the year 1913:

President: B. O. Peirce.
Vice-President: Ernest Merritt.
Secretary: A. D. Cole.
Treasurer: J. S. Ames.
Members of the Council: H. T. Barnes and W. J. Humphreys.

On behalf of the Council the secretary announced that the next meeting of the society, which would ordinarily be held in New York City on the Saturday nearest March 1, would be held in the new Sloane Physical Laboratory of Yale University at New Haven, the exact date to be announced later.

The secretary also announced that the Council had voted to continue the present arrangement with the publishers of Science Abstracts for the year 1913.

The meeting voted to approve the action of the Council in ruling that papers which contain chiefly matter that has already been published be not accepted for presentation before the Society.

In the absence of Mr. Ames, the treasurer's report for the year ending December 1, 1912, was read by the secretary. The report is printed at the close of these minutes.

The president announced to the society the action of the council at its meeting December 31, 1912, in regard to the proposed transfer of the Physical Review to the Physical Society, and gave an account of the steps which had led to this action. Owing to the indecisive result of the mail ballot of May, 1912, the editors of the Review had on their own account sent out a circular letter to all regular and associate members, a copy of which letter follows:

"*To the Members of the American Physical Society:*

"The question of the advisability of transferring the control of the Physical Review, and the responsibility for its maintenance, to the Physical Society has now been under consideration by the editors of the Review and the Council of the Society for nearly two years. In March, 1911, the editors made a statement to the Council in regards to the terms of such a transfer (page 593 of the Review for June, 1911) and in February, 1912, made a further statement looking toward a transfer of the Review to the Society on January first, 1913. There are reasons why, if the transfer is to be made, it should be made not later than that date.

"It will be recalled that in May, 1912, an effort was made by the Council to ascertain the sentiment of the Society by means of an informal mail ballot, in which all regular members were asked to participate. Of the 201 votes cast, 97 were in favor of the transfer; 87 were in favor of leaving the relations between the Society and the Review unchanged; and 16 were in favor of the establishment by the Society of a new journal. 115 members did not vote.

"Since less than a majority of the regular members (in fact less than one third) expressed themselves as in favor of the change, one of the essential conditions for the transfer has not been met, and technically the vote must be interpreted as indicating that the editors' offer has been declined. The ballot is, however, not convincing; for while it fails to give evidence of such enthusiastic approval of the plan as has been felt by the editors to be essential to its success, it nevertheless indicates a considerable sentiment in favor of the change.

"Through correspondence and discussion with individual members of the Society, the editors of the Review have been led to believe that the failure of the ballot to give convincing expression to the sentiment of the Society may have been due in part to a failure to understand the editors' attitude toward the proposed change. It is possible that our attempt to maintain an impartial attitude in the discussion has been interpreted as indicating that we are opposed to the plan. We are sending the following statement of our views to all members of the Society in the effort to make our position more clear, and in the hope of obtaining an expression of opinion from the members of the Society which we are certain is not based upon a misunderstanding.

"At the time when the transfer of the Review to the Society was first discussed, the expense of publication exceeded the income by a considerable amount; for, although the income was steadily increasing, the amount of material published had increased more rapidly. During the last two years, however, the situation has materially changed. With the effective coöperation of the advisory editors it has been possible, by urging a more compact style of presentation, to secure a considerable reduction in the bulk of material published. This fact, together with the various economies that have been introduced, has resulted in a decrease in the cost of publication. On the other hand the income has shown a very gratifying increase, with the result that during each of the last two years the income has exceeded the expense of publication.

"If the Society should undertake the maintenance of the Review the expense to the Society would be greater than it now appears to be by the amount of the editor's salary. But even taking this into consideration we believe that if the Review is managed in a conservative way, and if the amount of material published is not greatly increased, the transfer may not make necessary either the discontinuance of Science Abstracts or any great increase in the dues. No immediate increase in dues appears to be necessary. It must not be forgotten, however, that if the rapid increase in scientific activity continues, or if the transfer to the Society should result in the publication in the Review of the American material which now goes elsewhere—and this result is certainly to be desired—the expense of publication would be correspondingly increased. It might be that the very success of the change would make some increase in dues necessary. In considering the advisability of making the transfer, this possibility should not be left out of account. If the transfer is made the members should feel that they are taking the Review 'for better, for worse.'

"The conditions under which the editors are willing to make the transfer to the Society have been stated in our earlier communications, these conditions being made for the purpose of safeguarding not only the interests of the journal but also the interests of the members of the Society. The establishment of a scientific journal is a slow process. Even under the most favorable conditions, aside from the expenditure of money and effort, it requires a long time to build up a subscription list, to obtain the patronage of advertisers, and to acquire general scientific recognition. We feel that the Review is now so firmly established that its continued success is assured. On behalf of Cornell University we are offering to the Society a journal with an established position, with a subscription list that compares favorably with that of other similar journals, and with an annual income—in addition to that received from the Physical Society—of over $3,000. We feel that under the control of the Society, the Review can be made still more successful. But we cannot run the risk of losing what has already been gained. The conditions stated in our earlier communications appear to us to be essential; and of these the assured and hearty support of the members is the most important. Unless we are convinced that the members of the Society are prepared to take a vital interest in the success of the Review, and to insist that its policy and management are such as the physicists of the country desire, we feel that the surrender by the present editors of their control and responsibility would be little short of a breach of trust.

"If conducted by the Physical Society with the enthusiastic support of the members, we believe that the Review will be more successful, and more truly representative of American Physics, than it could possibly be under private ownership or when controlled by a single institution as at present. On the other hand, if the support of the members is lukewarm, we feel that lack of interest and divided responsibility might bring disaster upon the Review, and even upon the Society, and that, at the present time at least, the proposed transfer should not be made. The enthusiastic support of the members being assumed, we are heartily in favor of the transfer of the Review to the Society, and look upon this transfer as being to the best interests of both the Society and the Review.

"With this explanation of the situation and of the editors' views, may we ask that you indicate your preference on the enclosed postal card and return the card to us at once?

"If it appears that the transfer of the Review is strongly favored by the members, we intend to ask the advisory editors to assist us in preparing a definite plan for the control and management of the Review as the journal of the Society, and shall present this plan for consideration at the Cleveland meeting. If the members do not feel that the change is desirable we shall proceed to perfect and make more definite the present plan of a representative editorial board to coöperate with the present editors.

"We cannot too strongly urge that you mark and return the enclosed card at once whatever your opinion on the question may be.

"December 11, 1912."

"EDW. L. NICHOLS
ERNEST MERRITT
FREDERICK BEDELL."

The replies to this letter had shown so large a majority in favor of the transfer of the Review to the society that the editors felt that they were justified in approving the transfer and had therefore presented to the council the following proposition:

The Physical Society takes over the control of the Physical Review and assumes full financial responsibility for its maintenance under the terms proposed by the editors in their communications of March, 1911, and February, 1912 (printed in the Review June, 1911, and November, 1912).

The transfer shall be effective as of January 1, 1913, at which date a new series of the Review shall be begun, the January, 1913, issue being designated as "No. 1, Vol. I., Second Series."

The actual transfer of management shall be made as soon thereafter as the society can be incorporated and various necessary details can be arranged. This transfer shall not include the transfer of any tangible property, as office furniture, back volumes, etc., which property shall remain the property of the present management.

The complete control of the Review, including its business and editorial management, shall be vested by the Society in an editorial board elected by the Society.

This board shall consist of a managing editor and nine other members. The term of office shall be three years, and, except in the case of the managing editor, retiring members shall be ineligible for immediate reëlection. The council shall determine the salary of the managing editor and shall appoint a managing editor for the year 1913. The managing editor for 1914–16 and for subsequent terms shall be elected by the society.

The other nine members of the initial board shall be the present nine advisory editors, arranged in three groups as follows:

(*a*) B. O. Peirce, J. S. Ames, J. Zeleny;
(*b*) E. F. Nichols, J. C. McLennan, R. A. Millikan;
(*c*) W. F. Magie, K. E. Guthe, C. A. Skinner.

One group (determined[1] by lot) shall retire at the end of 1913; a second group (determined by lot) shall retire at the end of 1914; the third group shall retire at the end of 1915.

The board shall receive all money paid to the Review for subscriptions, sales, etc., shall receive from the treasurer of the society remittances of such sums as the Society may appropriate, shall make all disbursements for the maintenance of the Review, and shall make a report to the council each year of such receipts and disbursements. The treasurer shall place at the disposal of the board of editors one thousand dollars working capital and the sum of three dollars per year for each member of the Society until the council otherwise directs.

After extended discussion the council had voted to accept the proposition and to adopt the plan proposed by the editors of the Review subject to the approval of the meeting.

It was further announced that the council had voted, in case the action proposed above was approved by the meeting, to appoint F. Bedell managing editor of the Review for the year 1913.

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[1] The Board of Editors subsequently determined by lot that group *a* should retire at the end of 1913, group *c* at the end of 1914, and group *b* at the end of 1915.

It was moved and seconded that the meeting approve the action of the council, and after discussion by Messrs. Foley, Bedell, Carman, A. Zeleny, Guthe, Webster, Millikan, E. F. Nichols, McLennan, G. W. Stewart, Merritt, and others, the motion was adopted with no dissenting votes.

It was moved by Mr. Bedell that the managing editor be required to give bond for such amount as might be determined by the Board of Editors, the cost of such bond to be paid from the money appropriated to the Review, and that auditors be appointed annually by the president to audit the accounts of the managing editor. The motion was carried.

It was voted to place in the Review,—if possible in the first number of the New Series,—a statement in regard to the transfer of the Review to the Society, together with some account of the organization and history of the Review, and an expression of the society's appreciation of the work of the original editors. Mr. A. G. Webster was requested to prepare such a statement.

On motion of Mr. Webster the meeting voted to express to Mr. Merritt its appreciation of his services to the society as secretary during the past thirteen years.

The secretary called attention to the fact that the newly elected secretary, Mr. Cole, would be in Europe for several months and that the duties of the secretary's office could not be taken over by Mr. Cole until his return.

Tuesday afternoon, December 31, 1912, the Physical Society and Section B of the American Association for the Advancement of Science met in joint session with Section A of the Association. The program of the joint session was as follows:

Address of the retiring Vice-President of Section A, MR. E. B. FROST, on "The Spectroscopic Determination of Stellar Velocities."

Address of the retiring Vice-President of Section B, MR. R. A. MILLIKAN, on "Unitary Theories in Physics."

Henri Poincaré as a Mathematic Physicist. A. G. WEBSTER.

Some General Aspects of Modern Geometry. E. J. WILCZYNSKI.

Cosmical Magnetic Fields. L. A. BAUER.

The session of Wednesday morning, January 1, 1913, which was in charge of Section B was devoted to papers of general interest as follows:

Photographing and Analyzing Sound Waves. DAYTON C. MILLER.

The Reaction of the Room on the Source of Sound. WALLACE C. SABINE.

Some Points Concerning Absolute Measurements of Sound. ARTHUR G. WEBSTER.

In addition the following papers were presented during the meeting:

Notes on Problems Involved in Producing Three-color Transparencies. CHARLES D. HODGMAN.

Thermal Capacity of Carbon and Tungsten at Glowing Temperatures. A. G. WORTHING.

Resistance of Oxides at High Temperatures. A. A. SOMERVILLE.

A Micro-Pyrometer. GEORGE K. BURGESS.

Trial of a Method of Obtaining Equality of Temperature, at High Temperatures. C. F. Lorenz.

A Critique of Captain Kater's paper of 1818 on The Reversible Pendulum. John C. Shedd and James A. Birchby.

Some Diffraction Phenomena. C. F. Brush.

Effect of a Constriction in the Path of a Luminous Discharge. R. F. Earhart.

Arc and Spark Phenomena. E. S. Johonnott.

The Effect of Magnetization on Thermal Conductivity. N. F. Smith.

On the Origin of the Earth's Magnetic Field. L. A. Bauer.

A Study of the Joule and Wiedemann Effects in Nickel Rods. S. R. Williams.

On the Principle of Relativity. A. Macfarlane.

A Demonstration of the Relativity Concepts of Time and Space by Means of a Model. Reinhard A. Wetzel. (By title.)

Note on a High Tension Oscillatory Ignition System. M. E. Graber.

Reversal of the Rowland Effect. F. E. Nipher.

Application of Quaternions to Curvilinear Coördinates. L. B. Tuckerman, Jr.

Notes on the Hall Effect. Alpheus W. Smith.

The Series System in the Spectra of Ca, Sr, and Ba. F. A. Saunders.

Optimum Wave-length in Radio Telegraphy. A. H. Taylor.

Propagation of Signals in a Dispersive Medium. Walter Colby.

Absorption of β-rays by Air and Carbon Dioxide. Alois F. Kovarik.

Note on Some Anomalies of the Kerr Magneto-optic Effect. L. R. Ingersoll.

Standards of Wave-Length and Desirable Data for Them and for Other Lines. Charles E. St. John.

On Special Conditions in the Electric Furnace Which Produce a Spectrum Similar to that of the Spark. Arthur S. King.

Resonance in the High Frequency Oscillations of a Secondary Circuit Excited by the Lepel Arc. A. D. Cole.

The Similarity of Electrical Properties in Light-positive Selenium to Those in Certain Crystal Contacts. F. C. Brown.

The Effect of Increase of Temperature and Passage of an Electric Current upon the Elastic Properties of Soft Steel Wire. H. L. Dodge.

Vacuum Tube Discharge in a Magnetic Field. Norton A. Kent and Royal M. Frye.

Change in Electrical Conductivity Due to the Orientation of Oblate Spheroids Within the Conductor. S. R. Williams.

Spectra of Low Potential Discharges in Air and Hydrogen. (By title.) Gordon S. Fulcher.

The Brownian Movements in Gases at Low Pressures. R. A. Millikan.

The Free Vibrations of a Lecher System using a Lecher Oscillator. F. C. Blake and Chas. Sheard.

The Velocity of Electrons in the Photo-electric Effect. D. W. CORNELIUS.

On Magnetic Rays. L. T. MORE.

Certain Minimum Insulating Distances between Conducting Surfaces, the Method of Measurement, and Reference to Electronic Atmospheres. F. C. BROWN.

The Earth Inductor as an Inclinometer. N. E. DORSEY.

A Method for Charging the Needle of a Quadrant Electrometer. HENRY A. ERIKSON.

Line Structure and Shift. NORTON A. KENT.

Phase Change by Reflection. C. A. SKINNER and L. B. TUCKERMAN, JR.

Some New Diffraction Photographs. MASON E. HUFFORD.

The Constants of Spectral Radiation of a Uniformly Heated Enclosure or So-called Black Body. W. W. COBLENTZ.

Some Observations on the Electric Spark in Air. ARTHUR L. FOLEY.

The Fluorescence Spectra of the Uranyl Salts at Low Temperatures. EDW. L. NICHOLS and ERNEST MERRITT.

The Difference of Phase at the Ears Produced by a Source of Sound not Distant. G. W. STEWART.

Physical Characteristics of Standard Vowel Tones. DAYTON C. MILLER.

The Use of Sounding Boards in an Auditorium. F. R. WATSON.

ERNEST MERRITT,
Secretary.

TREASURER'S REPORT FOR THE YEAR 1912.

Disbursements.

Executive expenses:	
President, mileage	$ 78.63
Secretary, mileage	121.86
Secretary, office expenses	100.00
Treasurer, office expenses	50.00
Stationery, printing and postage:	
Orders of secretary	248.95
Orders of treasurer	42.00
Subscriptions to Journals:	
Physical Review, 602 members	1,806.00
Science Abstracts "A," 597 copies	1,194.00
Science Abstracts "B," 35 copies	70.00
Science Abstracts, back numbers	4.00
	$3,715.44

Receipts.

Annual dues, 578 members	$3,468.00
Payments for Science Abstracts "B"	70.00
Payment for life membership	50.00
Entrance fees, 11 members	33.00
Dues in arrears	12.29
Dues in advance	18.00

Interest and exchange	24.43
Refund from American Association for Advancement of Science	6.00
Old accounts	94.00
	$3,775.72

Summary.

In bank December 1, 1911	$1,017.17
Receipts	3,775.72
	$4,792.89
Disbursements	3,715.44
In bank December 1, 1912	$1,077.45

Membership November 30, 1912.

Honorary	6
Life	17
Regular and Associate	581

J. S. Ames,
Treasurer.

I have verified the above accounts, examined the bank-balance, and certify that they are correct.

A. H. Pfund.

December 23, 1912.

THE INTRINSIC BRILLIANCY OF THE GLOWWORM.[1]

By Herbert E. Ives and C. W. Jordan.

PREVIOUS work on the firefly has shown its radiant luminous efficiency to be very high. All its radiation is in the visible spectrum, and localized in the yellow-green portion.

A point which has not been investigated, to the authors' knowledge, is the intrinsic brightness of the light. This is a matter of far greater importance than the quantity of light emitted, for the latter is dependent in a large measure on the insect's size and not necessarily on the peculiar process of light production. If the chemical laboratory of the firefly could be copied, would the product be of use to man? This depends upon the intrinsic brightness, for that determines the size of light necessary for any lighting requirement.

A species of glowworm (*Photuris Pennsylvanica*) accidentally observed in Philadelphia had the great virtue, from the standpoint of study, of giving a steady glow from its luminous abdominal areas. The spectral character of the light appeared to be the same as that previously observed in the firefly. The brightness of these spots was compared directly with that of a white blotting paper illuminated by a 10-candle-power standard incandescent lamp. By proper manipulation of the glowworms their bright areas were visible directly against the white surface, with no dividing line, making possible a very close match by moving the incandescent lamp to and fro.

It was found that the luminous spots were as bright as the blotting paper under an illumination of 190 meter candles, or considerably brighter than white surfaces are ordinarily illuminated by artificial light (circa 25 meter candles). Taking the albedo of blotting paper as .76, it follows that the candle-power per cm^2 of the glow worm is .0046. In order, therefore, to obtain 25 M.C. on a plane 2 meters below a light source, the latter would have to be about 2 square meters in area. The source would give 100 c.p. which should be sufficient to illuminate a room of five times the area just given.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, Oct. 12, 1912.

A peculiarity of most instances of chemi-luminescence is their excessively low specific brightness; on the other hand most artificial illuminants are of such excessive brightness that the light must be spread over greater area by the use of diffusing glass or reflecting surfaces. The intrinsic brilliancy of the glowworm lies between these extremes. It appears, therefore, that the solution, not only of the problem of light efficiency, but of light intensity, may follow from copying the process exhibited by the firefly.

THE EFFECT OF DRAWING ON THE ELASTICITY OF CERTAIN WIRES.[1]

BY L. P. SIEG.

IN a recent paper[2] the writer has pointed out that the elastic constants of certain wires were modified to a remarkable extent by drawing. This work has been continued on six wires specially drawn for the purpose by Heræus.

It was specified that the wires should be from the same alloy, a 40 per cent. alloy of iridium with platinum. As far as possible the wires were to be treated in exactly the same manner during the process of drawing. Six wires were thus prepared, ranging in diameter from 0.20 mm. to 0.10 mm. It was hoped from results of previous work that there would be a progressive change in the torsional constants with the drawing, but while marked changes have been found, they have not been found to be uniform, in spite of the fact that careful efforts were made to have the conditions exactly the same in the six experiments.

Just one aspect of the matter will be presented. The wires were annealed under constant load to a yellow heat, care being exerted to get this annealing temperature nearly the same for all the wires. The supported weight could be adjusted so as to change its moment of inertia without changing the load. The periods in the six cases were adjusted so that the effects of air friction were the same. The following table gives the results of only one aspect of the work. In a way these results represent the relative amounts of internal

Diameter (mm.).	Vibrations.
0.250	65
0.200	186
0.165	171
0.130	98
0.115	100
0.100	56

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] PHYS. REV., XXXV., Nov., 1912.

friction in the six cases. In the first column is the diameter of the wire used, while in the second is the number of vibrations executed by the wire in question as it died down from an amplitude of 18° to an amplitude of 5° per cm. of length of the wire.

There is an indication from the above that with the successive drawings there is formed a wire with gradually increasing rigidity, although the results from the first wire are disconcerting. It is strange to find that even the heating of these wires to a bright yellow heat is not sufficient to bring them to the same elastic condition, even though their chemical composition is exactly the same. Results, which will not be given here, on the relation of period of these pendulums with their amplitudes tend to emphasize the above peculiarities. This may account for the wide variation of elastic constants of common wires as obtained by different investigators, whose attention has been fixed perhaps on the chemical constitution of their wires more than on the methods used in the drawing.

State University of Iowa,
November 26, 1912.

The Cooling Effect which Accompanies the Emission of Electrons from Hot Bodies.[1]

By H. L. Cooke and O. W. Richardson.

THE following experiments to detect and measure this effect have been made with filaments of osmium which were kindly supplied to us by the Deutsche Gasglühlicht Actiengesellschaft, of Berlin. The filament was placed in one arm of a sensitive Wheatstone's bridge which was actuated by the main heating current C. Two of the arms carried high resistances so that 99 per cent. of the current C flowed through the filament and the adjacent arm which carried a standard resistance A of 10 ohms. The filament was surrounded by a brass cylinder from which it was insulated. The cylinder was connected in series with a microampere-meter, suitably shunted, and a suitable source of electromotive force V, which could be reversed, to a point midway between two equal resistances. The two ends of these resistances were connected either (1) to the opposite ends of the filament (filament shunted) or (2) to the opposite ends of the standard resistance A (standard shunted).

The resistances were first adjusted so that there was no deflection of the galvanometer when the electromotive force V prevented the thermionic current escaping from the filament. The deflection of the galvanometer D_T and the magnitude T of the thermionic current were then observed when the potential difference V was reversed. This deflection is due to the following causes: (1) the change in the resistance of the filament which is due to the cooling effect under investigation, (2) the change in the Joule heating effect due to the superposition of the thermionic current in the wire on the original heating current and (3) a disturbance of the balance of the bridge circuit which arises from the thermionic current acting like a battery. The two last effects are different according as the filament or the standard is shunted. It can be shown that, to the first order of small quantities, the deflection which is due to these effects does not reverse when the main current is reversed, whereas the part which is due to (1) does reverse under these conditions. In this way these disturbing effects may be eliminated.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

To reduce the deflections to absolute values a known change in the Joule heating effect was produced by changing the total resistance R of the bridge circuit by a known amount. This was done by placing a shunt of 100 ohms across a standard 1 ohm resistance in series in R. If the galvanometer deflection thus caused is D_E it is easy to show that with the numerical values of the resistances stated above

$$\varphi = 9.9 \times 10^{-3}\left[\left(\frac{C^3R_fD_T}{ETD_E}\right)_1 + \left(\frac{C^3R_fD_T}{ETD}\right)_2\right],$$

where φ is the cooling effect per unit thermionic current expressed in equivalent volts, E is the electromotive force in the heating circuit and the suffixes 1 and 2 denote that the readings are taken with the main heating current in opposite directions in the two cases. The above formula is approximate and subject to certain corrections which it would take too much space to discuss here.

All the quantities C, V and R_f have been varied by factors of about 2 and D_E and D_T by much larger factors without changing φ by more than the error to be expected from the method of measurement. Experiments have been made with values of T varying from 5×10^{-6} amp. to 869×10^{-6} amp. The variation of T was produced by varying the temperature of the filament. There is no certain evidence that φ varies with the temperature, with the magnitude of the thermionic current or with the age of the filament; although the values of φ for the largest values of T are about 2 per cent. above the average and it is possible that this represents a real variation of φ with the temperature and is not due to experimental error.

Altogether 38 measurements of φ have been made. The mean of these gives $\varphi = 4.7$ volts. With the exception of three values, which were subject to large possible errors as they involved measurements of very small deflections, none of the remaining 35 values differ by more than 8 per cent. from the mean and most of them are very near it. This value of φ is in good agreement with these which have been obtained by two entirely independent methods: (1) from the heating effect when electrons are absorbed by metals and (2) from the variation with temperature of the emission of electrons from hot metals. The experiments therefore appear conclusively to demonstrate the existence of the effect under investigation.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

ON THE CAUSE OF THE APPARENT DIFFERENCES BETWEEN SPARK AND ARC SOURCES IN THE IMPARTING OF INITIAL SPEEDS TO PHOTO-ELECTRONS.[1]

BY R. A. MILLIKAN.

SOME months ago I reported in some detail[2] experiments which seemed to show that light of a given wave-length from a spark source possesses quite different photo-electric properties from light of the same wave-length from a quartz-mercury lamp.

In endeavoring to find a satisfactory explanation of these differences I made the following statement,[2] "The simplest assumption would be that these effects (of spark sources) are not purely photo-electric, but that the high initial velocities are produced by the action of long electromagnetic waves upon electrons already liberated by the light. It is to be observed that all the spark curves look like the simple superposition upon the ordinary photo-curve of some influence which throws part of the already freed electrons (*e. g.*, those ejected toward one side) away from the surface, thus producing the high velocity feature of the curve, while it drives the other part back against the surface, thus making necessary the application of strong accelerating potentials to overcome this backward push. Nevertheless, the (two) experiments previously mentioned seem to speak decisively against this assumption and to show that the inward and outward forces on the electrons must be looked for in some influence of the light itself." Of the two experiments referred to the one which was considered crucial was as follows. The spark and mercury arc sources were placed so that the light from both could fall simultaneously upon the electrode. Then, while the spark was kept running the light from it was prevented from falling on the electrode, while that from the mercury arc was allowed so to fall. This procedure, it was thought, would give opportunity for the electromagnetic disturbances to impart the high velocities characteristic of spark sources to the electrons liberated by the mercury arc. But instead of this being the case, *the velocities were always exactly those characteristic of the mercury source.* Just as soon, however, as the light from the spark, in addition to that from the mercury arc, was allowed to fall upon the electrode the high velocities characteristic of the spark source were obtained. This seemed to show conclusively that these high velocities were due to the light itself and not to secondary effects of any kind, since these secondary effects were all operating when the light itself was screened off. The second experiment which supported strongly this conclusion consisted in enclosing the whole spark-producing apparatus in an iron box *A*, Fig. 1, the walls of which were 2 mm. thick, without finding that the character of the curves obtained was materially altered. In view of the apparently conclusive evidence of these experiments I sought the explanation of the peculiar effect of the spark source in the extraordinary instantaneous intensity of spark sources and in the assumption of a dependence of initial velocity upon intensity when the latter was very great, but I promised further tests of this assumption as soon as opportunity could be found.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] Verb. d. D. Phys. Ges., 14, page 712, 1912.

Upon returning to Chicago two months ago I asked Mr. W. H. Kadesch to take the tube shown in Fig. 2, which was still set up as it had been in my earlier experiments and to first duplicate my curves with the arrangement of Fig. 1, the condensers *C* being close to the spark gap (very small self induction was found, in the earlier experiments, an essential condition of high velocities):

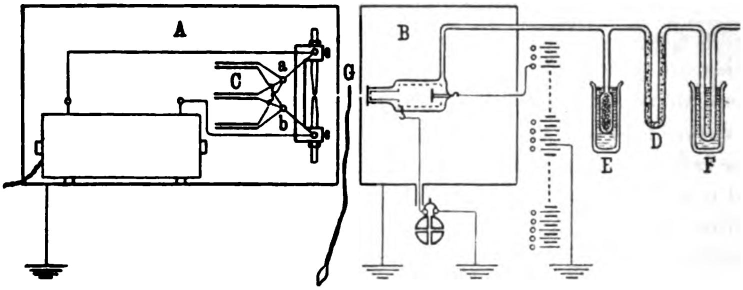

Fig. 1.

then to make a heavily insulated non-inductive spark circuit 30 ft. long and remove all the spark apparatus save the gap to another room. The spark circuit then led from the jars through the straight non-inductive circuit to the spark gap, all portions of this circuit being in the plane of the figure (Fig. 1). When this was done the positive potential fell from 60 volts to 10 volts and the photocurrent-potential curve was almost identical with that furnished by the mercury lamp. When, however, the spark gap was pulled six feet away from the photo-cell the positive potential slowly fell to 1.6 volts and the curves from the two sources became identical. When next the spark gap was kept in its position and the spark-producing apparatus was moved up toward the photo-cell the positive potential began to rise and ultimately reached over 100 volts. Shifting the position of the coil would change this markedly.

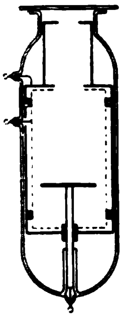

Fig. 2.

These experiments seem to furnish altogether conclusive evidence that the first assumption reported in the preceding paper was correct and that, despite the evidence to the contrary, the peculiarities of the photo-curves from spark sources are all due to disturbances having their origin in the spark circuit.

It becomes necessary, however, to explain the two experiments which led to the opposite conclusion. With regard to the second, viz., the lack of effect of all ordinary as well as extraordinary screening devices (both *A* and *B*, Fig. 1, were completely closed boxes of heavy sheet iron) in cutting down these disturbances, there is nothing to be said except that no effective screening from such disturbances seems to be possible with spark sources, at least under the circumstances here used, in which the coil had to be actuated by an alternating current led into its primary from an outside circuit. This may have important bearings on several other types of experiments.

With regard to the first, we must assume that the time during which the disturbances sent out from the continuously running spark circuit can act on the electrons liberated by the mercury light is less than 1/1,000 of the time interval between these impulses since otherwise the running of the spark circuit, when its light was screened off from the photo-cell, would have caused the positive potential to rise observably above that due to the mercury light.

These results are reported at once even though all the obscurities are not yet removed, in order that others who may have been interested in the preceding report may know that I am now convinced that the anomalous results obtained with spark sources must be explained by a consideration of electromagnetic disturbances rather than by the assumption of a dependence of energy of emission upon intensity. *In fact the final practical identity of the spark and mercury-arc potentials (the two sources were of about the same mean intensity) shows that the intensity may vary at least in the ratio 1,000 to 1 without influencing appreciably the velocity of emission of photo-electrons.*

Note added January 15, in correcting proof. — I have learned since the above paper was prepared that Pohl and Pringsheim have recently reached similar conclusions (Verh. d. D. Phys. Ges., Nov. 15, 1912), and have supported them by other types of evidence than that herewith presented.

THE STABILITY OF RESIDUAL MAGNETISM.[1]

By N. H. Williams.

A SIXTY cycle unidirectional current varying between zero and a maximum value is made to produce a demagnetizing influence upon a magnetized specimen of iron or steel. Demagnetization and recovery curves are plotted by an oscillographic method for different values of residual magnetism.

In a given specimen, for any value of residual magnetism, there is a definite elastic limit, which is the maximum field that can be applied without permanent demagnetization. Within this limit, magnetic stress and strain are approximately proportional.

The hysteresis for this kind of cycle is very small and is not greater for very hard steel than for soft iron.

The extreme points of the demagnetization curves lie upon a smooth curve which lies inside the maximum hysteresis curve and parallel to it.

The susceptibility to small demagnetizing fields increase slightly with decrease of residual magnetism, the maximum difference observed being about 8 per cent.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

The recovery from the effect of a demagnetizing field increases with decrease of remanence and for hard steel reaches a value as high as 24 per cent. of the maximum residual induction. For softer specimens, the recovery is less.

There is shown to be a well defined upper limit beyond which the residual magnetism of an open magnetic circuit cannot be made to pass.

SOME DIRECT MEASUREMENTS OF THE TEMPERATURE CORRECTION TO THE REFRACTIVE INDEX OF WATER AROUND 20° C.[1]

BY F. A. MOLBY.

THE interference apparatus, previously described before the Society, was used for making direct measurements of the change in the *optical length* of a column of water 1.3 cm. in actual length. A fused quartz ring, cut away to allow water circulation, was used for an annulus separating two partially silvered glass plates, between which the column of water was used as an optical medium instead of air or vacuum used by Scheel, Dorsey, and others when they were measuring the expansion of materials at various temperatures. A mercury thermostat was used to control the temperature at any desired value with a high degree of accuracy, and the temperature changes in the water were measured with five thermocouples connected in series.

Several different sets of measurements were made and second degree equations were obtained for the relation between temperature and refractive index correction. It is believed that for the wave-length of light used the correction to the refractive index has been determined to a degree of accuracy such that the errors are not greater than 0.5 per cent. of the whole correction, for ranges of several degrees above and below 20° C.

THE EFFECTS OF GAS CURRENTS ON SOUND, WITH APPLICATIONS TO VENTILATING SYSTEMS OF AUDITORIUMS.[1]

BY F. R. WATSON.

A NUMBER of recorded experiments indicate that gas currents have an effect on the progress of sound waves. Lord Rayleigh has shown[2] that reflection at the boundary of gaseous media depends on the difference in the velocities of sound in the two media. Taking $V = \sqrt{\gamma p/\rho}$, an increase in temperature or moisture decreases ρ and hence increases V. With constant temperature, changes in p do not affect V, since p/ρ remains constant. Since γ is not the same for all gases, the value for V is thus further likely to change. Total reflection may occur for oblique incidence. Refraction also occurs when the velocities of sound are different in the two media. Furthermore, air in motion carries sound with it. Experiments have been carried out that verify some of these conclusions. For instance, the sound from a Galton's whistle has been reflected from the sheet of hot gases rising above a straight row of gas burners. The receiver was a Barry sensitive flame. Reflection has also been obtained from the boundary between carbon dioxide and air. Another set of experiments has shown that hot currents of gas in a room absorb but little sound energy. It has also been shown that irregular air currents in a room have a pronounced action of the sound. This effect follows when a Rayleigh suspended disc resonator responds to the sound of an organ pipe across the room. With no air currents, the deflection is steady; with air currents, the deflection is erratic. The theory indicates that total transmission may take place for a certain angle of incidence. This conclusion is contrary to the case for other forms of radiation, and attempts are being made to test the statement. The application of these results to the case of a ventilating system in an auditorium shows that little effect is to be expected for ordinary conditions unless the air currents are very irregular. For extreme variation in temperature and other factors, an effect may be found. Examples may be cited to show these different effects of ventilating systems.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] Theory of Sound, Vol. II., section 270.

Air Resistance to Falling Bodies.[1]

By A. A. Somerville.

EXPERIMENTS have been performed to determine the time required for small spheres of various sizes to fall through a considerable distance. The falling body has been started by closing circuit through an electro-magnet thereby opening a trap-door. The time of fall is recorded on a chronograph, the stylus being connected with the same circuit as the magnet which releases the ball and also with another circuit which is closed when the ball strikes at

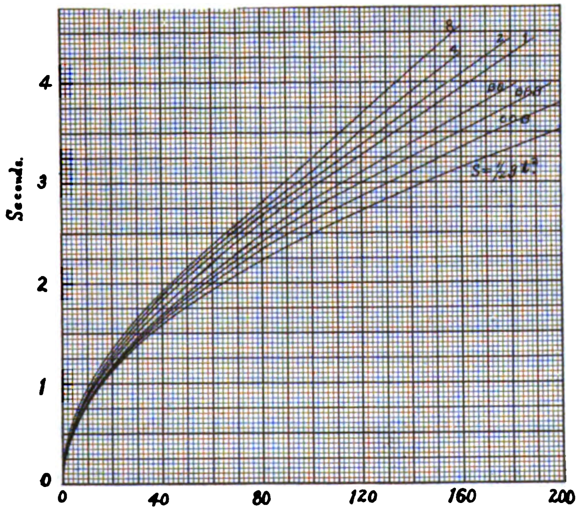

Fig. 1.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

the bottom or end of the distance fallen. A net as large as a pocket handerchief will generally catch the ball after a fall of 200 feet through quiet air. The experiments have been tried in three places: an elevator shaft, 70 feet; a street car bridge over a gorge 120 feet deep; a cable over a gorge 360 feet deep. Seven sizes of shot have been used, also marbles, steel balls and loaded phials. Curves are given, showing the time required for shot of various sizes to fall given distances.

DISTRIBUTION OF THE ELEMENTS IN THE SOLAR ATMOSPHERE.[1]

BY CHARLES E. ST. JOHN.

THE displacement of the Fraunhofer lines in the outer edges of the penumbra of sun-spots was first observed by Evershed, who attributed the displacements to the outward motion of the material. An investigation covering some five hundred lines has confirmed Evershed's observation and explanation. When one considers lines of intensity 0 to 10, the displacements for lines of the same element *increase* with *decrease* of the intensities and differ for lines of like intensity of different elements, and when the displacements are reduced to a common wave-length they vary as the wave-length. For the lines of intensities 1 and 2 of iron the result of the comparison between lines in the violet, green, and red is as follows:

	Absolute Displacements.		Reduced to λ 5000.	
	1.	2.	1.	2.
Violet	0.0220 Å	0.0200 Å	0.0262	0.0240
Green	0.0299	0.0259	0.0296	0.0259
Red	0.0318	0.0308	0.0268	0.0250

This relation between displacement and wave-length is the hall-mark of the Doppler effect and seems decisively in favor of motion as the cause of the phenomena. For lines of the intensity under consideration the displacements are such as to indicate an outward flow increasing with the depth, on the assumption that the weak lines are on the average at a low level. Assembling all the data for the 192 iron lines, we have the following scale connecting intensities and displacements reduced to λ5000.

Intensities	0	1	2	3	4	5	6	7	8	10
Displacements	0.0285 Å	0.0278	0.0249	0.0229	0.0209	0.0193	0.0165	0.0134	0.0095	0.0050

With this scale established it is possible to determine the level of the lines of other elements in terms of the iron scale.

For lines of intensity exceeding 10 on Rowland's scale the displacements for all lines become exceedingly small, change sign, and then increase in absolute value with the intensities of the lines reaching the highest negative value for the H_3 and K_3 lines of calcium. The negative sign indicates inward motion of the upper levels of the solar atmosphere.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

When the 27 substances are arranged in a series beginning at the left with the element appearing at the highest level, it is seen at a glance that high atomic weights occur more frequently toward the right, pointing to a low level even for the strongest lines of the heavier elements. This becomes more evident when taken in five groups from left to right, with seven elements in the first group, and the atomic weights summed for each group and the mean atomic weight of the group found.

Groups.	1	2	3	4	5
Sum of atomic weights	258	242	317	526	727
Mean atomic weights	37	48	63	105	145

On the Value of *e*.[1]

By R. A. Millikan.

THIS paper contains a summary of the results obtained in the work of the past two years in the determination of all of the factors entering into the oil drop method for the evaluation of *e*.

These results are as follows:

If μ_{23} is the coefficient of viscosity of dry air at 23° C. then

$$\mu_{23} = 1824 \pm .00015 \times 10^{-7}. \qquad (1)$$

Or if μ_t is the coefficient of viscosity at any temperature between 12° C. and 28° C. then

$$\mu_t = .0001824\{1 + .0027(t - 23)\}. \qquad (2)$$

If the mean free path of a molecule of air is computed from $\mu = .350 p\bar{c}l$ where $\bar{c}$ is the *average* molecular velocity, then, in the formula

$$x = 6\pi\mu a v_1 \left(1 + A\frac{l}{a}\right)^{-1}, \qquad (3)$$

x being the force necessary to impart to a spherical drop of radius a velocity v_1 then

$$A = .874.^{2}$$

But when $l/a > .4$ equation (3) must be replaced by

$$x = 6\pi\mu a v_1 \left\{ 1 + \frac{l}{a}\left(.874 + .32\epsilon^{-1.54\frac{l}{a}}\right) \right\}^{-1}.$$

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] Owing to a failure to see proof, incorrect numbers for these constants got into a note in the Physical Review, Vol. XXXIV., p. 398. These former numbers were never obtained in any experiments, but represented simply typographical errors.

If e denotes the value of the elementary electrical charge then the present investigation gives

$$e = 4.774^2 \pm .008 \times 10^{-10} \text{ E.S. units.}$$

A SUBSTITUTE FOR A BRONSON RESISTANCE.[1]

BY DAVID W. CORNELIUS AND J. G. KEMP.

IN the Philosophical Magazine for August, 1911, pp. 301, 302, Campbell describes certain liquid resistances which may be used for the same purposes as that generally known as the Bronson resistance.

The authors, in October, 1911, while working on photo-electric problems, found it very convenient to use a high resistance in order to measure electrical currents of the order of magnitude 10^{-11}, 10^{-13} amp. From the suggestions obtained from the article by Campbell liquid resistances were made using C.P. benzol and absolute alcohol, and C.P. meta-xylol and absolute alcohol. The mixtures were placed in straight glass tubes 1.7 cm. in diameter, about 15 cm. in length, with one electrode sealed in the bottom and the other electrode passed through an insulating stopper in the top. The upper electrode was made movable in order that the distance between the electrodes could be changed.

The material of the stoppers were in some cases hard rubber and in others amber. Owing to the action of the vapor of the mixture upon the hard rubber and the amber it was found impractical to use this type of insulating stopper for any other purpose than that of investigating the effect of change of distance between the electrodes.

In measurements for constant distance between the electrodes straight glass tubes of 1 cm. diameter, about 10 cm. in length were used with electrodes sealed in the lower end and in the side about 2 cm. and 3 cm. apart, and having a ground glass stopper in the upper end. This type of cell avoided the escape of any of the contents of the tube by vaporization, thus keeping the mixture constant.

Ohm's law was found to hold for voltage less than one volt and in some cases as high as 720 volts.

Tables of data are given below which show first the relation between the resistance and the distance between the electrodes for different percentages of alcohol; second, the relation between the resistance and the per cent. of alcohol in the mixture for benzol and meta-xylol.

The observations show that the relation between resistance and electrode distance is of the form $R = R_0 + \alpha x$ provided the distance between the electrodes does not exceed 3 cm.

R_0 varies from 6×10^{10} ohms for the mixture of benzol with 6.6 per cent. alcohol to about 5×10^8 ohms for a similar mixture containing 17 per cent. alcohol.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

Table I.

C_6H_6 + 6.6 per cent. C_2H_5OH.
Max. voltage used 406 *volts.*
Min. voltage used 81 *volts.*
Diam. of tube 1.8 *cm.*
Temp. between 23°–24° *C.*
Amber stopper.

$C_6H_4(CH_3)_2$ + 5.6 per cent. C_2H_5OH.
Max. voltage used 720 *volts.*
Min. voltage used 640 *volts.*
Diam. of tube 0.74 *cm.*
Temp. between 23°–24° *C.*
Hard-rubber stopper.

Resistance in Ohms.	Electrode Distance, Cm.	Resistance in Ohms.	Electrode Distance, Cm.
22.2×10^{10}	3.695	99.0×10^{10}	5.035
15.0×10^{10}	2.575	66.0×10^{10}	4.105
13.0×10^{10}	1.610	49.5×10^{10}	3.025
11.6×10^{10}	1.085	36.5×10^{10}	2.140
10.7×10^{10}	0.725	23.9×10^{10}	1.125
9.3×10^{10}	0.375	15.0×10^{10}	0.540
8.6×10^{10}	0.090	11.6×10^{10}	0.275

Table II.

C_6H_6 + 10.3 per cent. C_2H_5OH.
Max. voltage used 406 *volts.*
Min. voltage used 6 *volts.*
Diam. of tube 1.60 *cm.*
Temp. between 24°–25° *C.*
Amber stopper.

$C_6H_4(CH_3)_2$ + 10.1 per cent. C_2H_5OH.
Max. voltage used 640 *volts.*
Min. voltage used 200 *volts.*
Diam. of tube used 0.735 *cm.*
Temp. between 24°–25° *C.*
Hard-rubber stopper.

Resistance in Ohms.	Electrode Distance, Cm.	Resistance in Ohms.	Electrode Distance, Cm.
9.5×10^{9}	3.500	19.9×10^{9}	6.010
7.2×10^{9}	2.850	16.3×10^{9}	4.935
6.3×10^{9}	2.110	12.7×10^{9}	3.765
5.9×10^{9}	1.800	10.0×10^{9}	2.775
5.5×10^{9}	1.435	7.2×10^{9}	1.715
5.3×10^{9}	1.190	4.3×10^{9}	0.649
4.8×10^{9}	0.715	3.5×10^{9}	0.395
4.0×10^{9}	0.285	3.1×10^{9}	0.240

It was found that the resistances remained fairly constant for a period of about ten days. When evaporation was permitted to take place the resistance increased. This was due to the escape of alcohol.

The variation with the temperature was not investigated except that no change was noticeable for changes between 24° C. and 25° C. Since this work was completed, however, Campbell, in a note in the Philosophical Magazine for April, 1912, p. 668, shows that the resistance made of commercial xylol and absolute alcohol had a temperature coefficient. Between 15° C. and 30° C. the resistance at a temperature t is given by

$$R_t = R_{20}(1 + 0.014(t - 20)).$$

The method used for measuring the resistance was a simple circuit with the

TABLE III.

C_6H_6 + 17 per cent. C_2H_5OH.
Max. voltage used 41 *volts.*
Min. voltage used 6 *volts.*
Diam. of tube 1.68 *cm.*
Temp. between 24°–25° *C.*
Hard-rubber stopper.

$C_6H_4(CH_3)_2$ + 16.3 per cent. C_2H_5OH.
Max. voltage used 120 *volts.*
Min. voltage used 6 *volts.*
Diam. of tube 0.735 *cm.*
Temp. between 24°–25° *C.*
Hard-rubber stopper.

Resistance in Ohms.	Electrode Distance, Cm.	Resistance in Ohms.	Electrode Distance, Cm.
11.4×10^8	5.700	15.5×10^8	5.420
8.2×10^8	4.575	11.4×10^8	4.360
7.2×10^8	3.200	9.5×10^8	3.625
6.5×10^8	2.125	7.3×10^8	2.620
5.9×10^8	1.330	5.6×10^8	1.716
5.6×10^8	0.910	3.9×10^8	0.955
5.3×10^8	0.580	2.9×10^8	0.430
4.8×10^8	0.230	2.7×10^8	0.220

TABLE IV.

C_6H_6 + C_2H_5OH.
Max. voltage used 406 *volts.*
Min. voltage used 81 *volts.*
Diam. of tube 1.60 *cm.*
Temp. between 24°–25° *C.*
Electrode distance 1.435 *cm.*
Amber stopper.

$C_6H_4(CH_3)_2$ + C_2H_5OH.
Max. voltage used 720 *volts.*
Min. voltage used 560 *volts.*
Diam. of tube 0.735 *cm.*
Temp. between 24°–25° *C.*
Electrode distance 1.435 *cm.*
Hard-rubber stopper.

Resistance in Ohms.	Per Cent. Alcohol.	Resistance in Ohms.	Per Cent. Alcohol.
8.9×10^{10}	6.6	18.6×10^{10}	5.7
2.6×10^{10}	8.0	6.9×10^{10}	7.2
1.1×10^{10}	9.2	2.2×10^{10}	8.8
0.55×10^{10}	10.3	1.5×10^{10}	9.6
0.06×10^{10}	17.0	1.2×10^{10}	10.5
		0.05×10^{10}	16.3

resistance, storage cells, and a d'Arsonval galvanometer in series. The sensibility of the galvanometer was 3.7×10^{-10} ampere per mm. deflection.

A check on this method was made by using the leakage of a charged condenser through the resistance and measuring the rate of leak by means of an electrometer. It was found that the two methods gave values for the resistances which agree very closely.

The glass tubes were mounted on hard-rubber insulating stands to prevent leakage.

In addition to using these resistances in place of the Bronson resistance they may be used for teaching purposes in laboratory work in determining the figure of merit of a galvanometer, the comparison method of measuring insulation resistance of cables, and the experiment on measuring a high resistance by the leakage of a charged condenser.

THE RELATION BETWEEN CURRENT AND TIME RATE OF CHANGE OF ELECTRIC FORCE.[1]

BY MAX MASON.

A CONSISTENT atomic theory of electricity should imply that the electron acts as a unit, and terms should not be present in the fundamental equations which describe the action of parts of the unit. The introduction of the concept of density of charge as the number of electrons per unit volume times the electronic charge, is natural and advisable in the study of material bodies. It would be difficult to define density consistently with an atomic theory of electricity so that the concept could be applied to the electron itself. If the electron be assumed as a "point charge" a simplification of electrodynamic theory results, since it may then be shown that electric current appears as a manifestation of time rate of change of electric force. The equation

$$\text{curl } \mathbf{h} = \dot{\mathbf{e}} \tag{1}$$

is here assumed for the electron theory, in place of the usual form

$$\text{curl } \mathbf{h} = \dot{\mathbf{e}} + \rho\mathbf{v}. \tag{2}$$

By the application of statistical methods an equation for material bodies is derived from (1). If $\mathbf{E}$ and $\mathbf{H}$ be written for the mean values of $\mathbf{e}$ and $\mathbf{h}$ then the equation

$$\text{curl } \mathbf{H} = \dot{\mathbf{E}} + \rho\mathbf{v}$$

is obtained, the term $\rho\mathbf{v}$ for current, and the term $\dot{\mathbf{E}}$ both appearing as a consequence of the mean value of $\dot{\mathbf{e}}$. Thus the magnetic effect of electrons in motion is entirely accounted for by the time rate of change of electric force, and the term $\rho\mathbf{v}$ in equation (2) is redundant. The analytical reason for this is the fact that the time mean of the derivative of a discontinuous function differs from the derivative of the time mean of the function by a term which expresses the gain per second in the function due to its discontinuities.

THE EVIDENCE THAT SODIUM BELONGS TO A RADIOACTIVE SERIES OF ELEMENTS.[1]

BY F. C. BROWN.

CAMPBELL and Wood[2] could not detect in the compounds of sodium any activity that was definitely greater than that common to all matter and certainly no activity one thousandth as great as that of potassium. If therefore sodium belongs to a radioactive series of elements, it of itself must be undergoing disintegration and at the same time be inactive so far as measurable ionizing radiations are concerned, or sodium must be a relatively inactive product resulting from a radioactive parent. If the former is true then sodium left at rest a long period of time should diminish in amount. But if the latter presumption is true, sodium should in time form from one or more elements in appreciable quantities.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] Proc. Camb. Phil. Soc., 14, p. 15.

For evidence as to these presumptions the facts of geo-chemistry are used and it is found that two separate sets of facts favor the hypothesis that sodium has been accumulating radioactively over the land during geologic history. First an investigation of the sodium carried to the ocean by the rivers annually, and also an investigation of the total sodium content of the ocean, seems to show that the age of the ocean and of the earth is probably not more than 75,000,000 years old. Joly[3] and also F. W. Clarke[4] after a careful consideration of the possible sources of error deduce these figures. On the other hand if the age is based on the amounts of lead and helium associated with uranium in minerals, we find that the age is upwards of a billion years. Thus far the fact seems to be that there is not as much sodium in the ocean as expected. The apparently sufficient explanation is that sodium has been accumulating radioactively over the land but not so over the ocean. Thus the rivers formerly should not have carried as much sodium as they now carry.

An investigation of a second set of facts also makes it convenient to put forth the hypothesis of the radioactive accumulation of sodium over the land areas. If we compare the annual additions of sodium and chlorine by the rivers, it is found that more sodium than chlorine is carried to the ocean. While an examination of the contents of the ocean water reveals much more chlorine than sodium, these two elements are compared because both are soluble in all compounds and neither is deposited in appreciable quantities in the ocean sediments. Of course this comparison only indicates that sodium has accumulated more rapidly than chlorine. So far as the argument is concerned chlorine may also have accumulated radioactively.

No other common explanation has been offered for the two distinct sets of comparative data here used. It would seem therefore worth while to make a search for the possible parentage of sodium. The parent or parents should commonly exist over the land areas, and should in all compounds be relatively insoluble in water. If however the parent has become almost extinct this search would be futile.

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[3] Phil. Mag., 6, 22, p. 357, 1911.

[4] The Data of Geo-Chemistry, 2d edition, Bulletin 491, N. S. G. S.

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THE

PHYSICAL REVIEW.

SOME CHARACTERISTIC CURVES FOR GASES AT LOW PRESSURES.

BY ROBT. F. EARHART.

A LARGE amount of data on the potential required to produce and to maintain a discharge through gases under varying conditions of pressure has been published by numerous observers. The data upon the value of the current strength of the discharge and the effect of varying potential, pressure and spark length upon the magnitude of the current are more limited.

Townsend in his book on Ionization by Collision has gathered much of the available data on the relation between potential gradient and current. The data there presented and discussed apply to a different phase of discharge than the one described in this article. Townsend discusses the currents produced in a gas by the action of some agent like X-rays or ultraviolet light. The sweeping of the ions thus formed out of the ionized region constitutes the current, the current values produced in most cases lying far below the saturation value.

The term characteristic curve used here is similar to the technical term as applied to a dynamo or similar electrical device and merely shows in a graphic way the relation between current and potential under a stated condition.

The problem as it appeared to the author at the inception of the experiments resolved itself into several questions of a rather simple character. Given a luminous discharge at low pressure between two electrodes: what effect will changing the potential produce? To what extent will a characteristic curve be modified by altering the gas pressure and the distance between the electrodes? What effect will altering the size of the cathode have on the characteristic curve? To what extent will these be modified if a diaphragm with orifices of different sizes be placed in different parts of the discharge path? What, quantitatively, will be the effect of a constriction in the path of the discharge? So far as the author is aware the term characteristic curve used in the sense previously stated was first applied to gases by Kauffman.[1] The discussion is theoretical for the most part and the experimental data very limited. In 1901 Rieche[2] studied the relation between potential and current in gases under low pressures, but considered particularly the effect of a magnetic field on the characteristic curves. Both Warburg and Tamm have made experiments on the relation between current and potential for pressures of the order of atmospheric pressures and have stated their results in empirical formulæ. It may well be doubted whether their formulæ could be extrapolated to pressures of the kind used in a Geissler tube. In fact the observations recorded in this paper indicate that much larger currents are obtained at lower potentials as the pressure approaches the critical value, than would be anticipated from their formulæ. A digest of the experiments by Warburg and by Tamm is given by Sir J. J. Thomson in his Conduction of Electricity through Gases, pages 501–502. Two articles by Zeleny[3] constitute a comprehensive extension of the subject. His account of the variation of current and potential for the lower pressures is very brief. Mention is made of the fact that between pressures of 1 and .01 cm. large variations of current are produced with small changes in potential. The present article is concerned with pressures lying wholly within such limits. In a paper recently published by the author,[4] it appears that currents of considerable magnitude may be secured if the cathode is of generous dimensions. The apparatus used in this experiment is fully described in the previous article and will be but briefly mentioned here. A plate made up of ten concentric brass rings separated by an insulating material served as a cathode. The face of the cathode was in a horizontal plane and had a total conducting area of 85 sq. cm. The anode consisted of a hemisphere of Pt (of 1 mm. radius), mounted on a vertical brass rod. The brass rod was covered with a thin glass envelope leaving only the hemisphere exposed. The anode was capable of adjustment vertically and was always symmetrical with reference to the cathode. The discharge apparatus was contained in a bell jar of approximately 10 liters capacity. Measurement of the current which passed through any or all of the rings which together make up the cathode could be made. In the present case the rings were usually connected in parallel. The potential difference between the electrodes was produced by 450 storage cells and regulated by means of a cadmium-iodide-amyl-alcohol resistance. Potential was determined by means of a Weston voltmeter with multiplier in series and current measured directly with a Siemans and Halske milliammeter. The experiments may be grouped under four heads and will be taken up in that manner.

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[1] Gottingen Nach., 1899, p. 243.

[2] Annal. d. Phys., 1901, 4, p. 292.

[3] J. Zeleny, PHYS. REV., 25, p. 305, 1907; 26, p. 129, 1908.

[4] PHYS. REV., Mar., 1912, p. 188.

I. The characteristic curves between the sphere and plane for different pressures and for distances ranging from 1 to 4 centimeters.

II. Effect of changing the area of the cathode on the characteristic curves.

III. Effect of the introduction of thin diaphragms with apertures of different dimensions in the path of the discharge.

IV. Effect of a constriction. Length of constriction. Size of constricted portion. Location of the constricted portion.

I. Effect of Distance and Pressure upon the Characteristic Curves.

The cases studied experimentally in this connection were taken with the full cathode plate, 85 sq. cm., and with the anode at distances of 1 cm., 2 cm., 3 cm. and 4 cm. under various conditions of pressure. These are represented graphically in Figs. 1 to 12. In Figs. 1–4 the results obtained for air are given and in a general way are typical. Figs. 5 to 8 are for hydrogen and Figs. 9 to 12 for carbon dioxide. The form of the characteristic curve is the same for each gas. For hydrogen lower potentials than those necessary for air sufficed, while higher potentials were required for carbon dioxide. The apparatus as constructed made it possible to study the distribution of current density over the cathode. This was done in some cases, but has been discussed in a previous paper. The usual method of procedure was as follows: A discharge was produced, usually by making the sphere negative and then reversing the polarity of the electrodes, the current was raised to the maximum value and allowed to run for 10 or 20 minutes. The current was then reduced by increasing the resistance in the circuit by small steps. Frequent measurements were made upon the potential and current. These are represented in the graph. Many measurements were made at other pressures but are not incorporated in the figures. It may be noticed that for the lower pressures two current values are possible for the same potential, but the distribution over the plate differs in the two cases. When the current has the least value, the resistance in the circuit is a maximum and the potential difference between the electrodes exceeds the minimum value. The discharge is then confined to a limited portion of the cathode.

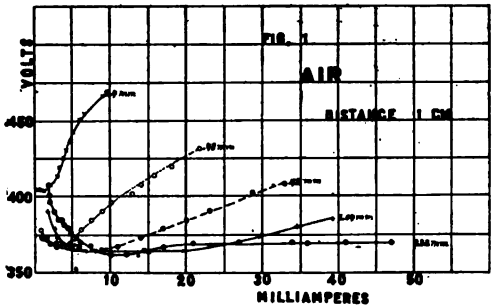

Fig. 1.

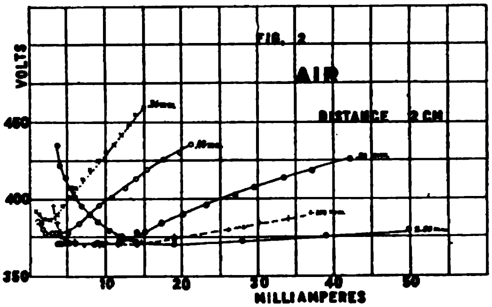

Fig. 2.

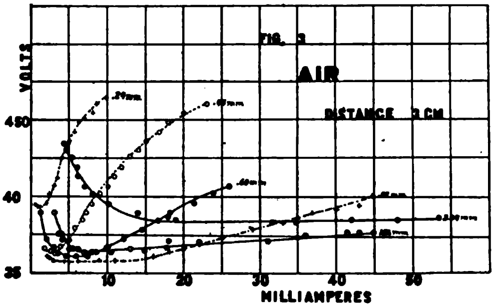

Fig. 3.

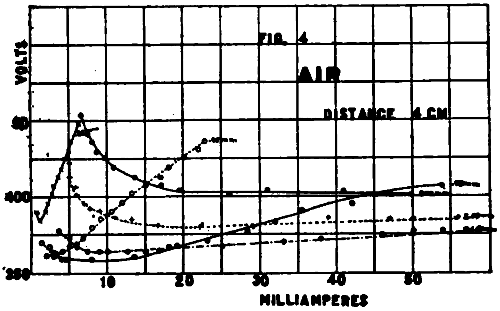

Fig. 4.

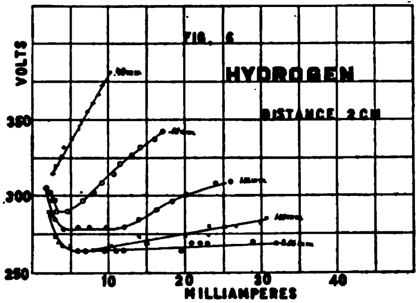

Fig. 5.

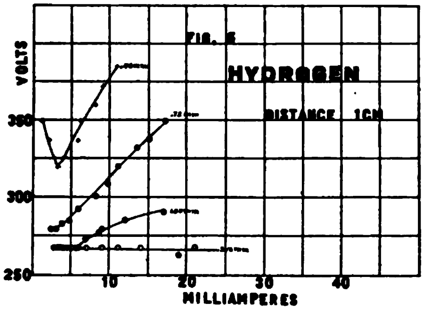

Fig. 6.

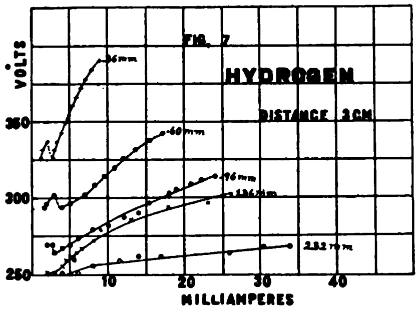

Fig. 7.

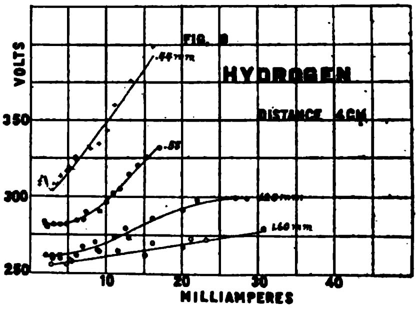

Fig. 8.

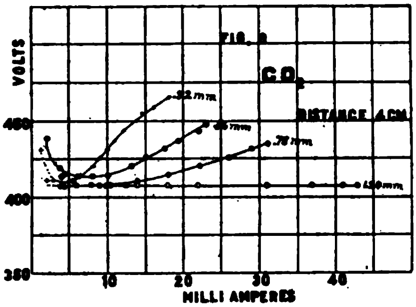

Fig. 9.

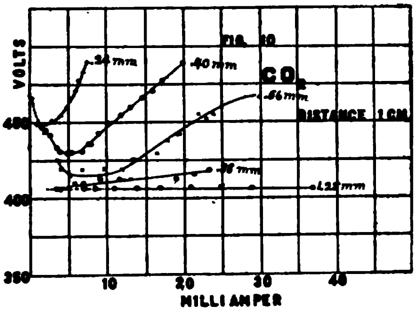

Fig. 10.

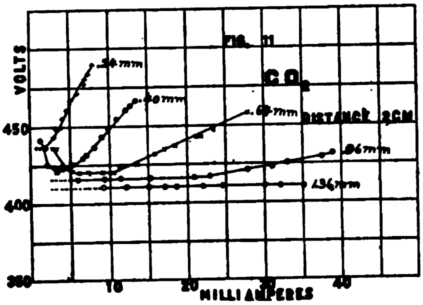

Fig. 11.

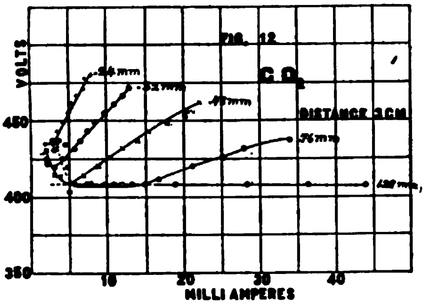

Fig. 12.

Should the pressure be above the critical value the discharge will be confined to the inner zones; should the pressure lie below the critical value, to the outer zones. When the resistance of the circuit is decreased the potential falls, the current increases and the luminosity spreads out over the plate. After the entire plate is functioning, decreasing the resistance causes a rise in current accompanied by a rise in potential between the electrodes. The increase in current density after this stage is reached is not uniform over the plate, but is greatest over the outer zone. It may be noticed that for the higher pressures, the largest currents obtained do not require a potential exceeding the minimum value. It was observed in such cases that the minimum discharge occurred between the inner zones and the anode and as the current increased the discharge spread over the cathode but never reached the edges of the plate. Near the critical pressure, as the current was diminished and the luminosity ceased to cover the cathode an unstable condition existed in which the luminous patch moved about over the plate. In general, while the redistribution takes place there is an unstable condition and the branch of the curves representing the smaller values of the current has a region of instability. This is particularly true for hydrogen which presented an anomaly. In obtaining the hydrogen curve the current was decreased in the usual manner. This was accompanied by rise in potential when the discharge drew in toward the center of the plate. At a certain stage, as the current was decreased the potential also fell, passing through an unstable region, but with further decrease in current the potential rose in a perfectly normal manner. This gives rise to the peculiar kink in the hydrogen curve in this region. Modification of the current control, by altering the batteries and control resistance indicated that it was not an error due to some peculiarity of manipulation but occurred each time under the same condition of pressure and potential. No explanation of this is vouchsafed. The hydrogen was prepared in a Kipp apparatus by the use of C.P. acid on aluminium turnings and was purified and dried. No similar kink was found for either air or carbon dioxide.

Before leaving this phase of the subject it may be of interest to indicate the extent to which the condition of the cathode surface influences the characteristic curve. In the experiments just described, care had been taken to have the cathode smooth, free from scratches and clean. The smooth cathode was covered with a handful of brass turnings and filings. These were spread out in such a manner as not to decrease materially the distance between the electrodes but served to give a rough surface. The characteristic curve was modified but slightly. The apparent effect was to increase the range of instability. The roughening of the plate had in general no effect on the form of the characteristic curve or on the numerical values which relate potential and current. The empirical formulæ based on measurements for high pressures and which connect pressure, potential, and current for a positively electrified point when extrapolated into this region, give values much smaller than those observed. No attempt has been made to develop a new relation for the variations in the region, for it will be seen later that these are further complicated by the size of the cathode.

II. EFFECT OF SIZE OF CATHODE ON THE CHARACTERISTIC CURVES.

Figs. 13 to 16 show the relation between current and potential when the distance between the anode and central part of the cathode was 4 cm. and air used as the dielectric. The characteristic curves shown are for pressures ranging from those less than to those greater than the critical pressure for this distance.

As noted previously, as soon as the cathode is completely covered with the luminosity and is functioning over the entire available area, there will be a rise in potential with increase of current. With the smaller cathode this condition will be attained for smaller values of current than with the larger cathode. The numerals accompanying the various curves refer to the cases given in the accompanying table.

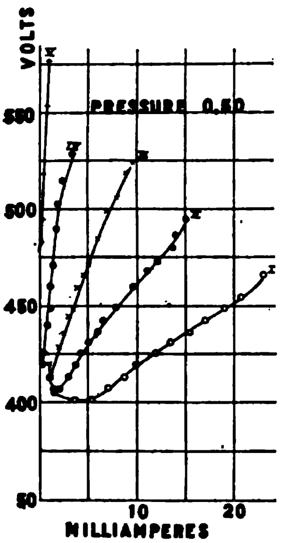

Fig. 13.

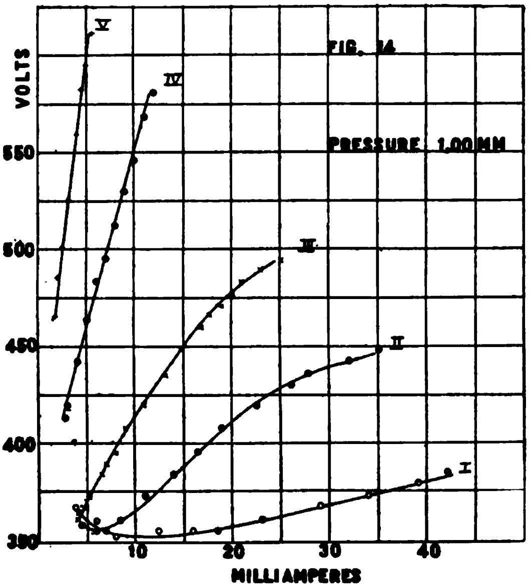

Fig. 14.

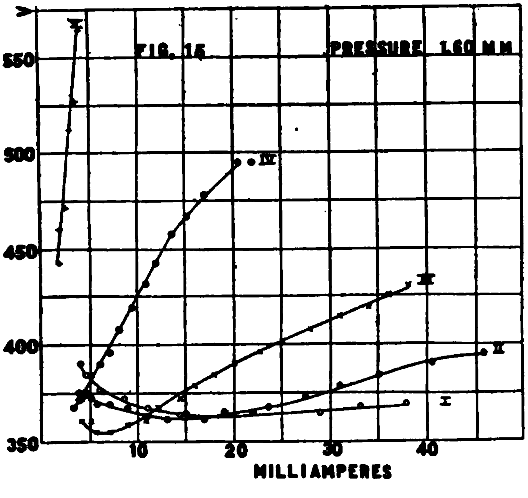

Fig. 15.

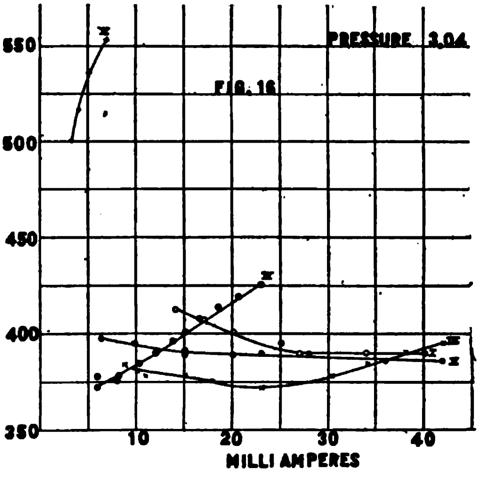

Fig. 16.

The area of the different plates is given. If a line be drawn from the midpoint of the cathode plate to the anode and another from the edge of the plate to the anode these lines will include the angle α.

Case.	α	Area of Cathode in Sq. Cm.
I.	54° 42′	85.0
II.	43° 54′	39.8
III.	33° 25′	19.0
IV.	12° 00′	5.9
V.	3° 36′	.2

III. EFFECT OF DIAPHRAGMS IN THE PATH OF THE DISCHARGE.

The full-sized cathode was used in this experiment. The anode was four centimeters from the plane of the cathode and placed symmetrically with reference to it. The outer zone of the cathode was surrounded by a ring of fiber. Over the top of this spacing ring a very thin diaphragm of mica was sealed. The part directly underneath the anode was pierced. Orifices of 5 mm., 2 mm. and 1 mm. diameter were used. By means of spacing rings of different heights these diaphragms were placed successively at distances of .5, 1, 2, 3, and 3.5 cm. above the cathode. Characteristic curves were taken for three pressures in each case; fifteen cases in all, making forty-five characteristic curves. The differences obtained between any of these and the curves secured when no obstruction was present, was so slight that it may be asserted that the introduction of the diaphragms had no effect on the characteristic curve. Their introduction however increased the stability of the discharge. The appearance of the discharge is also modified. In addition to the luminosity over the cathode and at the anode there is a luminous patch in the aperture. The intensity of the luminous patch is much greater for the small openings. The characteristic curve for the 2 mm. opening is the lowest one in Figs. 17, 18 and 19, and differs very little from the corre-

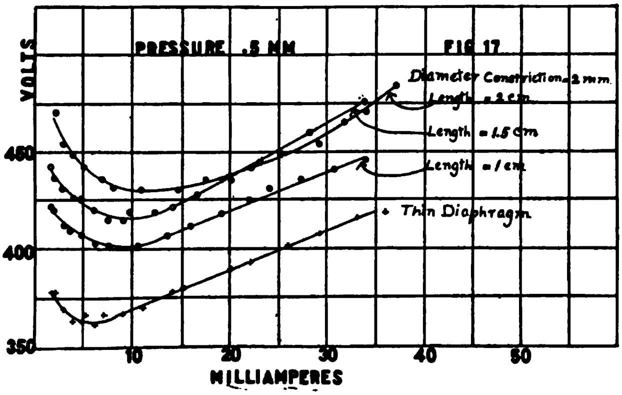

Fig. 17.

sponding curves in Fig. 4, which shows the case when no diaphragm is interposed. It is well known that the potential gradient along a discharge of this character is not uniform but that the rate of change of potential is greater near the electrodes, especially at the cathode. The diaphragms in all cases were interposed where the electric force is weak. It thus appears that the convergence of the stream lines for a short distance has little appreciable effect on the total potential fall.

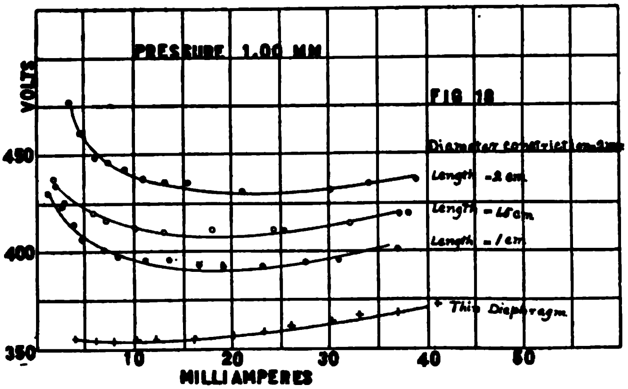

Fig. 18.

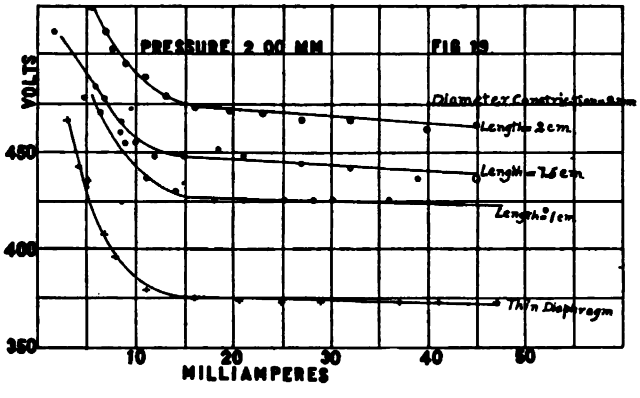

Fig. 19.

IV. Effect of a Constriction in the Path.

The modification of the apparatus for this portion of the experiment consisted merely in piercing a diaphragm and inserting glass tubes of different lengths and bores. The constriction tubes were placed directly underneath the anode and symmetrical with reference to the cathode unless otherwise stated.

The tube was sealed into the mica diaphragm with waterglass and all the discharge passed through the tube. By means of spacing rings this was placed in turn at different elevations above the cathode. The lower end was placed 2 mm. above the cathode, again with the upper end of the tube 2 mm. below the anode and in the midposition. The effect of the introduction of the tube was considerable, but the location of the tube so far as elevation was concerned introduced no further variation, at least within the limits mentioned. The fiber spacing ring which with the mica diaphragm served to enclose the cathode was removed and a glass ring substituted. This permitted an unobstructed view of the entire phenomena occurring within the apparatus. The diameter of the constriction tubes employed ranged from 5 mm. to 1.85 mm. Figs. 17, 18 and 19 show the effect of introducing constrictions of different lengths. They indicate that the increase in potential necessary to maintain a given discharge through the constriction increased directly as the length of the constricted portion. The tube employed in securing the data shown by the curve was 2 mm. in diameter. The characteristic curves in each case are displaced upward with increasing length of tube, indicating that the increase in potential required to maintain a given current through the constriction over that for the unobstructed discharge is similar to the increase of a wire with length. If the increase in the apparent resistance is due to the crowding together of the stream-lines then confining these to increasing lengths will cause an increase in the resistance varying directly as the lengths of the constricted portions. In Fig. 17, where the pressure used was .5 mm., the curve for the discharge through the longest tube cuts across the one representing the one of less length. This is due, in my opinion, to the fact that in this region small variations of pressure produce relatively large changes in current density. A very slight error in reading the pressure would account for this.

Varying the area of the constricted portion produced smaller changes than variations of length. Variations of area in the ratio of 10 to 1 produced relatively small variation as compared with the effect of varying the lengths. Tubes whose diameters were smaller and others larger were varied in length and gave results in agreement with those represented for the 2 mm. tube. The larger tubes had the corresponding characteristic curves shifted upward less than the smaller tubes but the shift was not proportional to the inverse ratio of the areas.

In the case of tubes of small bore it is difficult to start a discharge through them. For the smallest tubes employed there is a very limited range of pressure and potential for which a discharge may be induced to start. Once established, the pressure and potential could be changed through rather wide limits. The range of potentials at my disposal limited the experiments to tubes of about 1.5 mm. diameter if tubes longer than 1 cm. were to be employed, hence these observations were not extended to tubes of smaller bore.

A series of measurements was made with a tube of 2 mm. bore and 15 mm. length, the constriction being set our laterally. The constriction tube was first set out 1.5 cm. from the center of symmetry. The shift in the characteristic curves which resulted for this offset was inappreciable for the lower pressures, .5 mm. and 1 mm., but for the highest pressure, 2 mm., the curve was shifted upward, indicating an increase in the apparent resistance. When the same tube was set out 3 cm. from the line of symmetry the characteristic curves for the two larger pressures were shifted. Finally when the offset was 4.5 cm. all of the curves were displaced; the one corresponding to the highest pressure to a much greater extent than the others.

SUMMARY.

I. Characteristic curves for air, hydrogen and carbon dioxide in the region of the critical potentials have been obtained.

II. The strength of the current through a gas at low pressures is determined in part by the size of the cathode. Empirical formulæ for the discharge from a positively electrified point cannot be extrapolated from high pressure and potential regions to low pressure conditions.

III. Introducing a constriction into the path of a discharge has the effect of increasing the apparent resistance of the circuit. The increase in the apparent resistance varies directly with the length of the constricted portion.

Physical Laboratory,
Ohio State University,
May 26, 1912.

VISCOSITY AND FLUIDITY—A SUMMARY OF RESULTS.[1] II.

EUGENE C. BINGHAM.

FLUIDITY AND OTHER PHYSICAL PROPERTIES.

Fluidity and the Boiling-Point.—Thorpe and Rodger[2] have measured the viscosities of a very large number of liquids and with great care from 0° to nearly the boiling-point of each liquid. The corresponding fluidities have been calculated by Miss Harrison and the writer. When a line is drawn through the fluidities of a given class at the boiling temperatures as extrapolated, the resulting curve in a surprising number of cases turns out to be a straight line, as shown in Figs. 12–16. In those classes of

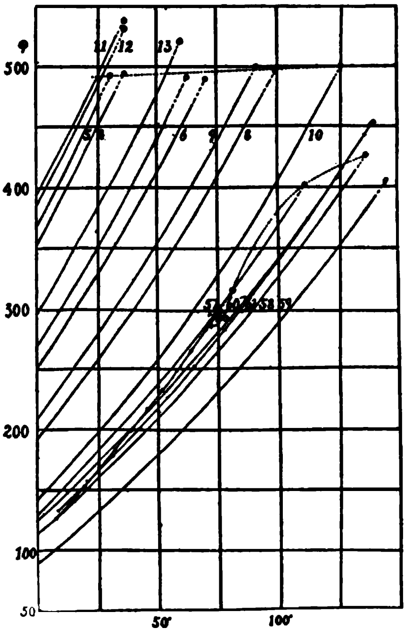

Fig. 12.

The fluidities of various hydrocarbons at different temperatures. 4, pentane; 5, isopentane; 6, hexane; 7, isohexane; 8, heptane; 10, octane; 11, trimethylethylene; 12, isoprene; 13, diallyl; 56, benzene; 57, toluene; 58, ethylbenzene; 59, (*o*)-xylene; 60, (*m*)-xylene; 61, (*p*)-xylene.

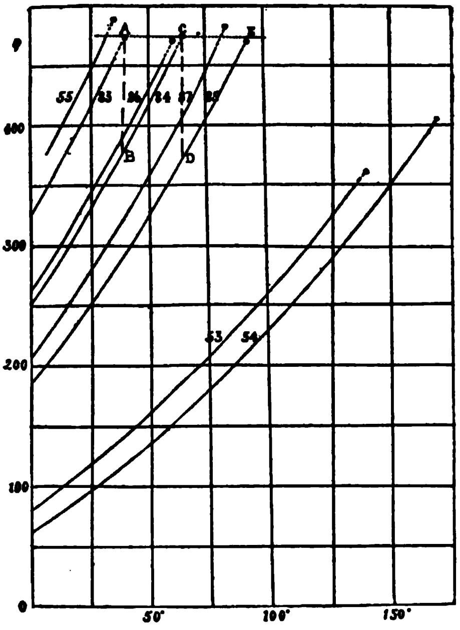

Fig. 13.

The fluidities of various ethers and acid anhydrides at different temperatures. 53, acetic anhydride; 54, propionic anhydride; 55, diethyl ether; 83, methylpropyl ether; 84, ethylpropyl ether; 85, dipropyl ether; 86, methylisobutyl ether; 87, ethylisobutyl ether.

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[1] For Part I., see the PHYSICAL REVIEW, December, 1912.

[2] Phil. Trans. (London), *185A*, 397 (1894); *Do.*, *189A*, 71 (1897).

compounds which are usually regarded as associated, as the aromatic hydrocarbons, acids, alcohols, esters, and ketones, the fluidities of a homologous series at the boiling-point still fall on a smooth curve, but this curve is no longer a straight line, as seen in Figs. 12, 16 and 17.

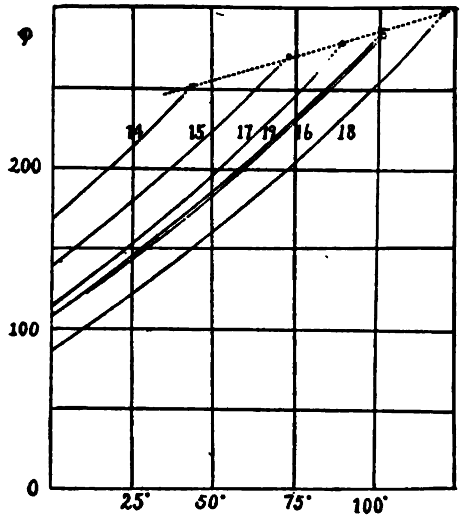

Fig. 14.

The fluidities of various alkyl iodides at different temperatures. 14, methyl iodide; 15, ethyl iodide; 16, propyl iodide; 17, isopropyl iodide; 18, isobutyl iodide; 19, allyl iodide.

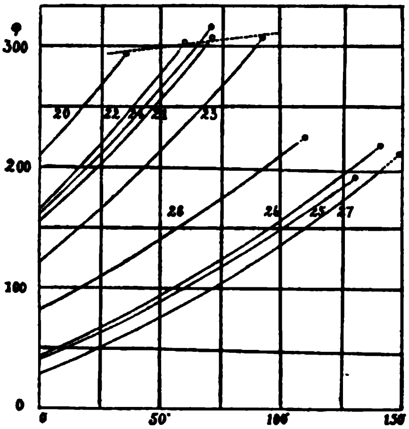

Fig. 15.

The fluidities of various bromine derivatives of the hydrocarbons at different temperatures. 20, ethyl bromide; 21, propyl bromide; [22, isopropyl bromide; 23, isobutyl bromide; 24, allyl bromide; 25, ethylene bromide; 26, propylene bromide; 27, isobutylene bromide; 28, acetylene bromide.

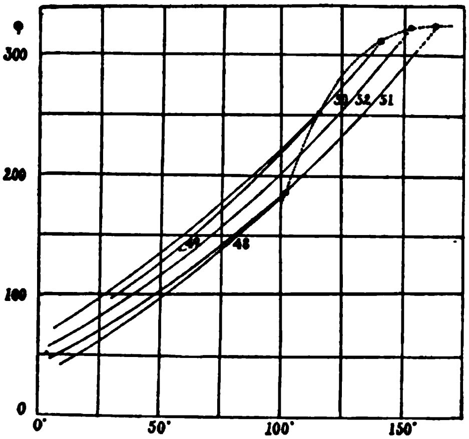

Fig. 16.

The fluidities of various organic acids at different temperatures. 48, formic acid; 49, acetic acid; 50, propionic acid; 51, butyric acid; 52, isobutyric acid.

It is quite improbable that these relationships are due only to accident. They have remained undiscovered for the most part, because of the hyperbolic nature of the viscosity curves and because the viscosities at the boiling-points are very small, making the relation uncertain. Finally the linear fluidity-boiling-point curves of the unassociated classes would themselves be hyperbolas when changed on to the viscosity basis, thus adding to the complication.

The effect of adding a methylene group to any compound is to raise the boiling-point and to lower the fluidity. The meaning of the relation which we have noted seems to be that the ratio between these effects is constant, or in Fig. 13 $AB/AC = CD/CE$.

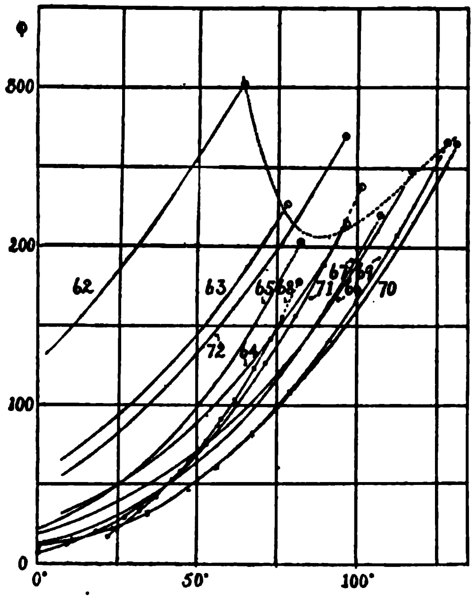

Fig. 17.

The fluidities of various alcohols at different temperatures. 62, methyl alcohol; 63, ethyl alcohol; 64, propyl alcohol; 65, isopropyl alcohol; 66, butyl alcohol; 67, isobutyl alcohol; 68, trimethylcarbinol; 69, active amyl alcohol; 70, inactive amyl alcohol; 71, dimethylethylcarbinol; 72, allyl alcohol.

Fluidity and Vapor Pressure.[1]—It is better perhaps to regard it merely as a coincidence that for the classes of aliphatic hydrocarbons and ethers the fluidity of every member of the class at its boiling-point is nearly identical. But it is quite unlikely that this coincidence is peculiar to the vapor-pressure of 760 mm., therefore it is desirable that it be shown that this relation holds for all vapor-pressures. If it be true for a given class that, at any given vapor-pressure, the fluidities are identical, it follows that the vapor-pressure-fluidity curves of the whole class should fall together into a single curve, for the fluidities of all of the different members of the class at any given vapor-pressure would form but a single point of the curve. How far this is the case is shown in Fig. 18 for the class of ethers so far as the requisite data have been obtained. The fluidities were measured by Thorpe and Rodger, the vapor-pressures were measured by the author by the method of Ramsay and Young.[1] A similar relation is found to hold for the aliphatic hydrocarbons.

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[1] Cp. ninth and thirteenth papers.

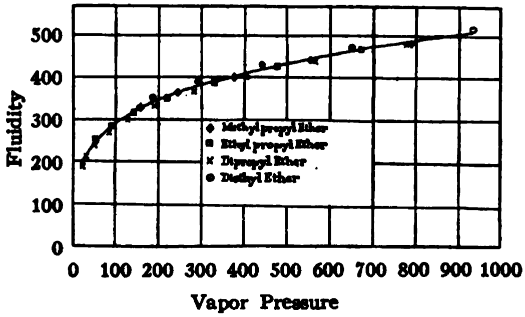

Fig. 18.

Fluidity-vapor-pressure curve of ethers.

This relation does not hold true for the other classes, but it has been pointed out that the other classes of unassociated substances also have linear boiling-point-fluidity curves and that it may be regarded as a coincidence that the fluidity of the classes of ethers and hydrocarbons are identical; hence it seemed desirable that a method be worked out for comparing the fluidity-vapor-pressure curves of these other classes.

The fluidity-vapor-pressure curves of all unassociated compounds are similar but not coincident. The fluidity of the aliphatic hydrocarbons at their boiling points is around 500 cm.g.$^{-1}$ sec. units. Taking the round number 500 as the standard, all other fluidities may be reduced to those of a substance which has a fluidity of 500 at the boiling-point—the general properties of the curve remain the same as before. Thus heptane has a fluidity at the boiling-point (98.4° C.) of 503.9. At 90° C. heptane has a fluidity of 468.3 absolute units and a vapor-pressure of 588.8 mm.

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[1] Professor Alexander Smith has kindly called my attention to the omission in my paper of the correction for the depression of the mercury in the thermometer due to the expansion of the glass under the diminished pressure. Probably the error in this case was relatively small because the pressures were not very low, but it should not have been neglected.

I may here call attention to a correction in my twelfth paper, Am. Chem. J., *46* (1911). On page 288, column three of Table XII. should read, "Volume per cent. of clay 1.015, 1.009, 1.003, 0.991, 0.977, 2.570, 2.565, 2.552, 2.542, 2.531, 4.603, 8.390."

Reducing this fluidity to standard we have 468.3 × 500 ÷ 503.9 = 464.7. By plotting the curves of a large number of substances we could obtain a composite curve which might then be regarded as the standard curve. There are obvious reasons, however, for choosing the curve of a known substance as the standard curve for reference, hence we have chosen heptane, since its fluidities and vapor-pressures are known over a considerable range and its fluidity-vapor-pressure curve is close to the composite curve.

We have elsewhere[1] given a table of vapor-pressures corresponding to the various fluidities of heptane, reduced as indicated above. By comparing the observed vapor-pressures of any substance with the values calculated by the use of this table, one can prove that the relation between fluidity and vapor-pressure in the different classes of substances is quite general, the highly associated substances alone being exceptional. The agreement is shown for a single substance, methyl isobutyrate, in the following table. The fluidity at the boiling-point (92.3°) is 397.3.

TABLE I.

The Vapor-Pressures (p) of Methyl Isobutyrate Calculated from Its Fluidities Reduced to 500 C.G.S. Units at the Boiling Point.

Temp.	ϕ Red.	p Obs. in Cm.	p Calc. in Cm.	Per Cent. Diff.	Temp.	ϕ Red.	p Obs. in Cm.	p Calc. in Cm.	Per Cent. Diff.
0	187.3	1.22	1.02	−17	50	338.3	16.20	16.20	0
10	214.7	2.24	1.98	−12	60	372.8	24.38	24.44	0
20	243.3	3.89	3.55	− 6	70	409.6	35.53	35.93	1
30	273.0	6.54	6.17	− 6	80	449.2	50.50	51.76	2
40	304.7	10.47	10.29	− 2	90	490.9	70.70	71.60	1

There is general agreement when the vapor-pressures are not too small and the compound not highly associated. From a study of a considerable number of compounds, we have found that for vapor pressures of more than ten centimeters, the calculated values differ from the observed by only a little over three per cent.

Since this relation exists, it is possible to calculate unknown vapor-pressures of unassociated compounds, when the boiling-point of the compound is known. Conversely, if the vapor-pressures of a substance and its fluidity at the boiling point are known, it becomes possible to construct the fluidity curve. Thus suppose that the fluidity of ethyl-propyl ether at 20° C. is desired, it being known that the fluidity of this substance at the ordinary boiling-point is 479.9 and that its vapor-pressure at 20° is 14.26 cm. The corresponding fluidity interpolated from the table for heptane, the standard substance, is 328.0; but these fluidities are reduced, so that they must be corrected back. Thus $328.0 \times 479.9 \div 500 = 314.8$, which is practically identical with the observed value of 314.9.

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[1] Am. Chem. J., 47, 187 (1912).

A priori, one would hardly expect to find any relation at all existing between the fluidity of a liquid and the pressure of the vapor above it. Yet it is proved that a relation does exist of a very simple character. This is doubtless the result of both of these properties being similarly dependent upon another property of the liquid, which is probably its molecular volume.

As has been already hinted, the calculated values of the vapor-pressure differ very widely from the observed values in the case of highly associated liquids. A discussion of these interesting cases would carry us beyond the limits of this paper.

Finally it is important to point out that the above relation could hardly have been discovered on the assumption that viscosities are additive. The viscosity-vapor-pressure curves must be hyperbolic in character, although like the viscosity curves some of them might appear to be linear, as would indeed those of the ethers and aliphatic hydrocarbons.

Fluidity and Chemical Composition and Constitution.[1]—As soon as any considerable amount of viscosity data had been collected, it became evident to the early workers in this field that there is a relation between viscosity and chemical composition. At the hands of Pribram and Handl[2] the number of empirical relations became quite extended, although their experimental material lacked great precision. This seemed to Thorpe and Rodger to indicate that results of great importance might be obtained if very accurate data were available for a sufficient variety of substances of unquestioned purity. Consequently these investigators began the accumulation of viscosity data considering carefully every detail of the purification of material, precision in measurement, and accuracy in the calculation of results. Their monumental work[3] finally embraced some 85 substances. To bring out new relationships, it became necessary to find the proper basis of comparison. For this purpose Thorpe and Rodger studied their data from every point of view which occurred to them. They compared viscosity coefficients, molecular viscosities, and molecular viscosity work, at the boiling-point, critical temperatures, temperatures corresponding to a given slope of the viscosity curves, and viscosities corresponding to a given slope of the viscosity curves. They confirmed the conclusion that different atoms and groupings have, in general, an additive effect upon the viscosity, but as a whole their results are disappointing, for the value of an atom or grouping is sometimes arbitrarily assumed to vary from class to class. Thus oxygen has one value for ethers, another for alcohols, and still another for acids. But even with these assumptions, a considerable number of substances fail to give satisfactory results upon any basis of comparison. They obtained best results in comparisons at equal slope, although their argument to prove that this should be the case is scarcely conclusive.

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[1] Cp. sixth paper.

[2] Sitzungsber. der math.-naturwiss. Klasse der Kais. Akad. d. Wiss. Wien, *78*, II., 113 (1878); *Do.*, *80*, II., 17 (1879); *Do.*, *84*, II. 717, (1881).

[3] *Loc. cit.*

If the fluidities are additive, and not viscosities, it seems probable that the true basis of comparison should be worked out in comparing fluidities and not viscosities as has been done uniformly heretofore. Inspection of Figs. 12–16 shows that in homologous series, the fluidity-temperature curves form a family of approximately straight and parallel lines. Were they actually linear and parallel, their slopes and intercepts would of course completely define them, and give us therefore a true basis of comparison. Hence it seems certain that we should either compare fluidities at a given temperature or temperatures of equal fluidity together with the slopes of the fluidity curves. Since the curves in a given class are nearly parallel, it is evident that the slope is characteristic of the class to which a compound belongs. The molecular weights of the compounds determine the values of the intercepts. But the fluidity-temperature curves are neither perfectly linear nor parallel. The lack of linearity may be ascribed to association, using that term broadly. It should be recalled at this point that metallic mercury gives a fluidity curve which is quite linear and apparently cuts the temperature axis at absolute zero. It is probable that all fluidity curves would unite at absolute zero, so that exact parallelism is not to be expected even within a given class.

There are several reasons for comparing the temperatures of equal fluidity rather than fluidities at a given temperature.

1. The slopes are more nearly equal when the fluidities are the same.

2. At a given fluidity the members of at least two classes have the same vapor-pressure, and experience has proved that substances are comparable at temperatures corresponding to equal vapor-pressure.

3. The fluidity curves for associated compounds depart widely from linearity at low fluidities, but approach linearity at high fluidities, as do the curves of other compounds.

4. A yet more cogent reason grows out of the fact, mentioned above, that exact parallelism in the curves of a given class is not to be expected. A methylene group added to pentane, for example, lowers the fluidity a

certain amount, but a methylene group added to decane cannot lower the fluidity by the same amount, since as the molecular weight increases the fluidity must approach zero asymptotically. Otherwise, it would require no very high molecular weight to give a negative fluidity, which is inconceivable.

The fluidity of 200 was chosen for the basis of comparison since most substances could then be included. The temperatures and slopes of several unassociated compounds corresponding to the fluidity of 200 are given in Table II.

Table II.

Absolute Temperatures and Slopes of Non-Associated Substances Corresponding to a Fluidity of 200 C.G.S. Units.

Substance.	Abs. Temp. ($\phi = 200$) Observed.	Diff. CH_2.	Slope at ($\phi = 200$).	Abs. Temp. ($\phi = 200$) Calc.	Per Cent. Diff.
Hexane	(255.1)[1]	(21.0)	(2.88)	254.6	0.2
Heptane	276.1	23.0		277.3	0.4
Octane	299.1		2.44	300.0	0.3
Isohexane	(249.0)	(20.2)	(2.79)	247.0	0.8
Isoheptane	269.2		2.68	269.7	0.2
Methyl iodide	290.2	19.0	1.92	287.4	1.0
Ethyl iodide	309.2	23.5	1.80	310.1	0.3
Propyl iodide	332.7		1.82	332.8	0.0
Isopropyl iodide	324.5	21.0	1.92	325.2	0.2
Isobutyl iodide	345.5		1.86	347.9	0.7
Allyl iodide	330.5		1.82	328.8	0.5
Ethyl bromide	268.7	27.9	2.22	273.5	1.8
Propyl bromide	296.6		2.08	296.2	0.1
Isopropyl bromide	289.4	25.6	2.22	273.5	1.8
Isobutyl bromide	315.0		2.08	311.3	1.1
Ethyl propyl ether	(255.0)	(24.0)	(2.70)	256.1	0.5
Dipropyl ether	279.0		2.62	278.8	0.1
Methylissbutyl ether	(251.1)	(19.0)	(2.75)	248.5	1.0
Ethylisobutyl ether	270.1		2.68	271.2	0.4

Table III.

The Value of the Iso-Grouping.

Substance.	Temp. Obs. Normal Grouping.	Temp. Obs. Iso-grouping.	Diff.
Hexane	255.1	249.0	6.1
Heptane	276.1	269.2	6.9
Propyl iodide	332.7	324.5	8.2
Propyl bromide	296.6	289.4	7.2
Propyl chloride	261.5	255.2	6.3
Butyric acid	381.6	371.6	10.0
Methyl butyrate	304.2	295.8	8.4

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[1] Values in parentheses are extrapolated.

Column 3 of Table II. shows that the value of a methylene group varies around a mean value of 22.7. The effect of an iso-grouping is to decrease the fluidity of a compound some 7.6 units, as shown in Table III.

The value of the hydrogen atom is calculated as follows:

TABLE IV.

The Value of the Hydrogen Atom.

Substance.	Temp. Obs.	$n \times CH_2$ Calc.	Diff.
Hexane	255.1	136.2	118.9
Heptane	276.1	158.9	117.2
Octane	299.1	181.6	117.5
Isohexane	249.0	128.6	120.4
Isoheptane	269.2	151.3	117.9

The mean value for H_2 is 118.4. Hydrogen has therefore a value of 59.2 and carbon of − 95.7.

The value of the "double bond" in allyl compounds is obtained from Table V.

TABLE V.

The Value of a Double Bond.

Substance.	Temp. Obs. Normal Propyl.	Temp. Obs. Allyl.	Diff.
Iodide	332.7	330.5	2.2
Bromide	296.6	292.2	4.4
Chloride	261.5	256.0	5.5

To raise the fluidity of an allyl compound to 200 it is therefore only necessary to raise it to a temperature which is some four degrees lower than is required for the corresponding normal compound, containing two more hydrogen atoms. Thus the "double bond" has a value of 114.4, the absence of the hydrogen atoms being nearly compensated for by the "condition of unsaturation."

Assuming that the ethers are unassociated, we may obtain the value of the oxygen atom.

TABLE VI.

The Value of the Oxygen Atom.

Substance.	Temp. Obs.	C_nH_{2n+2}.	Oxygen.
Ethylpropyl ether	254.9	231.9	23.0
Dipropyl ether	279.0	254.6	24.4
Methylisobutyl ether	251.4	224.3	27.1
Ethylisobutyl ether	270.3	247.0	23.3

This gives an average value for oxygen of 24.2. The values of sulphur and the halogens are obtained similarly but need not to be given here.

From these values, the temperatures corresponding to a fluidity of 200 may be calculated. Some of these calculated values are given in the fifth column of Table II. A comparison between the observed and calculated values for 35 substances gives an average percentage difference of less than 0.8 per cent. The constants found at a fluidity of 300 are entirely consistent with these found at a fluidity of 200. The association is smaller at the higher temperatures as would be expected, and thus the temperature coefficients of association have been calculated,[1] but need not be given here.

In comparing viscosities, it has been customary to give an important special value to a ring grouping, but we have found that in comparing fluidities the value of the ring grouping is of small importance and may perhaps be neglected in a first approximation; for it would diminish the immediate usefulness of the method if a special value had to be assigned for each constitutive difference.

From the constants at different fluidities it is evidently possible to construct the entire fluidity curve of any unassociated substance.

Fluidity and Association.[2]—In the above calculations of constants it has been assumed that the compounds chosen are non-associated. This is probably not entirely warranted, but they must be associated to nearly the same extent since the agreement between the calculated and observed

TABLE VII.

Absolute Temperatures and Slopes of Some Associated Compounds Corresponding to a Fluidity of 200 C.G.S. Units.

Substance.	Abs. Temp. for ($\phi = 200$) Obs.	Abs. Temp. for ($\phi = 200$) Calc.	Slope for ($\phi = 200$).	Association.
Water	328.9	142.6	3.04	2.31
Formic acid	(380.2)	185.5	(2.18)	2.05
Acetic acid	363.8	208.2	2.06	1.77
Propionic acid	362.0	230.9	1.92	1.57
Butyric acid	381.6	253.6	1.92	1.57
Isobutyric acid	371.6	246.0	2.00	1.51
Methyl alcohol	305.2	165.3	2.78	1.84
Ethyl alcohol	343.4	188.0	3.24	1.83
Propyl alcohol	365.6	210.7	3.76	1.74
Butyl alcohol	377.0	233.4	3.44	1.62
Ethyl formate	273.8	230.7	2.40	1.19
Ethyl acetate	284.0	253.4	2.50	1.12
Ethyl propionate	298.1	275.1	2.44	1.08

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[1] Cp. ninth paper.

[2] Cp. sixth and ninth papers.

values is very good in most cases, and it is a general belief that some of these compounds are indeed unassociated. If we now consider the manifestly associated compounds, the method of comparison outlined above gives us a means of calculating the magnitude of the association. For we have seen that the absolute temperature required to produce the given fluidity is directly proportional to the molecular weight. The assumption that this holds true as molecules unite with each other is rather a deduction than an assumption. Its contradiction would involve

TABLE VIII.

A Comparison of the Values of Association as Determined by Different Investigators.

Substance.	R. & S.[1] 16–46°.	R. & S. Corr. by Traube.	Traube.[2] 15°.	Longinescu.[3]	B.&H.[4], Temp. of ($\phi = 200$).
Water	3.55 1.64	1.79	3.06	4.67	2.31
Ethylene chloride	——	——	1.46	1.00	1.21
Dimethyl ketone	1.26	1.18	1.53	1.60	1.23
Diethyl ketone	——	——	——	1.25	1.16
Methyl propyl ketone	1.11	1.10	1.43	1.25	1.14
Formic acid	3.61	2.41	1.80	1.80	2.05
Acetic acid	3.62 2.13	2.32	1.56	1.75	1.77
Propionic acid	1.77	1.45	1.46	1.55	1.57
Butyric acid	1.58	1.35	1.39	1.36	1.51
Isobutyric acid	1.45	1.28	1.31	——	1.51
Benzene	1.01	1.05	1.18	——	1.17+
Toluene	0.94	1.01	1.08	——	1.08+
Methyl alcohol	3.43 2.32	2.53	1.79	3.17	1.84
Ethyl alcohol	2.74 1.65	1.80	1.67	2.11	1.83
Propyl alcohol	2.25	1.70	1.66	1.67	1.74
Isopropyl alcohol	2.86	2.00	1.53	——	1.75
Butyl alcohol	1.94	——	——	1.47	1.62
Isobutyl alcohol	1.95	1.53	1.54	——	1.66
Active amyl alcohol	1.97	1.54	1.53	——	1.54
Allyl alcohol	1.88	1.50	1.55	1.80	1.69
Methyl formate	1.06	1.07	(1.60)	1.12	1.25
Ethyl formate	1.07	1.08	1.39_0°	——	1.19
Methyl acetate	1.00	1.04	1.48_0°	1.09	1.17
Ethyl acetate	0.99	1.04	1.25	1.00	1.12
Propyl acetate	0.92	1.00	1.31	1.00	1.11
Ethyl propionate	0.92	1.00	1.27	0.94	1.08
Methyl butyrate	0.92	1.00	1.30_0°	1.00	1.10

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[1] Ramsay and Shields, Zeitschr. f. physik. Chem., *12*, 464 (1893); Do., *15*, 115 (1894).

[2] Traube, Ber. d. deutsch. chem. Gesell., *30*, 273 (1897).

[3] Journ. Chim. Phys., *1*, 289 (1903).

[4] Bingham and Harrison, *loc. cit.*

an assumption for which we do not at present see any justification. The average molecular weight or association of a liquid is therefore measured by the ratio of the observed absolute temperature to the calculated absolute temperature at the fluidity chosen as the basis of comparison.

The observed and calculated absolute temperatures corresponding to the fluidity of 200 and the calculated association of some supposedly associated compounds are given in Table VII. The slopes of these compounds are also given in the fourth column.

In Table VIII. is given a comparison of the association as calculated by the fluidity method with that obtained by other methods.

So far as one is able to judge, the result seems to be all that could be desired. We believe that the fluidity method is freer from assumptions to which question may be raised, than the other methods, and we hope that it may prove of use in calculating association. However that may be, it seems certain that the above concordance ought to add very greatly to the weight of the reasoning of this paper as a whole; for it is especially to be noted that similar results have not and probably cannot readily be obtained from a comparison of viscosity data as such. Lest this statement appear unsupported, we shall attempt to prove it as follows. Let *AB* and *A'B'* in Fig. 19 represent two fluidity curves, parallel to each other and therefore presumably representing members of the same class of substances, and let a third fluidity curve *CD* be at an angle to the other two to represent a substance in another class. Since we have elected to compare absolute temperatures at a fluidity of 200, this amounts to comparing the intercepts of the curves on the line *AD*, whose equation is $\varphi = 200$. The corresponding viscosity curves obtained by taking the reciprocal values of the above fluidities and multiplying by 10,000 are represented by *ab*, *a'b'*, and *cd*; and *ad* is of course the reciprocal of the line *AD*. But the point *a* and the point where the curve *a'b'* crosses *ad* are points of equal slope on the viscosity curves, hence we conclude that within a given class it makes no difference whether we compare temperatures corresponding to a given fluidity or temperatures corresponding to a given slope on the viscosity curves. The latter is exactly the method of comparison which Thorpe and Rodger found very advantageous. But between different classes they found difficulties.

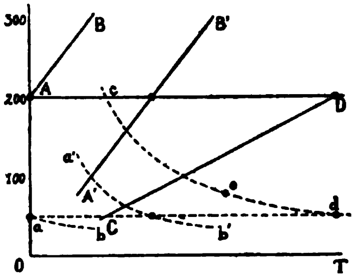

Fig. 19.

Diagram illustrating the relationship between viscosity and fluidity-temperature curves.

This is now easily explained, for *d* is the reciprocal of the point *D* which we believe should have been used; but they selected the point *e*, which has the same slope as the point *a*, and for this choice we think that there is not adequate reason. However it is advantageous for our point of view that they unwittingly proved that a comparison of temperatures corresponding to a given fluidity gives the best results even if they did not make it perfectly general.

Fluidity and Hydration.[1]—It is but a step from the consideration of the complexes which exist in homogeneous liquids to that of the loose combinations in solution, which are most common in aqueous solutions and are called hydrates. An increase in the average molecular weight usually, but not necessarily invariably, takes place and this increase should under favorable circumstances be capable of detection and measurement by means of the fluidity method.

The queston of hydration was in the minds of the earliest investigators in the field of viscosity, hence it is useless to hope to add anything of moment to their work without an advance in the underlying theory. Our study of the problem has led to the belief that there are three misconceptions prevalent, which have stood in the way of its solution.

1. It has invariably been assumed that viscosities and not fluidities are additive, causing the simplest mixtures to appear abnormal, as already discussed.

2. By common consent the concentrations of solutions have been reckoned on the weight-percentage or molecular percentage basis. But we have already pointed out that concentrations must be reckoned in terms of volume percentage, since it is the total space occupied by a liquid of given fluidity which is of importance, and not its density or molecular volume.

3. It has been assumed that a maximum, or minimum, in a viscosity curve had peculiar significance and that it coincided with the composition of the hydrate formed. Neither part of this assumption appears to be true, as we shall now attempt to prove by the consideration of the following simple cases.

Case I. The fluidity-temperature curves of two similar substances are represented by *A* and *B* in Fig. 20*a*. Let it be supposed that a combination takes place between the two substances on mixing, which is a maximum in the 50 per cent. mixture; so that for this mixture the fluidity is lowered from the curve *C* (dotted), which would be expected were there no combination, to the curve .5*B*. Plotting these points, the fluidity-concentration curves of these same substances for the various temperatures t_1, t_2, t_3, etc., may be interpolated as shown in Fig. 20*b*. The curves $.25B$ and $.75B$ may be added to Fig. 20*a* for the 25 per cent. and 75 per cent. mixtures respectively. In this case there is a well-defined minimum in the fluidity-concentration curves in the 50 per cent. mixture corresponding to the composition of the hydrate.

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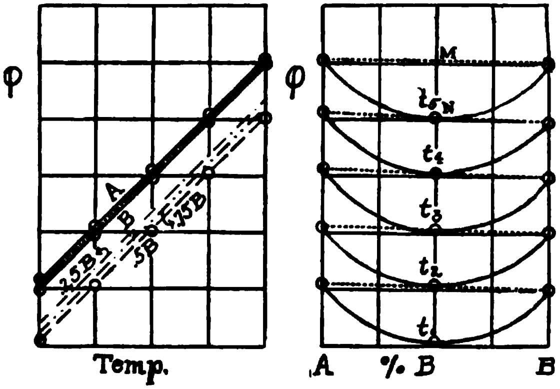
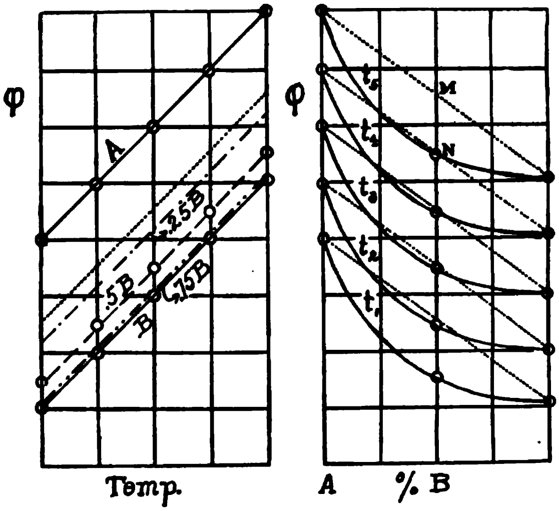
[1] Cp. fourteenth paper.



a *b*

Fig. 20.

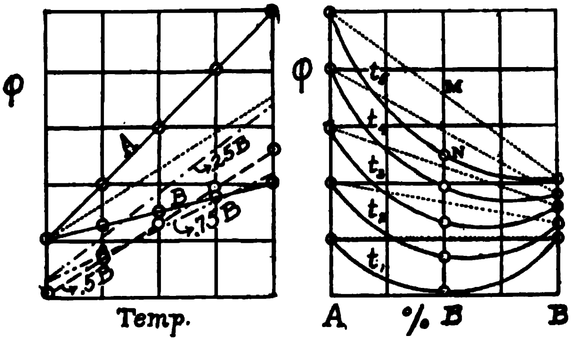
Case II. In Fig. 21*a*, the same conventions are employed, the curves *A* and *B* being still parallel and the maximum lowering of the fluidity being in the 50 per cent. mixture and the same in amount as in Case I. The only difference is that the two curves *A* and *B* are sufficiently



a *b*

Fig. 21.

distant from each other so that the curve .5*B* falls above the curve *B*. The result is that the fluidity-concentration curves, shown in Fig. 21*b*, no longer give a minimum, although by assumption the hydration is the same as before both in relative composition and in amount. However it is clear that the deviation of the fluidity-concentration curves from the linear curves, which would be expected were there no combination, and as measured vertically *MN*, is the same as in the preceding case.



a *b*

Fig. 22.

Case III. In Fig. 22*a* and *b* is depicted the case where the fluidity-temperature curves of the components are no longer parallel so that the result is practically a combination of cases I. and II. At low temperatures there is a good minimum in the fluidity-concentration curves, but it gradually shifts to the right as the temperature is raised, until at the highest temperatures there is no minimum at all. It is manifestly erroneous to assume that the composition of the hydrate changes on this account. On the other hand the deviation from the expected linear curves as measured vertically is everywhere the same as in the simpler cases.

In practice the hydration will generally be less at the higher temperatures so that the deviation should grow smaller as the temperature is raised, but the cases given are perhaps sufficient to show that the *deviation* of the observed fluidity-concentration curve from the linear curve, which would be expected were there no combination between the components of the solution, can alone furnish trustworthy information.

If the above reasoning is correct it seems possible that the way may now be open to use the property of fluidity to throw important light upon the question of hydration and other similar combination, in solution. For example, certain puzzling contradictions may be immediately disposed of. Thorpe and Rodger[1] measured the viscosities of mixtures of methyl iodide and carbon disulphide and of carbon tetrachloride and benzene at several temperatures. In each case they found sagging in the viscosity curves, which they accounted for by assuming that the components of the mixtures united with each other to some extent. But they themselves noted that when methyl iodide and carbon disulphide are mixed, quite a marked expansion takes place, with the absorption of heat. With carbon tetrachloride and benzene there is no expansion but perhaps a very slight contraction. The evidence is therefore puzzling if not absolutely contradictory. When the fluidities of these pairs are plotted against volume concentrations, on the other hand, we find that the curves are slightly bent upward in the case of methyl iodide and carbon disulphide, and slightly sagged downward in the case of carbon tetrachloride and benzene. Moreover the sagging in the latter case is greatest at the lower temperatures where the conditions are most favorable to combination. This is shown by the continuous curves in Fig. 23. The curves indicated by dashes represent the result of plotting fluidities against the weight concentrations as given by Thorpe and Rodger. The result is contradictory, because an increase in fluidity would not be expected to accompany even a very slight contraction. Moreover the departure from the linear (dotted) curve, which we should expect if there were no combination, should not increase as the temperature is raised as is apparently the case. For an increase in fluidity in the mixture would seem to be due to one component causing

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[1] J. Chem. Soc., *71*, 366 (1897).

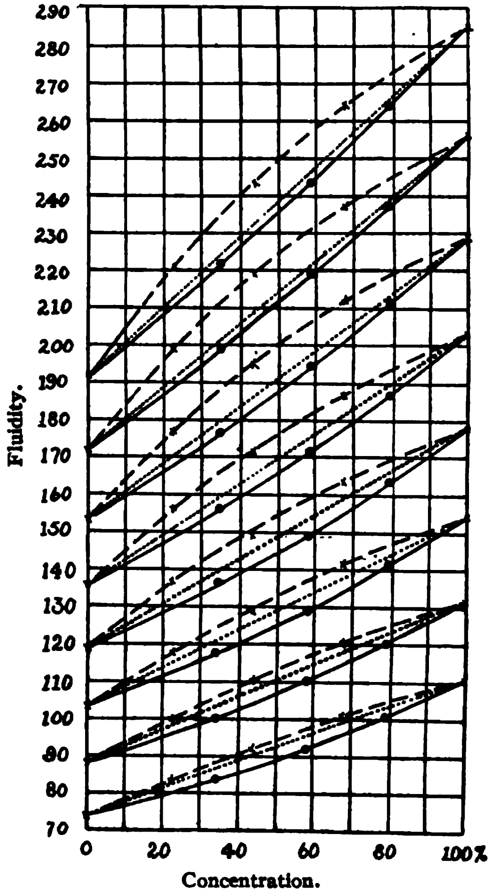

Fig. 23.

The fluidities of mixtures of carbon tetrachloride and benzene plotted against volume concentrations (continuous lines) and weight concentrations (dashes) as compared with the linear curves (dotted).

the destruction of the association of the other, and this effect would naturally be greatest at low temperatures where the association is greatest. Fortunately these contradictions are cleared up as we have seen by plotting fluidities against volume-concentrations, for which there is ample justification.

The viscosity curves for chloroform and ether as measured by Thorpe and Rodger[1] show a point of inflection. To explain this they were forced to assume that at some concentrations one of the liquids destroyed the association of the other, while at other concentrations the two components united with each other to form complexes. On its face this assumption seems somewhat improbable and this improbability is further increased by other facts obtained by the same experimenters. For in the case of chloroform and ether there is "a considerable evolution of heat and a notable contraction," the greatest contraction taking place in the mixture containing 40.14 weight percentage of ether. But the point of inflection in some of the viscosity curves occurs in this mixture, the exact point varying considerably as the temperature is raised. So viscosity tells a tale which is different from that of any of the other properties, and this fact tells heavily against viscosity comparisons as an aid in investigation. When rightly viewed the testimony of all of the properties must agree, since all of the properties of a substance must be the outgrowths of its chemical composition and constitution together with the physical conditions imposed upon it. In the case under discussion, if we plot fluidities against concentrations, we obtain curves for the different temperatures, which no longer show a point of inflection, but are considerably sagged, the greatest deviation from the linear curve occurring in the mixture where the greatest contraction and presumably the greatest heat evolution also occur. Thus the different kinds of evidence agree in indicating that there is some kind of a combination between the components of the mixture.

Were the components of the mixture non-associated, it would not be difficult to calculate the average molecular weight of the mixture by the fluidity method. But substances which form the feeble combinations on mixing which we have under consideration are usually associated and it is quite likely that this association is altered in the mixture, so that the result is considerably complicated thereby. This change in the association of the components may indeed shift the point of greatest deviation from the linear curve, so that this point will not exactly correspond to the composition of the hydrate. The familiar problem of hydrates in aqueous solution is therefore a complicated one, which we are not yet in a position to handle very satisfactorily, although it does seem possible to clear the way for a solution. To fix our attention upon this problem more closely, let us consider the case of ethyl alcohol and water. The former at the fluidity of 200 requires an absolute temperature of 343.6 and has an average molecular weight of 1.83 times the empirical formula. The latter requires an absolute temperature of 328.9 and the association is 2.31. With ethyl alcohol and water there is a definite minimum in the fluidity curves, which at some temperatures occurs in the mixture corresponding to the composition $C_2H_6O.3H_2O$, *i. e.*, about 52 volume per cent., hence earlier workers on the viscosity of these mixtures concluded that this hydrate is actually formed. The greatest deviation from the linear curves, which would be expected were there no combination, occurs in the mixture which corresponds to the hydrate $C_2H_6O.4H_2O$, or in the mixture containing 44.79 volume per cent. of alcohol. It has been objected that the maximum in viscosity (or the minimum in fluidity) migrates as the temperature is changed, but so far as we have been able to observe, this is not true of the maximum deviation from the linear curves.

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[1] Loc. cit.

On mixing, it seems altogether likely that the diminished frequency of collision between molecules of the same kind will lessen the tendency toward association. Whether this action occurs and whether it obeys the law of mass action can probably be determined and will be discussed later. But for the moment let us consider this breaking down of association as negligible.

The temperature corresponding to a fluidity of 200 in the 52.08 volume per cent. mixture which we should expect were there no combination, would be $.4479 \times 343.6 + .5521 \times 328.9 = 335.5$. On the other hand, from the constants already given, the temperature required to give the pure hydrate $C_2H_6O.4H_2O$ a fluidity of 200 would be $14 \times 59.2 + 5 \times 24.2 - 2 \times 95.7 = 758.4$. But the observed absolute temperature at which the 44.79 volume per cent. mixture has a fluidity of 200 is 362.3. Hence if we let x represent the fraction of the volume of the mixture combined as $C_2H_6O.4H_2O$, the rest remaining unchanged, we have $335.5(1 - x) + 758.4x = 362.3$ or $x = 0.06337$. But this value has no great significance for the following reasons.

1. It has been assumed that the hydrate has the simplest possible formula, but it may be a polymer of the value given. This would make the percentage of hydration lower than that given.

2. The association of the components may be less after mixing. This would raise the percentage.

3. In mixtures other than the one considered, the percentage of hydration would certainly be different, if the law of mass action applies.

It may now be shown that the hydrates in solution are qualitatively at least subject to the law of mass action. The mixture of alcohol and water in the ratio corresponding to $C_2H_6O.3H_2O$ has at 25° C. a fluidity of 42.5 and a mixture of acetic acid and water corresponding to $C_2H_4O_2.H_2O$ has at 25° C. almost exactly the same fluidity; so that were there no chemical change, these two mixtures might be combined with each other in any proportion without altering the fluidity. But we have noticed that when acetic acid and ethyl alcohol are mixed there is a drop in the fluidity and we may remark in passing that this cannot be due to the formation of ethyl acetate, because this substance has a high fluidity—236.3 at 25°. However, the dilution of the mixture of alcohol and water by adding acetic acid will cause a marked decrease in the amount of combination between the alcohol and water, because according to the law of mass action four molecules of water must meet a molecule of alcohol in order to form a hydrated molecule, and the presence of the acetic acid lowers the concentration of the water. As a matter of fact, the fluidity of the mixture is raised by 10 per cent. above the expected 42.5 or less, if this law had been inoperative. This is not the only possible explanation of the increase in fluidity in this case. It may be due to the dissociation of the acetic acid on dilution.

The same point may have light thrown upon it from another source. The values of μ_∞ measure the sums of the velocities of the cations and anions $u + v$ in ionic migration. In mixtures of alcohol and water these values suffer a gradual falling off as the percentage of alcohol is increased, but there is no minimum to correspond with the minimum in the fluidity of the mixtures. The reason seems to be that in the mixture with the smallest fluidity, the components of that mixture are united with each other to their very fullest extent, hence the "atmosphere" around the ion is reduced to the smallest amount and the migration velocity is therefore relatively large. If the atmosphere about the ions were uniform in size, conductivity would be directly proportional to the fluidity of the solution.

Since writing the above, an instance has occurred to us where two apparently slightly associated substances unite with each other with considerable vigor to form complex molecules, which seems to give us the simplest possible case for determining whether the formation of solvates follows the Law of Mass Action. Thorpe and Rodger[1] in studying mixtures of ether and chloroform, noted that there is a considerable heat evolution and contraction on mixing. So far as we can learn from their measurements, the maximum contraction occurs in a mixture containing less than 40 per cent. of ether by weight and perhaps less than 39 per cent. The maximum deviation of the fluidity-volume-concentration curve from the linear curve occurs in the 58 volume per cent. mixture ± 3 per cent. This corresponds to about 39.8 per cent. by weight. A mixture corresponding to the composition $C_4H_{10}O.CHCl_3$ contains 38.30 per cent. of ether by weight. Guthrie[1] has noted that the greatest heat evolution occurs in the mixture of equimolecular proportions and that the vapor-pressure curve of the mixtures gives confirmatory evidence of the formation of a compound with the formula corresponding to $C_4H_{10}O.CHCl_3$.

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[1] *Loc. cit.*

Since the evidence afforded by the different properties of mixtures of ether and chloroform was formerly conflicting, it is a source of great satisfaction and confidence that so many lines of evidence are absolutely in agreement in pointing to the formation of a definite compound. The reluctance which chemists have toward recognizing this class of compounds has no doubt been due partly to the fact that the compounds themselves cannot usually be isolated, and furthermore the mixture does not act like a chemical compound in that it maintains the properties of all of the original components. This however is doubtless due to there being a dynamic equilibrium between the compound and the original components. Nevertheless recognition will not be given to this large class of compounds, until their presence and properties can be quantitatively established. Fortunately in the case of ether and chloroform, this information seems available.

In the mixture containing 56.26 per cent. by volume of ether corresponding to one molecule of ether to one molecule of chloroform we may calculate the percentage combined as follows. From the atomic constants we find that the compound $C_4H_{10}O.CHCl_3$ should have a fluidity of 200 at the absolute temperature of 538.6°. But actually we find that a mixture of this composition has a fluidity of 200 at 282.9° absolute. Pure ether and pure chloroform have fluidities of 200 at 216.5° and 305.3° respectively, so that if the mixture were wholly uncombined, the absolute temperature necessary for a fluidity of 200 would be $216.5 \times 0.5626 + 305.6 \times 0.4374 = 255.4$. Letting x represent the fraction of the volume of the mixture which is combined, we obtain the equation

$$538.6x + 255.4(1 - x) = 282.9,$$

$x = 0.0971$. Since less than ten per cent. of the volume of the mixture is actually in combination, it seems reasonable to assume that a dynamic equilibrium exists between the combined and the uncombined portions. If the Mass Law holds, we have

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[1] Phil. Mag., *18* (5), 495 (1884).

$$\frac{[C_4H_{10}O]\,[CHCl_3]}{[C_4H_{10}O.CHCl_3]} = K,$$

where the concentrations are no longer volume but molecular concentrations.

In the above equimolecular mixture, if we let y represent the number of cubic centimeters of ether which are combined in every one hundred cubic centimeters of mixture, the volume of the chloroform combined will be $0.7366 \times 193.4y/74.08 \times 1.526$ where the specific gravities of ether and chloroform are taken as 0.7366 and 1.526 respectively and their molecular weights 74.08 and 193.4. Since the sum of the two volumes is 9.71 we find that the volume of the ether combined is 4.30 and of the chloroform 5.41 c.c. Substituting the molecular concentrations in the above formula we obtain for the value of the constant K

$$K = \frac{\left[\frac{(56.26 - 4.30)0.7366}{74.08}\right]\left[\frac{(43.74 - 5.41)1.526}{193.4}\right]}{\left[\frac{9.71 \times 1.1016}{277.52}\right]} = 4.0633.$$

With thsi value of K it becomes possible to calculate the absolute temperature corresponding to a fluidity of 200 for any mixture on the assumption that only one compound is formed and that the Law of Mass Action is obeyed. Thus for a mixture containing 28.21 per cent. by volume of ether, we have, if z is taken to represent the fraction of the volume of the ether which is combined,

$$\frac{\left[\frac{28.21 \times 0.7366}{74.08}(1 - z)\right]\left[\frac{71.79 \times 1.526}{193.4} - \frac{28.21 \times 0.7366}{74.08} \cdot z\right]}{\left[\frac{28.21 \times 0.7366}{74.08} \cdot z\right]} = 4.0633,$$

$z = 0.1162$. The volume of ether combined in 100 c.c. is therefore $28.21 \times 0.1162 = 3.278$ and the volume of the chloroform is $3.27 \times 0.7366 \times 193.4 \div 74.08 \times 1.526 = 4.131$ and the total volume in combination is 7.409 c.c. The volume of the uncombined ether is 24.94 c.c., and of chloroform 68.52 c.c. Hence the calculated absolute temperature corresponding to a fluidity of 200 is $0.2493 \times 216.5 + 0.6766 \times 305.3 + 0.07409 \times 538.6 = 300.4°$. The value read from the curve is 297.4°. Similar remarkable agreement is obtained for other mixtures whose fluidity was measured by Thorpe and Rodger.

We are not quite justified in stating that this proves that the mass law applies to all of those loose combinations which are believed to exist in solution, but it points the way toward a solution of this interesting question. In the case of highly associated liquids the problem is complicated by the breaking down of this association upon dilution. Other constants are thereby introduced, but it is believed that this difficulty may be overcome. Finally it may be pointed out that this method seems suited for the solution of various problems which have vexed the organic chemists such as the mechanism of esterification. Our experiments seem to indicate that there is a loose combination between the alcohol and acid before the ester is actually formed.

Fluidity Formulas.[1]—Since fluidities are normally additive, the formula expressing the change of fluidity with the temperature in its simplest form should be $\varphi = AT$ or $\varphi = \alpha + \beta T$. This latter form may be deduced from the empirical viscosity formula of Meyer and Rosencranz,[2] $\eta = c/(1 + at)$. But most liquids do not expand equally on heating and it may be that almost all liquids are associated to some extent. However that may be, mercury is the only liquid known which has a linear fluidity curve within the limits of experimental error, over a large range of temperature. The curves of other substances must not cut the temperature axis since we cannot conceive of negative fluidities, yet they must all approach the temperature axis at absolute zero. Therefore we conclude that the fluidity curves must be asymptotic to that axis. At high temperatures the association is broken down and the fluidity curves become more and more nearly linear as we observe to be the case in Fig. 17. A general fluidity equation must therefore incorporate the following properties.

1. It must reduce to a linear equation for the ideal substance.
2. One branch of the curve in its general form must be asymptotic to the temperature axis.
3. At high temperatures the curve must be linear, so that the other branch of the curve must be asymptotic to a line which forms an acute angle with the temperature axis. This will not be true to the critical temperature as indicated later.

These conditions are fulfilled by the equation.

$$t = A\varphi - \frac{B}{\varphi} + C, \tag{9}$$

where A, B, and C are constants. This is the equation of a hyperbola, $t_1 = A\varphi$ representing the equation of the ideal substance and $t_2 = C - B/\varphi$ representing the breaking down of association which is complete at the temperature C. An extended study has shown that the average percentage difference between the observed and calculated values for 85 substances with some 1,000 duplicate observations is 0.17. If the alcohols are omitted from the calculation, the percentage difference falls to 0.09 for 70 substances.

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[1] Cp. seventh paper.

[2] Wied. Ann., 2, 387 (1877).

The best formula heretofore proposed is perhaps that of Slotte,[1]

$$\eta = \frac{c}{(a + t)^n},$$

or in the slightly simplified form of Poiseuille,[2]

$$\eta = \frac{c}{1 + \alpha t + \beta t^2}.$$

From this latter is derived the formula

$$\varphi = A + Bt + Ct^2, \tag{10}$$

which is the equation of a parabola, and hence may be debarred at once since it does not meet the conditions of a general equation that it must be asymptotic to the temperature axis and to a line at an acute angle to this axis As a matter of fact, it has been found that this equation does not apply at all to the alcohols where the fluidity curves depart most widely from linearity. The average percentage deviation between the observed and calculated values with Slotte's equation applied to 64 substances is 0.15. The alcohols were excluded because Slotte's formula breaks down entirely when applied to them.

By introducing a fourth constant into our equation a much better agreement may be obtained, while keeping the general properties of the equation the same. Thus with the equation

$$t = A\varphi - \frac{B}{\varphi + D} + C \qquad (11)^3$$

eight substances which gave an average percentage difference between the observed and calculated values of 0.77 with equation (9), with equation (11) give a percentage difference of only 0.07.

Fluidity and Volume.[4]—Evidence from several different quarters points to a relation between fluidity and volume in liquids.

1. We have seen that the association of liquids may be calculated from the fluidity, and Traube[5] has calculated the association from the volume. There is therefore a connection between the two and it is

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[1] Wied. Ann., *14*, 13 (1881).

[2] Pogg. Ann., *58*, 424 (1843).

[3] Cp. seventh and ninth papers.

[4] Cp. third, sixth, and ninth papers.

[5] *Loc. cit.*

evidently possible to calculate the volume of a compound from its fluidity and *vice versa*.

2. Again we have seen that in mixtures a contraction in volume is usually accompanied by a decrease in fluidity.

3. In pure liquids an increase in the temperature is usually accompanied by an increase in both the volume and the fluidity. Water and sulphur are exceptional.

4. Pressure usually decreases both volume and fluidity. Water is again exceptional at certain temperatures.

5. If we regard the pressure as constant as is the case in ordinary viscosity measurements, then van der Waals' equation

$$\left(p - \frac{a}{v^2}\right)(v - b) = RT$$

may be written

$$T = \alpha v - \beta - \frac{\gamma}{v} + \frac{\delta}{v^2} \tag{12}$$

where α, β, γ, and δ are constants. This equation expressing the relation between the volume and temperature is the same in form as our equation (9) for expressing the relation between the fluidity and the temperature, except for the term δ/v^2, which is of minor importance. It follows therefore that fluidity and volume are either dependent upon some more fundamental property or upon each other, and that quite similar formulas may be used to represent the effects of the temperature upon both. Further work needs to be devoted to such relationships.

Nature of Viscous Resistance.[1]—As regards fluidity, gases and liquids afford a sharp contrast. In liquids the fluidity generally increases with the temperature, while in gases the fluidity decreases with the temperature, so that at the critical temperature the fluidity apparently passes through a maximum. In gases the fluidity is independent of the density, while in liquids the density, as we have already seen, is of prime consideration. This contrast suggests that the origin of these losses of energy must be different in the two cases.

In gases, the particles of one layer penetrate another layer with lower translational velocity and thus the uni-directional motion becomes changed into irregular molecular motion or heat. This loss increases with the temperature because the motion of the molecules is thereby increased. So long as the volume of the molecules themselves is negligible and the molecular free path is small in comparison with the dimensions of the containing vessel, it appears that as the density of the gas is increased, the increase in the loss of energy due to the greater number of molecules crossing a given plane surface in a unit of time, is exactly compensated by the decrease in the loss of energy due to the smaller depths from which the molecules have come, so that the viscosity is independent of the density.

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[1] Cp. tenth paper.

In liquids the molecules themselves have such large volume in comparison with the spaces between them, that the excursions of the molecules are short and the cause of the loss of energy which is powerful in gases is of relatively small significance. On the other hand, as one layer of liquid moves over another, collisions will frequently be caused by the molecules of one layer overtaking and colliding with the molecules of the more slowly moving layer, and these collisions will cause a loss in translatory motion. Increasing the density either by lowering the temperature or increasing the external pressure will increase this effect.

It has been customary to ascribe the viscous resistance of liquids to the attraction between the molecules. In the opinion of the author, cohesion and fluidity are indeed related but not as cause and effect. Since the cohesive forces within the liquid are balanced, it is difficult to see how work can be done against them as one layer of liquid moves over another. Space will not now permit going further into this interesting and important subject.

The Fluidity of Solids.[1]—It has been assumed by some investigators that the damping of the vibrations of a wire, after allowing for the friction of the air and the loss of energy to the support, is a measure of the viscosity of the solid. To the present writer, it seems probable that while this loss of energy is a true viscous effect, the amount of the damping cannot for a moment be considered *directly* proportional to the viscosity of the substance. Such an assumption would lead us to the absurd conclusion that lead is less fluid than steel or quartz, since the damping in lead is very rapid. Moreover, since the rate of damping increases with the temperature, it would force us to conclude that solids are analogous to gases, in that the fluidity decreases as the temperature is raised, even though we recognize that amorphous solids are nothing but undercooled liquids. The truth is probably quite the opposite of this common assumption. Lead is of course more fluid than steel and the fluidity of both increases with the temperature, and the rate of damping is rather a measure of the *fluidity*. In bending a "perfectly elastic substance," the molecules do not move over one another, but merely move about in the spaces between the molecules, causing strains to be set up. Therefore as soon as the stress is removed the strain disappears without loss of energy by means of internal flow. But since these spaces are small, the elastic limit is soon reached, beyond which rupture or viscous flow must take place. In ordinary solids there is some flow of the molecules over each other as stress is applied, so that when the stress is removed the material does not fly back at once to its old position, but appears temporarily to be considerably distorted—the "elastic after-effect"—and *is* permanently distorted to a very much slighter extent. The rate of damping is then a measure of the viscous flow or fluidity. This conception may be used to explain many of the curious facts in connection with elasticity.

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[1] Cp. tenth paper.

The Molecular Weight of Solids.[1]—Amorphous solids are to be regarded as liquids and hence there is no inherent reason why their molecular weights may not be obtained by the fluidity method as soon as sufficient data are accumulated for the purpose. When such undercooled liquids change to crystalline solids, the fluidity usually drops to but a small fraction of its former value. This indicates a great increase in apparent molecular weight which it seems possible to calculate by the method already given, but the viscosity data necessary for such calculation are lacking.

Conclusions.—Upon the relations between fluidity and physical and chemical properties we may add the following to the conclusions already given:

17. The fluidities of unassociated substances of a homologous series at their boiling-points are a linear function of their boiling temperatures.

18 The above relation is true of other vapor-pressures than that of the ordinary boiling temperature, and it may be made evident in various ways. For example, the fluidities of the ethers are nearly identical at their boiling-points, so by plotting fluidities against vapor-pressures corresponding to the same temperatures, a single curve is obtained for the whole class of substances. Or by taking one substance as a standard and multiplying the fluidities of other substances by the ratio of the fluidity of the latter at their boiling-points to the fluidity of the standard at its boiling-point, "reduced fluidities" are obtained. Employing the fluidity-vapor-pressure curve of the standard substance, vapor-pressures may be calculated from the reduced fluidities. It is found that the calculated values of vapor-pressures of over 10 cm. for about 20 substances differ from the observed values by a little over 3 per cent.

19. There is shown to be a relation between fluidity and chemical composition. The fluidity-temperature curves of a given class of compounds consists of a family of nearly straight and nearly parallel lines.

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[1] Cp. tenth paper.

Hence the slope of the curves is characteristic of the class and their intercepts are indicative of the molecular weight, and of constitutive influences to a very much lesser degree. Evidence is given to show that absolute temperatures corresponding to a given fluidity form a proper basis of comparison. Constants obtained for a given fluidity enable us to calculate the absolute temperatures of many other substances at that fluidity. For 35 substances, the calculated values differ from the observed by less than 0.8 per cent.

20. Since fluidity gives a means for calculating the molecular weights of liquids it is evidently possible to use the method for calculating association, the association being the ratio between the observed absolute temperature corresponding to the fluidity used as a basis of comparison and the absolute temperature as calculated from the constants. A comparison of values of association by the fluidity method and the methods of Ramsay and Shields, Traube, and Longinescu for 27 substances shows as satisfactory an agreement as can well be expected.

21. Reasons are given why similar relations to those here given between fluidity and vapor-pressure, fluidity and chemical composition, and fluidity and association have not been and cannot readily be worked out by comparing viscosities instead of fluidities.

22. The fluidity method of calculating molecular weights may evidently be extended to the case of mixtures where the components unite in loose combinations as in hydration. The theory demands that the solutions be reckoned in terms of volume percentages and not in weight or molecular percentages, as is often done. It is proved furthermore that the occurrence of a maximum or minimum in a viscosity or fluidity curve does not necessarily indicate the composition of the solvate, and the shifting of a maximum or a minimum does not necessarily indicate a change in the composition of the solvate. Theory indicates that the point in the fluidity-concentration curve where the deviation of the fluidity curve from the linear curve, which would be expected were there no solution, is a maximum, may give us knowledge of the composition of the solvate. Evidence is given to show that solvation follows the law of mass-action, and the method is given of calculating the percentage of the mixture which is combined.

23. The properties of a general formula to represent the changes in the fluidity of a substance with the temperature are given, and the formula

$$t = A\varphi - \frac{B}{\varphi} + C$$

where t = temperature absolute, φ = fluidity, and A, B, and C are constants, is proposed, which has these general properties. This formula may be used to reproduce the observed values of 85 substances tested with an average percentage difference of 0.17. Another constant added to this equation reduces this already small percentage difference very much further.

24. Reasons are given for the belief that there is a close relation between the fluidity of a liquid and its volume:

(*a*) When liquids are mixed, a decrease in volume is generally accompanied by a decrease in fluidity, and the greatest decrease in both appears to coincide in the same mixture. The opposite seems to be true when liquids expand on mixing.

(*b*) Pressure generally decreases both volume and fluidity, and increase in temperature generally increases both.

(*c*) Association may be calculated from fluidity data, and Traube has also calculated association from volume.

(*d*) The above equation is almost exactly the relation between temperature and volume implied in van der Waals' well-known equation.

25. The nature of viscous resistance is discussed, and it is pointed out that while the kinetic theory explains the cause of viscosity in gases, it does not explain it in liquids. The usual assumption, that the viscosity of liquids is due to the attractions between the molecules, is insufficient. A theory is outlined which makes the volume of the molecules themselves the most important factor, the loss of energy arising from the collisions, due to their translatory motion, of the molecules of one layer with the molecules of adjacent but more slowly moving layers.

26. It is indicated that many of the phenomena of elasticity in solids may be explained on the basis of their fluidity. The rate of damping of a wire is probably directly proportional to the *fluidity* of the solid, and *not* to the viscosity, as has often been assumed. The "elastic after-effect" is, according to this view, caused by partial flows in the solid which cause a state of strain to remain after the deforming stress has been removed. This state of strain has been observed in the viscous flow of very viscous liquids like pitch.

27. A possible method for calculating the molecular weights of solids is suggested.

Richmond College,
Richmond, Va.,
June 27, 1912.

AN ABSOLUTE DETERMINATION OF THE VISCOSITY OF AIR.

By Lachlan Gilchrist.

IN view of the recent work of Professor R. A. Millikan, of the University of Chicago,[1] the accuracy with which the elementary charge can be determined is limited only by the accuracy obtainable in the measurement of the absolute value of the coefficient of the viscosity of air. It is of the utmost importance therefore for this reason, and also in view of the recent work that has been carried out on the relation between temperature, pressure and viscosity, that the absolute value of this constant be determined with all possible precision. Professor Millikan had already been led by a careful study of existing data to the conclusion that the present uncertainty in η for air at 25° C. could be still further reduced by new work with a constant deflection method since this method is exceedingly direct and simple and seems more suitable for the absolute determination of η than are either the capillary tube or oscillating body methods. At his suggestion therefore and with his assistance the following apparatus was designed. The results show that the method is capable of measuring η with an uncertainty which does not exceed .2 per cent.

The constant deflection method does not appear to have been used carefully in a direct determination of η for air except by Zemplen.[2] He used concentric spheres separated by a small distance and his results are much higher than those which have been obtained in any trustworthy experiments in this field.

In the present work it was decided to use concentric cylinders for two reasons.

1. It appeared to be the simplest method and the one in which the sources of experimental error could be most readily reached and their effect minimized.

2. The theory associated with the method is simple and the formula which is derived is exact.

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[1] Phys. Rev., Vol. XXIII., No. 4, April, 1911.

[2] Annalen d. Phys., 29, 5, pp. 869–908, Aug. 10, 1909; 38, 1, pp. 71–125, May, 1912.

A. Theory.

The theory shows that

$$\eta = \frac{\pi\varphi I(b^2 - a^2)}{a^2b^2T^2\omega l},$$

where η = the coefficient of viscosity.
I = the moment of inertia of the suspended body about the line of suspension.
T = the period of oscillation of the suspended cylinder.
φ = the angular displacement of the suspended cylinder.
a = the radius of the inner cylinder.
l = the length of the inner cylinder.
b = the radius of the outer cylinder.
ω = the constant angular velocity of the rotating cylinder.

The inner cylinder itself was used to determine I and T, and φ was determined by a mirror and scale and a telescope for observation.

Then

$$\eta = \frac{stK}{dT^2},$$

where d = the distance of the scale from the mirror.
$s = 2\varphi d$, that is, the observed deflection of the scale image.
$t = 2\pi/\omega$, that is, the time of one rotation of the outer cylinder.

Now it was hoped from the use of a bifilar suspension to diminish such errors as are due to fatigue or stretching of the suspension.

With such form of suspension the expression for the period is

$$T = \frac{2\pi k}{\alpha}\sqrt{\frac{l}{g}},$$

for small oscillations, where k is the radius of gyration of the cylinder.

α = one half the distance between the strands of the suspension.
l = the length of the suspension.
g = the gravity constant.

A consideration of these formulæ suggested the form of construction and the dimensions used in the following apparatus:

B. Experimental Arrangements.

Figure 1. Main Apparatus.

F, the inner cylinder with suspension and mirror M attached.
The length of the inner cylinder = 24.88 cm.
The outside radius of the inner cylinder = 5.342 cm.
The length of the suspension = about 25 cm.

This cylinder was made of brass, the wall being made as thin as possible to diminish the mass. The wall was supported by sheets of aluminium as shown in Figs. 5 and 4, *ff*.

The wall of the cylinder was made parallel to the line of suspension in the following manner: The center of the upper aluminium sheet was carefully determined to within .1 mm. A fine steel wire suspension was temporarily attached at the center of the upper sheet and the cylinder weighted until the wall at all points was parallel. This was tested with a plumb square to which a very sensitive level was attached. Then three aluminium supports for the permanent suspension were fastened to the upper aluminium sheet and just above their junction the point of attachment of the suspension was placed. This point could be readily adjusted to make the line of suspension parallel to the wall of the cylinder.

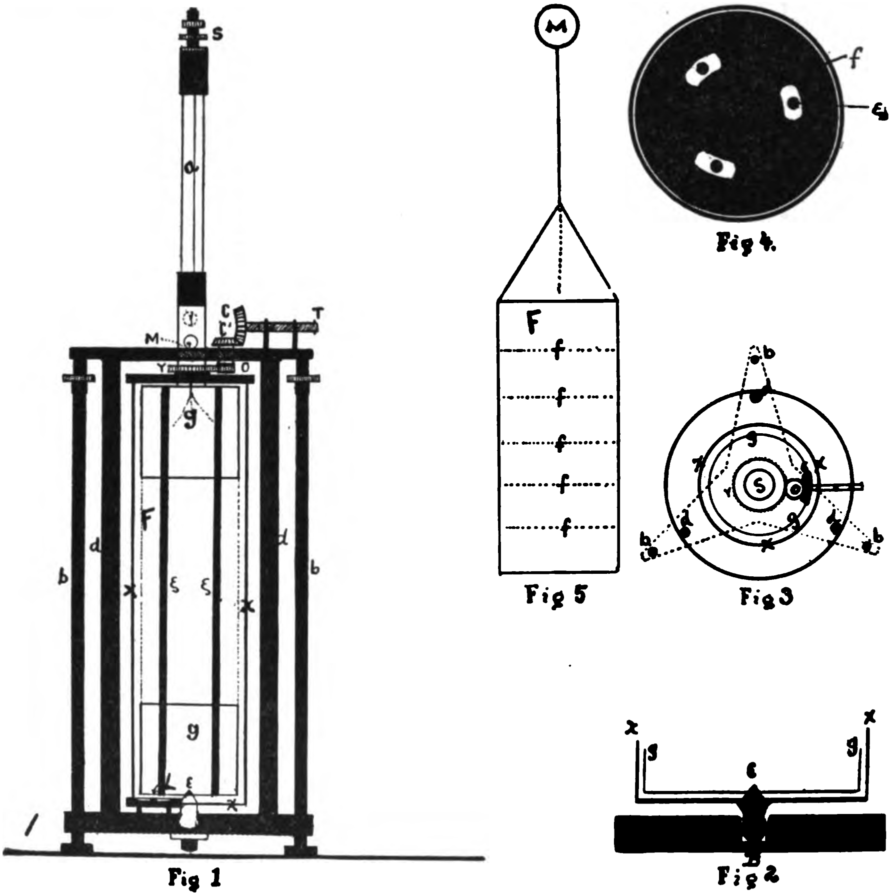

Fig 1 Fig 2 Fig 3 Fig 4. Fig 5

Figs. 1–5.

The mass of the inner cylinder was 320 grams.

S = the screw head to which the suspension was attached.

gg were guard cylinders about 10 cm. long and rigidly attached by three brass rods. These were also rigidly attached to the tube supporting the screw head S. The distance between the guard cylinders was 24.92 cm., leaving .4 mm. between the suspended cylinder and the guard cylinder. The lower guard cylinder engaged the outer cylinder at ϵ as is shown.

a is a glass portion of the tube so that the suspension thread was always in view.

XX = the outer cylinder, *i. e.*, the rotating cylinder. The inner radius = 6.06357 cm.

This cylinder engaged the frame at Y and ϵ as is shown.

The lower bearing is shown in fuller detail in Fig. 2.

$CC'OY$ represent the transmission gear connecting the driving shaft T with the outer cylinder.

L = a level. There were two at right angles to each other.

d = the supports of the frame

b = the levelling rods on which the instrument was supported.

Fig. 2 shows the lower bearing of inner guard cylinder g, the outer cylinder X and the frame B.

Fig. 3. A plan of the top of instrument.

S = the opening through which the tube which contained the suspension passed.

$CC'Oy$ the transmission gear.

g = the inner guard cylinder, as given in Fig. 1.

x = the outer cylinder as given in Fig. 1.

b, d = etc., as given in Fig. 1.

The dotted lines indicate the supporting frame at the top of the instrument.

Method of Measuring Dimensions.

1. *Inner Cylinder.*—The length and diameter were obtained by means of calipers reading to .001 cm. and the average of a large number of readings was taken.

The mass was obtained by means of a balance reading, .1 mg.

2. *Outer Cylinder.*—The diameter was obtained by filling the cylinder with water, weighing the water, noting the temperature and using density tables to get volume, and then calculating the diameter.

3. The time of rotation of the outer cylinder and the period of oscillation of the inner cylinder were obtained by means of a stop watch. This watch was compared with a chronometer which had been regulated to

accord very closely to the clock in the Canadian Government's Observatory at Toronto.

4. The thermometer which was graduated to .1 dg. was compared with the standard thermometer in use at the observatory. A Beckmann thermometer was also immersed in the bath to indicate changes of temperature.

Fig. 2. Other Apparatus.

1. A clock-driving apparatus was used to drive the rotating cylinder. This instrument was of superior type and simple construction. It was driven by weights and could be directly connected by a shaft to the cylinder. It was made by Wm. Gaertner & Co., of Chicago, and was kindly loaned for the experiment. The average error over one hour in the clock did not exceed 1 part in 3,600. When it was loaded and set running the tests for speed agreed to this degree of accuracy.

The period of vibration of the inner cylinder was such that an average error in the rotation of the outer cylinder much larger than this would not seriously affect the steadiness of deflection.

2. A constant temperature bath. This was a water-bath of the kind described in several text-books in physics.

It was heated by incandescent electric lamps and was regulated by a mercury thermostat, a telegraph relay being used to control the make and break of the heating circuit. For temperatures within 10° C. of the temperature of the room when making a determination of η, the bath did not vary .01° C. and not more than .1° C. at 20° C. above the temperature of the room.

3. *The Telescope and Scale.*—The scale was ruled carefully for the experiment by Wm. Gaertner and Co. and the telescope was such that readings to .1 mm. could be made fairly readily.

The complete apparatus was set up in the first place on a stone pier in Ryerson Physical Laboratory, University of Chicago, during the summer of 1910 and subsequently on a stone pier in the basement of the Physics Building, University of Toronto. In this way rigidity and constant temperature were obtained and drafts and other disturbances were eliminated.

C. THE METHOD OF MEASUREMENT AND OBSERVATION.

1. The following was the general plan of work including the difficulties which were encountered, and the methods of dealing with them.

The inner suspended cylinder was made plumb with the line of suspension by use of a plumb square. A shift of approximately five divisions in the level corresponded to .05 mm. out of plumb between top and bottom. One tenth of one division on the levels could be observed.

The levelling rods on the instrument were adjusted so that the outside surface of the guard cylinders, *g*, Fig. 1, was aligned with the surface of the inner cylinder. Two short focal length telescopes which were adjustable on the same base were used to observe this condition. The levels on the instrument were then adjusted. There were scales as well as cross hairs in the telescopes. One half of one division on the scale was easily recognized and the distance between the divisions represented .05 mm. motion of the telescope. The cylinder was found to be from .05 to .15 mm., less in diameter than the guard rings, being smaller at lower end than at upper.

The suspended cylinder was set very accurately symmetrical to the guard cylinders in this way. The upper end of the suspended cylinder was made symmetrical with the upper guard cylinder by adjusting the horizontal screws at the upper end of the suspension. Then the lower end was made symmetrical with the lower guard cylinder by the use of the levelling screws. This threw the upper end out a little. It was re-adjusted, and again the lower end, and so on. It will be observed that this plan really adjusted the instrument to the suspended cylinder which had previously been made plumb. The levels were then made to read properly. Then the suspended cylinder was lowered to rest on the lower guard cylinder one quarter to one half turn of the screwhead being sufficient for this purpose. The mounted telescopes were also used to test the symmetry of the guard cylinder about the axis of rotation. The average variation was about .05 mm.

2. The accuracy of centering of the outer cylinder was tested thus. The surfaces of the guard cylinders and the inner cylinder were put in alignment as in (3) above and the levels *L* adjusted. The suspension tube *a* and the inner cylinder *F* were then removed. An upright rod with arms at each end which were attached perpendicularly to it was inserted through the opening that was left by the removal of the suspension tube. This rod was arranged so that it could be raised or lowered or rotated in a horizontal plane, and by means of a swivel any motion of the lower arm was communicated to the upper arm. The extent of the motion was indicated by a sensitive level attached to the upper arm. The end of the lower arm was held lightly in contact with the inside wall of the outer cylinder by a small weight attached to the upper arm and the cylinder was rotated by the clock driving apparatus. The test for different places in the cylinder showed an average variation of less than .05 mm., the range of variation being very small. The instrument was adjusted and fixed solidly in the bath and the following observations were made, viz., the zero position on the scale, the period of oscillation of the

inner cylinder, its deflected position, the time of rotation of the outer cylinder, and the distance from the scale.

Some difficulty was found with the suspension owing to its elastic properties and the effect of temperature and moisture—silk, rolled and unrolled phosphor bronze wire, and rolled and unrolled steel wire of different sizes were tried. The use of silk was not successful. The others were used both as single strand and as bifilar suspensions and fair results were obtainable with several of the suspensions used, but the phosphor bronze ribbon rolled from wire, .0762 mm. in diameter and used as a bifilar suspension, gave the best results. It was necessary to obtain a suitable deflection for accurate measurement and still maintain constant zero and deflected positions of the cylinder. With several of the suspensions there was a slow drifting of the zero position or a slight unsteadiness of the deflected position. It was found that upon making the suspension bifilar and such that the period of oscillation of the cylinder was long, a very steady deflected position was obtainable. By leaving the instrument in the constant temperature bath for several hours with the inner cylinder suspended the zero position became more steady. It was also arranged that the driving apparatus could be damped so that it commenced slowly and gradually reached the condition of uniform speed. In this way the deflected position was quickly reached and oscillations that otherwise would exist, were eliminated. The measurement of the deflection was then made and the cylinder was gradually let back to the zero position. With this proceedure deflections of more than 600 mm. could be repeatedly duplicated with an error of no more than .3 mm.

In each determination all the observations and measurements were checked by repetition.

II. *The Measurement of the Moment of Inertia of the Inner Cylinder about the Line of Suspension.*

(*a*) The cylinder was suspended by a strand of steel piano wire .36 mm. in diameter from a firmly built support resting on a stone pier. It was set up in a room which was free from drafts and whose temperature was maintained constant to within 5° C. The cylinder was set oscillating for some time. The period was then obtained by counting a large number of oscillations of small amplitudes, noting the full time, and then calculating the time of a complete oscillation.

(*b*) An annular ring with a cross bar of aluminium was made in the workshop of Ryerson Physical Laboratory with dimensions uniform to a high degree of precision. This was placed symmetrically on the cylinder and the period of oscillation was again obtained, the amplitude of the oscillations being almost the same as those considered in (*a*).

The expression for the period in (*a*) is

$$T_1 = 2\pi\sqrt{\frac{I_1}{G}}, \tag{1}$$

and in (*b*)

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{G}}, \tag{2}$$

where T_1 = period of oscillation of the cylinder alone.
T_2 = period of oscillation of the cylinder and the aluminium ring and bar.
I_1 = moment of inertia of the cylinder.
I_2 = moment of inertia of the ring and bar. This was calculated from the dimensions.
G = the couple constant of the suspension.

From (1) and (2) I_1 was obtained.

(*c*) *Calculation of* I_2.—For the aluminium bar

$$I_2' = M\frac{c^2 + d^2}{3},$$

where M = the mass of the bar = 22.157 grams.
c = the half width of bar = 6705 cm.
d = the half length of bar = 9.2265 cm.

For the aluminium ring

$$I_2'' = M'\frac{e^2 + f^2}{2},$$

where M' = the mass of the ring = 97.3771 grams.
e = the radius of outside edge = 9.2075 cm.
f = the radius of inside edge = 8.1608 cm.

$$I_2 = I_2' + I_2'' = 8002.409.$$

The periods of free oscillation of the cylinder without and with the ring and bar were 19.7183 secs. and 28.239 secs. respectively.

$$\therefore I_1 = 7615.8.$$

Then

$$K = \frac{I(a^2 - b^2)}{4a^2b^2}$$

$$= 7615\cdot 8\,\frac{[(5\cdot 342)^2 - (6.06357)^2]}{4 \times (5\cdot 342)^2 \times (6.16357)^2 \times 24.88} = .60027.$$

D. RESULTS.

I. *Measurements of η.*

The coefficient of viscosity was then calculated from the formula

$$\eta = \frac{stK}{dT^2}.$$

In Table I. are given the values obtained with different speeds of rotation. It shows that the experimental method that has been used gives results which are very concordant. Save for the last two observations which deserve less weight than the others owing to the smallness of the deflection, the maximum divergence of any observation from the mean is but .22 per cent. and the average divergence but .1 per cent.

TABLE I.

	t	s	d	T	$st/dT_2 \times 10^7$	$\eta \times 10^7$	Temp.	$\eta \times 10^7$ at 20.2° C. (Calc.).
1	30.89	70	316	150.33	3,027	1,817	20.2	1,817
2	30.88	69.86	316	150.32	3,021	1,813	20.205	1,813
3	30.79	59.92	302.55	142.06	3,021	1,813	20.2	1,813
4	61.74	34.87	316	150.32	3,015	1,810	20.2	1,810
5	61.66	31.74	315.35	147.46	3,015	1,810	20	1,811
6	61.60	30.01	302.55	142.11	3,025	1,815	20.2	1,815
7	61.58	30.04	302.55	142.11	3,027	1,817	20.2	1,817
8	123.44	16.72	315.35	147.26	3,018	1,812	20.1	1,812.5
9	123.22	16.75	315.35	147.32	3,015	1,810	20.1	1,810.5
10	184.96	11.09	315.35	147.31	2,997	1,800	20.1	1,800.5
11	184.88	13.86	301	188.25	3,007	1,805	20.1	1,805.5

These results for η have been reduced (see Table IV., last column) to the value for the common temperature 20.2° C. by the use of the formula used by Professor Millikan[1] for small differences in temperature.

$$\eta_t = .00017856\,[1 + .00276(t - 15)].$$

The most probable value of η at 20.2° C. is obtained as follows: Since the deflections for the higher speeds of rotation are proportionally greater than for the lower speeds the values should be weighted accordingly. Considering also the steadiness of the deflected and undeflected positions in each case it was decided to give the readings the following weights: 1, 2, 3, 4 each 2; 5, 6, 7 each 1.5; 8, 9, 10, 11 each 1.

This gives as a mean value $\eta = 1{,}812\ (2) \times 10^{-7}$, with a probable error less than 1.3×10^{-7}. The total probable percentage error may also be fairly closely estimated. The probable error of observation as here calculated is about .14 per cent. Since however for the different measurements which are given the apparatus was in most cases replaced and readjusted this would include the errors of observation in the measurement of s, t, d, T, and the effect on these of the error of adjustment. The errors in the measurement of a, b and l are exceedingly small and together with that in the measurement of the moment of inertia, do not, I believe, exceed .06 per cent. Then the total probable error is not greater than .2 per cent.

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[1] Reference already given.

II. In order to determine whether this method would give the same temperature coefficient of viscosity as that which has been obtained by the capillary tube method the following observations were made (see Table II.). For the purpose of comparison the values which are obtained by calculation from the formula of S. W. Holman[1] $\eta \times 10^7 = 1715.50 (1 + .00275t - .00000034t^2)$ are also given. This formula has been shown by several experimenters to be very satisfactory and was found by Tomlinson[1] to accord with his results perfectly.

Table II.

Temp.	t	s	d	T	$st/dT \times 10^7$	$N \times 10^7$	$N \times 10^7$, Using Holman's Formula (Calc.).
20.2						1,812.2	1,810
24.58	30.86	66.58	316.5	146.28	3,034.5	1,821.5	1,831
31.9	30.86	67.8	316.5	146.25	3,090.7	1,855.2	1,865.4
37.9	30.88	68.15	316.5	145.84	3,126.2	1,876.5	1,892
43.5	30.88	69.49	316.5	145.84	3,126.2	1,913.4	1,919.6

The greatest variation of the observed from the calculated values is .82 per cent. and the curve showing the relationship between η and temperature is not quite so steep as that given by Holman's formula, or by the formula obtained by most of the experimenters with capillary tubes. The range of temperature for my observations is not great enough, however, to give the relationship as closely as is desirable nor are the determinations at the higher temperatures as accurate as that at 20.2° C.

III. *The Effect of Moisture on the Viscosity of Air.*

The following observations were made in order to determine whether the formula for mixtures of gases which is based on the kinetic theory holds for a mxture of water vapor and air.

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[1] Reference given in Table IV.

Air which was dried by being passed slowly through phosphorous pentoxide and sulphuric acid was passed for some time into the instrument through a tube reaching to the bottom. Then a determination of the viscosity was made. Some of the air from the instrument was then drawn slowly into a Crova[1] dew point apparatus and the dew point observed.

The results obtained are shown in Table III. For the last three determinations water was passed into the outer cylinder for the purpose of saturating the air. In number 5 only 2 to 3 c.c. were put in and in numbers 6 and 7 water was put in to a depth of about 2 cm.

The Crova dew point apparatus could not be used to show that the air in the instrument was saturated since the temperature of the room was about 18.3° C. and that of the air in the instrument about 20° C.

TABLE III.

	t	s	d	T	Temp.	Dew Pt. Point.	$st_2/dT \times 10^7$.	$\eta \times 10^7$.
1	3,086	6,557	316.5	145.66	20.02°C.	6.6	3,015	1,810
2	3,088	6,552	316.5	145.34	20.05	11.6	3,026	1,816
3	3,088	6,557	316.5	145.57	20.05	14.1	3,017	1,811
4	3,087	6,546	316.5	145.62	20.02	17	3,010	1,807
5	3,081	6,432	316.5	143.84	20.02		3,026	1,816
6	3,083	6,432	316.5	144	20		3,021	1,813
7	3,086	6,438	316.5	143.82	20		3,034	1,821

Temperature of room = 18.2° C.

From the results in Table III. it does not appear that the presence of unsaturated water vapor in the air has any measurable effect on the coefficient of viscosity at normal pressure. It is noteworthy, however, that the average of the first four results gives $\eta = 1{,}811 \times 10^{-7}$ and the average of the last three, for which the maximum quantity of water was present, and saturation apparently existed, gives $\eta = 1{,}816.5 \times 10^{-7}$. This seems to show an increase of about .3 per cent. This result is smaller than that obtained by Zemplen[2] (viz., .8 per cent.) but agrees more closely with that obtained by Tomlinson. From the Puluj[2a]

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[1] See Preston, Theory of Heat.

[2] Reference already given in paper. (See 2 above.)

[2a] $$\eta = \eta_1 \frac{\sqrt{p_1 + \frac{m_2}{m_1} p_2}}{\left[p_1 + \left(\frac{\eta_1}{\eta_2}\sqrt{\frac{m_2}{m_1}}\right)^{\frac{2}{3}} p_2\right]^{\frac{3}{2}}},$$

where η = coefficient of viscosity of the mixture, $\eta_1\eta_2$ = coefficient of viscosity of the components, p_1p_2 = partial pressures of the components, $p_1 + p_2 = 1$, m_1m_2 = molecular weights of the components.

formula for mixtures of gases there should be a decrease of about 2 per cent. A modification of this formula by Thiesen[1] has recently been shown by Tanzler[2] and Thomsen[3] to hold fairly well for certain gaseous mixtures. For saturated water vapor the formula is not sustained by the results of my experiments. These accord much better, however, with the calculations of Tomlinson by the method of Stokes[4] based on the experiments of Crookes according to which there should be an increase in the coefficient of viscosity of about .2 per cent. Crookes[5] found that the presence of water vapor did not affect the coefficient of viscosity of air until the total pressure was less than half of the normal pressure and for pressures lower than this it was diminished but not as much as is required by the kinetic theory on which the Puluj formula for gases is based. The results of recent experiments on mixtures of gases by Tanzler and by Kleint[6] are as much as 1.3 per cent. higher than the values calculated from the Puluj formula, that is, the variation is in the same direction as that which appears to exist for mixtures of water vapor and air. It is very desirable then to have the work on mixtures of gases carried out with the same proportional partial pressures as have been investigated, but through a range of lower total pressures until at least the critical pressure is reached. Thomsen shows that in the case of the mixtures of gases investigated by Tanzler and by Kleint, the results which were obtained were approximately such as were to be expected from the formula of Puluj, and Schmitt[7] succeeds in making the approximation much closer by means of a correction factor made necessary by the method of experiment which was used, the formula for which had been developed by Knudsen[8] from kinetic theory considerations. This however makes more imperative the necessity for the investigation which has been suggested above.

E. The Work of Other Experimenters.

The results of recent work by the oscillating body and capillary tube methods are given in Table IV. Those of Tomlinson by the former method and of Holman by the latter method, though of more remote date, are also given, as their work shows every evidence of great care and a high degree of precision. The values at 20.2° C. are calculated from the

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[1] M. Thiesen, Ver. der Deut. Phys. Ges., 8, 12, 1906, p. 237.

[2] P. Tanzler, Ver. der Deut. Phys. Ges., 8, 12, 1906, pp. 222–235.

[3] E. Thomsen, Ann. d. Physik, 36, 1911, p. 815.

[4] G. F. Stokes, Phil. Trans., Part II., 1889, p. 440.

[5] Crookes, Phil. Trans., Part II., 1889, p. 427.

[6] F. Kleint, Ver. der Deut. Phys. Ges., 7, 1905, pp. 145–157.

[7] K. Schmitt, Ann. d. Physik, 4, 30, 1909, pp. 393–410.

[8] M. Knudsen, Ann. d. Physik, 4, 28, 1909, pp. 75–130.

TABLE IV.

Experimenter.	Method.	Temp.	Result $\times 10^7$.	Value at 20.2° C. $\times 10^7$.	References.
1. S. W. Holman	Capillary tube			1,810	Phil. Mag. (5), 1886, p. 199. $\eta = .00017155.(1+.002751t-.00000034t^2)$
2. W. J. Fischer	Capillary tube			1,807	PHYS. REV., 28, p. 104.
3. J. H. Grindlay and A. H. Gibson	Capillary tube	18.94	1,802.7	1,809	Proc. Roy. Soc., A, 80, 1908.
4. A. O. Rankine	Capillary tube	10.6	1,767	1,814	Proc. Roy. Soc., A, Vol. 83, 1910, p. 522.
5. Rapp	Capillary tube	26.	1,838	1,810	Not yet published.
6. Breitenbach	Capillary tube	15.	1,817	1,833	Wied. Ann., LXVII., 1899, p. 803.
7. Schultze	Capillary tube	15.	1,811	1,837	Ann. d. Phys., Vol. 5, 1801, pp. 166–169.
8. Markowski	Capillary tube	15.9	1,814	1,835	Ann. d. Phys., Vol. 14, 1904, p. 742.
9. Tanzler	Capillary tube	15.	1,812	1,838	Ver. der Deut. Physik. Gesell., 8, 12, 1906, pp. 222–235.
10. H. Tomlinson	(*a*) Oscillating pair of cylinders.	12.65	1,774	1,811	Phil. Frans., Vol. 177, Part II., 1886, pp. 767–789.
	(*b*) Oscillating pair of spheres.	9.97	1,762	1,812	
	(*c*) A single oscillating cylinder.	11.78	17,711	1,812	
11. J. L. Hogg	....	17.33	1,794	1,808	Am. Acad. Proc., 40, 18, 1905, pp. 611–626.

values actually obtained. For all except that of Holman and Fisher the mean values for temperatures near 20.2° C. were taken and the transformation was made by means of the formula used by Millikan.

The Holman and Fisher values are calculated from their final formulæ.

The results are grouped as follows:

(*a*) Those which are obtained by the capillary tube method in which the tubes that were used were fairly large and the results obtained were concordant and agree well with the result obtained by me. (See nos. 1–5.)

(*b*) Those which were obtained by the capillary tube method in which the results, though fairly concordant differ considerably from those of the others. (See nos. 6–9.)

(*c*) Those which were obtained by the damping of a simple pendulum in which all or nearly all of the loss of energy is due to the pushing of the air. (See nos. 10, *a* and *b*.)

(*d*) The damping of torsional vibrations where the complete loss of energy is due to *dragging*. (See nos. 10, *c*, and 11.)

It will be seen from the table that in the work with capillary tubes the results obtained by Breitenbach, Schultze, Markowski, Tanzler and Schmitt are somewhat higher than those of Holman, Fisher, Grindley and Gibson, Rankine, and Rapp, through the determinations by both groups seems to have been made with equal care. With the oscillating body method exceedingly careful work has been done by Tomlinson and recently by Hogg. Their results are concordant but differ considerably from that of Reynolds[1] who used a similar method. In view of the care with which the work has been done it would appear that the discrepancies can only be accounted for by the fact that there are factors in these methods, the effects of which are somewhat indefinitely determinable and therefore have not been uniformly estimated.

It may be well to point out some of the difficulties and sources of error associated with each method and which may to some extent account for the difference in results.

1. *Variations in the Form of the Bore.*—The following will illustrate. The equation giving the coefficient of viscosity for a tube of circular bore as developed by O. E. Meyer[2] is

$$\eta = \frac{\pi g d r^4}{16 L V} \frac{P^2 - p^2}{P} t, \tag{1}$$

where d = density of mercury at 0° C.

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[1] F. G. Reynolds, Phys. Rev., 19, 1904.

[2] O. E. Meyer, Kinetische Theorie der Gase, 1899.

g = acceleration due to gravity.
r = radius of the capillary in cm.
L = length of the capillary in cm.
P = pressure on entering the capillary.
p = height of barometer, *i. e.*, the pressure at 0° C. at exit.
y = time in secs.
V = volume of gas transpired.

This is the expression usually made use of. Now in case of a tube of elliptical bore the formula developed by Émile Mathieu[1] is

$$\eta = \frac{\pi d g}{8LV}\frac{a^3b^3}{a^2+b^2}\frac{P^2-p^2}{P}t, \tag{2}$$

where a and b are the semi-major and semi-minor axes respectively. Now for two tubes one with circular and the other with elliptical bore through which the same volume of gas was transpired and which had the same rate of shear, the areas of cross section would be the same; *i. e.*,

$$r^2 = ab. \tag{3}$$

No. (1) is greater than (2) if

$$\frac{r^4}{2} > \frac{a^3b^3}{a^2b^2},$$

or by (3) if

$$\frac{a^2+b^2}{2} > \frac{a^3b^3}{a^2b^2},$$

then

$$a^2 + b^2 > 2ab,$$

$$(a - b)^2 > 0,$$

and therefore the square of the difference between the semi-major and the semi-minor axes is a measure of the error introduced.

If a tube with an elliptical bore is used and the calculation is made for a tube with a spherical bore of the same cross section the result will be too high. This is illustrated in the results of experiments with a tube of circular bore and a tube of elliptical bore carried out by F. M. Pedersen.[2]

2. The difference in the form of the lines of shears at the entrance and exit of the capillary from the form of these lines in the body of the capillary. This source of error may be eliminated by using different lengths of the same tube but the plan has not been generally used.

3. There is considerable uncertainty in measuring the bore of the tube and when it is not uniform, which usually obtains, in reducing the bore to its proper value.

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[1] Émile Mathieu, Comptes Rendus, 1863, Tome 57, p. 320.

[2] F. M. Pedersen, PHYS. REV., October, 1907, pp. 225–254.

4. On account of the smallness of the bore, in the case of metal tubes at least, one cannot be certain that the action of the gas on the wall of the tube does not affect the result.

5. The possibility of distortion of the lines of shear in the body of the tube because of eddies or turbulence is inherent in the capillary tube method.

The effect of all these errors is to give an apparent value of η higher than the actual value. On this account therefore it is probable that the lower values which have been obtained by the capillary tube method are closer to the correct value particularly since there is close agreement among several experimenters whose work was carried out under conditions that were considerably different.

II. *The Damping of the Oscillations of a Suspended Body.*

There are two modifications of this method.

(*a*) A spherical or cylindrical bob attached to a fine suspension has been used as a simple pendulum.

(*b*) The damping of torsional vibrations of spheres and cylinders has been used; in some cases these bodies vibrated in free air and in other cases adjacent to or confined in similar bodies. This was the method used by Tomlinson, Hogg and Reynolds. The formulæ which have been derived to connect the coefficient of viscosity and the damping of the oscillations is approximate and it is difficult to calculate or determine experimentally the allowance that must be made for the suspending parts, or the inner part and edge of the suspended body. The agreement of the results obtained by Tomlinson and Hogg seems to show that they have been very successful in estimating these factors and does much to justify the theories giving rise to the mathematical development of the formulæ which have been used. A criticism of Reynolds's work is given by Hogg in an endeavor to account for the somewhat higher value which was obtained by the former.

SUMMARY.

1. The value of η for air has been determined for a temperature of 20.2° C. to be $1,812 \times 10^7$ with a probable error less than .2 per cent.

2. The variation of η with temperature has been found to agree fairly well with the relationship found by S. W. Holman and other experimenters.

3. The presence of water vapor in the air up to 60 per cent. saturation at 20° C. has no effect on η and at saturation the effect is an increase not greater than .3 per cent.

4. The method which has been used for an absolute determination of η for air at normal temperature has been shown to be exceptionally reliable.

In conclusion I desire to acknowledge my indebtedness to Professor A. A. Michelson and the Department of Physics of the University of Chicago for placing at my disposal the facilities necessary for this research, to Professor R. A. Millikan, who suggested the problem and under whose direction the work was done; to Mr. Wm. Gaertner for the loan of the clock-driving apparatus and to the University of Toronto for providing some of the incidental expenses connected with the completion of the work.[1]

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[1] A fairly full list of references to the work on the viscosity of gases to 1904 is given by F. G. Reynolds, PHYS. REV., Vol. XVIII., 1904, pp. 427–431, and by F. M. Pederson, PHYS. REV., Vol. 25, 1907, pp. 249–254.

The following may be added to complete the list to date:

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DETERMINATION OF THE HEAT OF SOLUTION OF METALS BY MEANS OF A VACUUM FLASK CALORIMETER.

By E. E. Somermeier.

THE reliability of the results obtained in calorimetric work depends essentially upon the skill of the operator, the conditions under which the experiments are conducted, the precision with which the apparatus is calibrated, and the accuracy with which correction constants can be determined. A great deal therefore depends upon the reliability of the balance and weights, the thermometers, the radiation correction, and the water equivalent of the calorimeter.

Balance and Weights.—The analytical balances and weights usually available in a well-equipped laboratory, if properly adjusted, are entirely adequate to the requirements of the calorimeter operator, and the errors of weighing are much smaller than some of the other unavoidable errors of the determination.

Thermometers.—Thermometers graduated to 1/50° and 1/100° are obtainable without difficulty, and it is quite possible to have such instruments calibrated by the Bureau of Standards or by other authorities to 0.002°. The operator can himself determine the graduation errors by means of divided threads, and in experiments in which a high degree of accuracy is to be attained this is very desirable.

The operator must assume that the values given by the Bureau of Standards or the Reichsanstalt for the upper and lower readings of the thermometer are correct. The difference between these readings, divided by the number of degrees included between them, gives the true value for one degree. The determination of the true value of the zero mark of the thermometer (when this is needed) should be made by the usual method of determining the zero point, namely, by surrounding the bulb and stem of the instrument with closely packed, crushed ice, moistened with distilled water. Many thermometers designed for calorimetric work do not have a zero reading, but are graduated only through the working range, as for example, from 15° to 20°. In this case, if it is necessary to know the true reading for the 15° mark, an auxiliary thermometer, graduated from zero to as high as 15°, is required. The two thermometers are immersed in water at 15° and the relation of the two 15° marks is read by means of a telescope, a series of readings for each instrument being taken. The two bulbs should be very close together, the stem should be tapped between readings, the water kept well stirred, and the temperature of the room in which the work is done should, if possible, be very close to that of the water; otherwise radiation changes are likely to be so large that high accuracy in results is rendered doubtful.

For comparison of thermometers the author has recently used a 500-c.c. Dewar flask for holding the water. The two thermometers are inserted through a stopper into the flask, and as many readings as may be desired can be taken at any temperature. The flask should be inverted between readings to insure a uniform temperature, and corrections should be made in each case for the mercury thread outside of the flask, the magnitude of the correction being proportional to the length of the thread. The changes in temperature of the water in the flask are so slow that as many readings as may be desired may be taken at any temperature The average of a series of readings is taken as the true reading at a given temperature. Violent shaking of the flask is to be avoided as the extra strain may cause the flask to collapse with the possibility of disastrous results to the thermometer. As an added precaution the flask should be covered with cloth or metal to protect the experimenter in case of possible accident.

Radiation Correction.—Accuracy in correction for radiation depends upon the particular apparatus employed, the conditions attending the experiment, and the method of correction adopted. In bomb calorimeters where the actual time of combustion is very short (only a few seconds), the error involved in the radiation correction is probably not greater than 5 or 10 calories, if the conditions for making the determination are favorable. With unfavorable conditions the error may be considerably in excess of these values. In other types of calorimetric measurements, where the reaction period is from 15 to 30 minutes, as in the solution of a metal in an acid, using an ordinary form of jacket calorimeter, the corrections may be so large that high accuracy is difficult to attain, and the values for the corrections become uncertain. Richards and Burgess endeavor to eliminate radiation entirely in their adiabatic calorimeter,[1] and their values for the heat of solution of zinc, magnesium, aluminium, iron and cadmium apparently should be very accurate. The adiabatic calorimeter is, however, very elaborate, expensive, and slow in operation, and it occurred to the author that a Dewar flask, in which radiation and conduction are reduced to so small an amount that the correction is a minor one, should be very efficient for this kind of work.

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[1] Jr. Am. Chem. Soc., Vol. 32, p. 431.

The following table gives the variations in the rate of radiation and conduction for the particular 2,000-c.c. vacuum flask used as obtained from about one dozen tests ranging in time from 24 to 130 minutes and with a difference between the interior of the flask and the room temperature of from − 6 to + 9° and with an average water equivalent for the series of about 1,700 calories for each degree change in temperature.

Temperature Difference.	Average Loss in Calories per Minute.
8	1.05
7	.88
6	.72
5	.56
4	.43
3	.30
2	.19
1	.09

During shorter periods of time the observed gains or losses were occasionally several times as large as the rate shown above due to several reasons:

1. The accuracy with which the thermometer can be read is certainly not greater than ± .001° and occasionally the error may be ± .002° and for short periods of time when the observed temperature change is only .002° or .003° the possible reading error is a very large percentage of the total observed change.

2. The rate of conduction of heat from the neck of the flask into the solution or from the solution into the neck of the flask with a considerable initial temperature difference between the two is larger during a short period of observation (10 to 15 minutes) than during a longer period (30 to 60 minutes) as the two tend to reach a condition approaching equilibrium and the longer the period the more nearly the condition of equilibrium is approached.

In an actual determination when the radiation and conduction corrections are based on comparatively short periods of time (15 to 30 minutes) this larger effect of conduction enters into the actual radiation-conduction correction. But aside from making this correction larger it does not necessarily make the correction any less exact. The actual temperature change in degrees for a given determination is shown by the typical determination given later in this paper.

The idea of the use of a Dewar flask for calorimeter work is not new. Ostwald and Luther in Physiko-Chemischer Messungen call attention to its possibilities and Bogorodskii[1] in an article published since the writer performed his experiments used a Dewar flask for the determination of the heat of solution of Na_2HPO_4.

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[1] J. Russ., Phys. Chem. Soc., 43, 12628.

The experiments shortly to be described on the heat of solution of some of the metals in acid were undertaken by the author primarily to obtain more data as to the adaptability of the vacuum flask to such work, rather than with the object of obtaining any new values.

Water Equivalent.—The fourth essential for accuracy in calorimetric work is a correct water equivalent of the calorimeter, and if a Dewar flask is to be employed its water equivalent under working conditions must be determined. The author accomplished this with very satisfactory results by the addition of hot and cold water to the flask. The values used for the thermal capacity of water at different temperatures are those of Barnes as tabulated by Loeb.[1]

A 2-liter Dewar flask was used as the calorimeter, and in the determination of its water equivalent the method of procedure and the apparatus employed were as follows: The thermometer in the calorimeter flask read from 15° to 25° and was graduated in 1/100°. The graduation errors of this thermometer were determined by the author by means of divided threads. About 50 separate points were determined and a curve of intermediate positions was plotted. This calibration is accurate to within ± 0.002° and for most readings to within ± 0.001°. The Reichsanstalt certificate for this thermometer gives readings to 0.01° for each whole degree, and these values in all cases coincided with the curve obtained by divided threads. The true value for a degree on this thermometer, computed from the Reichsanstalt value for 15° and 25°, is 1.002°. Comparison of the readings on this thermometer with those on a thermometer calibrated by the Bureau of Standards confirm this value to within 0.001°. The two thermometers, used in measuring the temperature of the hot and cold water, were graduated to 0.005°, and the graduation covered ranges of from 0° to 26° and from 24° to 50°. Each was calibrated by the Reichsanstalt for the true value of the degrees. The relation of the readings on each of the three thermometers to the true zero point was ascertained by the author by determining the value of the zero point of the one reading from 0° to 26° and comparing the readings on this thermometer with points on the other two.

The hot or cold water to be added to the calorimeter was kept in a 500 c.c. vacuum flask provided with a cork stopper through which was inserted a long glass tube extending nearly to the bottom of the flask (for admitting air while emptying the flask), and a short sleeve of test tubing, 2 cm. long, through which the thermometer was introduced when temperature readings were being taken, and by means of which the flask was emptied into the calorimeter. The flask serving as calorimeter was provided with a three-hole cork stopper. Through one hole the thermometer was inserted; through another was fitted a short piece of thin test tube, 2 cm. in length and of sufficient diameter to permit the ready introduction of the emptying tube with which the small flask was provided. The third hold in the stopper served merely to permit the escape of air while pouring the hot or cold water into the flask, and at other times was kept loosely stoppered, as was also the larger opening.

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[1] Ind. and Eng. Chem., Vol. 3, p. 175.

In making a determination of the water equivalent, about 1,200 grams of water at any convenient temperature was weighed in the large calorimeter flask, the stopper adjusted, and the thermometer slipped into such a position that the bulb was entirely immersed. About 450 grams of water, approximately 20° hotter (or colder) than the water in the calorimeter flask, was weighed into the smaller flask, the stopper tightly inserted, and the thermometer slipped into place, being held in the glass sleeve by a short piece of close-fitting rubber tubing. A series of readings was then taken on each thermometer, the large flask being shaken by a rotary motion between each reading, and the small flask being completely inverted. The readings on the thermometer in the calorimeter were made through a telescope, while those on the thermometer in the hot (or cold) water flask were made with a reading lens such as is used with a Junker calorimeter.

When a satisfactory series of readings had been obtained, the thermometer was slipped out of the small flask and its contents quickly poured into the larger one, care being taken to properly insert the delivery tube of the small flask into the receiving tube of the calorimeter. The time required for the process was about four seconds. The weight of glass tubing heated (or cooled) in adding the water was only 1.6 grams, or for a difference of temperature of 15° the correction is about 4.8 calories. Experiments on the heat radiation of water from tubes of the same size gave a radiation loss in four seconds, and for a temperature difference equal to that used in the experiment, of only 1 calorie. As the larger portion of the four seconds during which water is poured into the flask is doubtless required to heat (or cool) the tubing, the actual radiation gain (or loss) is practically negligible. The heat carried off by the escaping air when cold water is added, or gained when hot water is added, amounts to less than 1 calorie and does not require correction.

From the temperature and weight of water in the calorimeter, and the temperature and weight of the water added, together with the observed changes in temperature, the following values were obtained for the water equivalent of the calorimeter flask:

Adding hot water:	**38.2**	**35.0**				
Adding cold water:	**37.9**	**36.8**	**37.0**	**35.6**	**33.8**	**35.6**

The average value, 36.2, was taken as the water equivalent of the flask and thermometer.

As a check on this value it is possible to calculate the water equivalent in the following way. The water equivalent of the portion of the thermometer in the flask was 4.2 calories. The weight of the calorimeter flask was 555 grams. Allowing 75 grams for the upper portion of the neck leaves 480 grams of the weight of the outer and inner flasks. Assuming the outer flask to weigh twice as much as the inner, gives 160 grams as the weight of the inner flask. Multiplying this weight by 0.2 (the specific heat of glass) gives 32 calories, which, added to the water equivalent of the thermometer, gives a calculated value of 36.2 calories as the water equivalent of the apparatus—in very close agreement with the experimental value.

With an average rise (or fall) of about 5° in the water in the calorimeter, and a fall (or rise) of about 15° in the water added, and with about 1,650 grams of water in the calorimeter at the conclusion of the experiment, the probable errors of a single determination are of about the following magnitude:

	Calories.
Error in weight of water added (± 0.3 gram)	± 4.5
Error in weight of water in calorimeter (± 0.2 gram)	± 1.0
Error in transferring correction (calculated)	± 5.0
Error in initial reading (calorimeter thermometer), ± 0.002°	± 2.5
Error in final reading (calorimeter thermometer), ± 0.002°	± 3.4
Error in reading hot (or cold) thermometer, ± 0.005°	± 2.2
Stem correction (calorimeter thermometer), ± 0.001°	± 1.6
Stem correction (hot or cold water thermometer), ± 0.001°	± 0.5
Error in radiation correction, ± 0.002°	± 3.2
Total	± 23.9

If all of these errors should have the same sign, the error in the water equivalent for a single determination would be 23.9 divided by 5, or 4.8 calories. But with the chances of some of the errors being of opposite signs, the probable error in a single determination should not exceed 10 calories, giving a value of 2 calories for the error in the water equivalent. Inspection of the values obtained for the water equivalent shows 6 out of 8 of them to fall within this limit.

The magnitude of the constant errors of a determination is about as follows:

	Calories.
Error in calibration of calorimeter thermometer, ± 0.002°	± 3.4
Error in calibration of hot (or cold) water thermometer, ± 0.005°	± 2.2
Error in adjustment of the two thermometers, ± 0.01°	± 4.5
Error in specific heat of water, probably less than	± 2.0
Total	± 12.1

If all of these errors should have the same sign, the maximum constant error in the water equivalent would be 12.1 calories divided by 5, or ± 2.4 calories. But the chances that some of the error are of opposite signs are about 7 to 1; so the probable error is not greater than ± 1 calorie. Taking into account the combined magnitude of variable and constant errors, the value 36 calories obtained for the water equivalent should be accurate to within less than ± 2 calories.

Heat of Fusion of Ice.—As a preliminary test of the possibilities of the calorimeter, some experiments were made on the heat of fusion of ice. The mode of procedure was to add weighed quantities of ice at 0° to weighed quantities of water in the calorimeter at a convenient observed temperature, and determine the fall in temperature of the water. From these data, together with the water equivalent of the calorimeter, the heat of fusion is readily calculated. Three determinations gave values as follows: 79.98, 79.88, 79.89, or an average of 79.92, which agrees almost exactly with Smith's value for the same constant.[1] These results are regarded as tentative, however, as it was later found that the ice sticks added may have been moist instead of dry. The results of experiments on this point have been described in detail elsewhere.[2]

Heat of Solution of Metals in Acid.—The manipulation involved in determining the heat of solution of a metal in an acid is very simple. A convenient quantity of acid of the desired strength is weighed in the vacuum calorimeter, the temperature of the acid being adjusted before it is put into the flask. The stopper and thermometer are inserted and readings are taken with a telescope at intervals of from 3 to 5 minutes, until a satisfactory series is obtained. The flask is shaken and the thermometer tapped between readings. A weighed quantity of the metal is then poured into the flask through the test-tube sleeve, which is again loosely stoppered. The flask is shaken from time to time and readings are taken at intervals of from 5 to 10 minutes, until they indicate complete solution of the metal, and a regular rate of gain or loss is established. The heat evolved is then calculated from the data obtained.

The temperature range during determinations was between 16.3° and 21.5°, which is very nearly the same range chosen by Richards and

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[1] Phys. Rev., Vol. 17, p. 193, Oct., 1903.

[2] Phys. Rev., Vol. 35, p. 128, Aug., 1912.

Burgess. The ratio of acid to metal was in all cases practically the same as in their experiments. By actual weight, about two thirds more of each was used, these quantities being better adapted to the size of the vacuum flask calorimeter. The three metals selected were zinc, aluminium and magnesium.

The temperature observations obtained during a determination of the heat of solution of magnesium in dilute hydrochloric acid are as follows:

Time.	Temperature.	Room Temperature.	Difference between Temperature of Flask and Room Temperature.
4:40	16.300	—	—
4:43	16.301	—	—
4:45	16.302	19.8	—
4:50	16.307	—	—
4:53	16.309	—	—
4:56	16.311	—	−3.5
Zinc added			
4:60	17.400	—	−2.3
5:04	19.150	—	−0.4
5:08	20.300	19.6	+0.7
5:12	20.530	—	+1.1
5:16	20.556	19.4	+1.1
5:20	20.572	—	+1.1
5:25	20.588	—	+1.2
5:30	20.587	19.4	—
5:33	20.586	—	+1.6
5:55	20.583	19.0	—
6:15	20.581	18.8	+1.8

The temperature 20.588 at the time 5:25 was taken as the final temperature of solution. The radiation and conduction corrections during the first ten minutes of the solution period practically balance those of the last 19 minutes as may be seen from the column of differences, the average of the first ten minutes, 4:56 to 5:06, being about − 1.9, the average of the last 19 minutes, 5:06 to 5:25 being about + 1.1. The average rate of gain per minute before the addition of the zinc with a temperature difference of 3.5° is .00066°. After the solution in the zinc with a temperature difference of 1.4°, the average rate of loss per minute is .00018°. The actual radiation-conduction correction applied in this determination by the author from the above data was + .0005°, a correction amounting to only .8 of one calorie.

Metals Used.

Zinc.—The zinc was presumably of the same grade as that used by Richards and Burgess (Merck's drop-shot zinc). It was washed with alcohol and dried before weighing. Tests as to its purity indicated the presence of 0.1 per cent. of iron. One gram of iron is equivalent to 0.79 gram of zinc in heating value when dissolved in acid; consequently for the present purposes the zinc was taken as 99.98 per cent. pure.

Magnesium.—Ribbon magnesium was selected. It was well polished and burnished before weighing by means of Tripoli powder and a piece of heavy cloth. Tests as to its purity resulted in a precipitate of iron and aluminium oxides equal to 0.1 per cent. of the magnesium, and equivalent to 0.06 per cent. of iron plus aluminium. The purity of the magnesium was therefore assumed to be 99.9 per cent.

Aluminium.—Sheet aluminium was used, being first washed with dilute hydrochloric acid and afterwards with alcohol. Tests for the presence of impurities gave 0.27 per cent. of iron, equivalent to 0.02 per cent. of aluminium; 0.16 per cent. of silicon, equivalent to 0.21 per cent. aluminium; 0.06 per cent. of carbon. The heat of evolution of the hydrocarbons is small and this carbon is assumed as having no heating value or as equivalent to 0.00 per cent. of aluminium in heating value. The total of impurities present amounted to 0.49 per cent., equivalent to 0.23 per cent. of aluminium. The purity of the aluminium was therefore assumed to be 99.75 per cent.

At the outset of the experiments it was the intention of the author to make use of acids of the same strength as those used by Richards and Burgess, namely, $HCl.20H_2O$ and $HCl.200H_2O$. In preparing the acids the preliminary standardization was made against a one tenth normal solution of sodium hydroxide which had been on hand for some time, methyl orange being used as the indicator. The final standardization, based upon the weighing of chlorine as silver chloride, was delayed until determinations of the heat of solution of each of the metals had been made. When this final standardization was made, it developed that the acid was about 4 per cent. stronger than had been intended, and investigation showed that this was due to the fact that the sodium hydroxide had acted upon the glass of the containing bottle, and had gained strength to the extent of about 4 per cent. As some determinations had been made before this fact was discovered, the series was completed with this stronger acid, but with the intention of preparing a second set of acids of exact strength and making a second series of measurements. A new solution of hydrochloric acid of the strength $HCl.20H_2O$ was actually prepared, and two determinations of the heat of solution of aluminium were completed, when the calorimeter flask went to pieces and ended the experiments until it is possible to secure and standardize another flask. Meanwhile it seems advisable to publish the results so far obtained.

In the calculation of the results, Richards's values for specific heats were taken as a basis, and the following values were used:

Specific heat, $HCl.20H_2O$ = 0.8486 (Richards).
" " $HCl.19.15H_2O$ = 0.8420 (Interpolated).
" " $HCl.200H_2O$ = 0.9814 (Richards).
" " $HCl.191.5H_2O$ = 0.9806 (Interpolated).

The temperature range through which Richards and Burgess worked was from 16° to 20°, while that employed by the author was from 16.5° to 21°. This slight difference can cause very little change in the specific heats, and the error in the assumed specific heats based on Richards's values cannot be large enough to materially affect any of the results.

The correction for the evolution of hydrogen was 213 calories for each gram molecular volume, which is practically the value adopted by Richards. The correction for the heat absorbed by the added metal was obtained by multiplying the weight of the metal by its specific heat, and by the difference between the final temperature of the solution and the temperature of the room at the time of the addition of the metal. The specific heats adopted were as follows: zinc = 0.098; aluminium = 0.202; magnesium = 0.245.

In determining the heat of solution of aluminium, the reaction between the metal and the acid was accelerated by the addition of a few drops of a solution of chloro-platinic acid to the hydrochloric acid solution. The heat evolved by the liberation of the platinum was calculated from the weight of the platinum precipitated, making use of the same data employed by Richards.

The atomic weights used were as follows: zinc, 65.37; magnesium, 24.32; aluminium, 27.10. All weighings of acid and metals were corrected to vacuum conditions.

The values obtained are those of the heat of solution of zinc in $HCl.19.15H_2O$; of magnesium in $HCl.191.5H_2O$; of aluminium in $HCl.19.15H_2O$; and in $HCl.20H_2O$. The data for these values are as follows:

From data given by Richards, Rowe and Burgess,[1] the heat of dilution of zinc in $HCl.20H_2O$ to zinc in solution in $HCl.200H_2O$ is 6,400 calories. The heat of dilution of zinc in $HCl.19.15H_2O$ to zinc in $HCl.200H_2O$ will evidently be greater than this—probably by as much as several hundred calories. 250 calories may be assumed as a conservative value. The dilution of the solution of magnesium in $HCl.191.5H_2O$ to magnesium in $HCl.200H_2O$ is probably attended with very little thermal change, and the value obtained for solution in $HCl.191.5H_2O$ is therefore prac-

[1] Jour. Am. Chem. Soc., Vol. 32, p. 1185.

No.	Metal.	Grams Taken.	Grams Acid.	Temperature Rise.	Water Equivalent of Acid + 96 for Calorimeter.	Calories of Heat Developed.	Correction for Platinum.	Heat Absorbed by Metal Added.	Correction for Hydrogen.	Heat per Gram Atomic Weight of Metal.	Heat per Gram Atomic Weight Corrected to 100 Per Cent. Purity.
1	Zinc.............	13.0758	1641(*a*)	4.2620	1417.7	6042.2	00	0.0	+42.6	30403	
2	Zinc.............	13.0756	1641(*a*)	4.2634	1417.7	6044.2	00	+3.8	+42.6	30434	
3	Zinc.............	13.0762	1641(*a*)	4.2633	1417.7	6044.1	00	+6.2	+42.6	30439	
	Average........									30425	30430
4	Magnesium.......	1.6225	1726(*b*)	4.2866	1728.5	7409.4	00	+1.3	+14.2	111310	
5	Magnesium.......	1.6227	1726(*b*)	4.2866	1728.5	7409.4	00	+0.3	+14.2	111294	
6	Magnesium.......	1.6225	1726(*b*)	4.2870	1728.5	7410.1	00	+0.3	+14.2	111306	
	Average........									111303	111410
7	Aluminum........	1.3554	1721(*a*)	4.2910	1485.1	6372.6	−0.4	0.0	+16.0	127802	
8	Aluminum........	1.3555	1721(*a*)	4.287	1485.1	6366.7	−0.4	0.0	+16.0	127693	
9	Aluminum........	1.3553	1721(*a*)	4.288	1485.1	6368.1	−0.4	+0.4	+16.0	127705	
10	Aluminum........	1.3552	1721(*a*)	4.290	1485.1	6371.1	−0.4	0.0	+16.0	127752	
	Average........									127738	128060
11	Aluminum........	1.3555	1707(*c*)	4.3038	1484.6	6389.4	−0.4	0.0	+16.0	128053	
12	Aluminum........	1.3555	1707(*c*)	4.3034	1484.6	6388.8	−0.4	0.0	+16.0	128043	
	Average									128048	128360

a = 19.15 H_2O, HCl. *b* = 191.5 H_2O, HCl. *c* = 20 H_2O, HCl.

tically the same as for solution in $HCl.200H_2O$. The following table gives a comparison of the values obtained by the author with those obtained by Richards:

	Author.		Richards.	
	Solution in $HCl.19.15H_2O$.	Solution in $HCl.20H_2O$.	Solution in $HCl.20H_2O$.	Difference.
Zinc	30,430	(30,680)	30,180	400
Aluminium	128,060	128,360	126,000	2,360
	Solution in $HCl.191.5H_2O$.	Solution in $HCl.200H_2O$.	Solution in $HCl.200H_2O$.	
Magnesium	111,410	(111,410)	110,150	1,260

The values for zinc and magnesium which are included in parentheses are based upon experiment together with data already given.

In making the determinations the author used one fifth of a gram-atom of zinc; one fifteenth of a gram-atom of magnesium; and one twentieth of a gram-atom of aluminium. With these weights, the differences tabulated above, computed to a single determination, are as follows: zinc, 80 calories; aluminium, 118 calories; magnesium, 84 calories. In the case of aluminium a fraction of this discrepancy may be due to an actual difference in the materials used, and the corrections applied for the impurities in the metal. A small fraction of the discrepancy in the case of zinc and magnesium may also be due to the same causes. It does not seem possible, however, that any large fraction of the discrepancies can be accounted for in this way.

In regard to errors in a determination which might be attributed to the apparatus, it may be said that all of the apparatus used by the author was checked at every point. The corrections to the thermometers have already been described. The weights were all adjusted very carefully to one another, and to a standard set secured through Troemner, and double checked by him against standards supplied by the Bureau of Standards. Errors in the thermometers or weights cannot, therefore, have been of any appreciable magnitude. The error in the water equivalent of the calorimeter is believed to have been safely within ± 2, which amounts to less than 10 calories in a determination. The errors in reading the thermometers amount to only 1.6 calories for each 0.001°, so that these cannot account for any great difference in results. The total radiation corrections in most cases amounted to less than 5 calories and in no case to more than 10, and it does not seem possible that such errors are accountable for any appreciable fraction of the discrepancy.

The heat obtained during a determination must have come from the reaction as no complicated stirring apparatus was used, and experiments showed that the heat produced by shaking the solution during a determination amounted to less than 3 calories; moreover, the greater part of this error is eliminated in the initial and final rate correction.

In reviewing the work of Richards and Burgess the author notes the fact that the water equivalent of the adiabatic calorimeter, used in the solution of the metals in acid, is calculated and is not verified by experimental determinations. As to whether the use of a calculated water equivalent, not verified by experimental means, accounts for any of the discrepancy in results, the author is unable to venture an opinion.

In the opinion of the author it is entirely possible that the radiation correction for the adiabatic calorimeter, instead of being zero, may have an actual value; and that for the same operator, the same calorimeter, and the same set of conditions, it may be practically a constant. Such a constant, if it really exists, would, in the case of a calorimeter with a calculated water equivalent, cause the results to be uniformly low or high; whereas in the adiabatic bomb calorimeter used by Richards and Jesse,[1] in measuring the heat of combustion of octanes, etc., the water equivalent is experimentally determined against cane sugar. Any systematic error is absorbed in this experimentally determined water equivalent, and is eliminated from the final results. So far as the author is aware there are no published data giving comparisons between calculated and experimental water equivalents for an adiabatic type of calorimeter, and as to whether or not they agree, he has at present no means of knowing.

The results described in this paper are to be regarded as preliminary only, and the author hopes to find time and opportunity for continuing the investigation. Meanwhile he hopes that the publication of the results already secured may stimulate other investigators to take up thermal measurements.

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[1] Jour. Am. Chem. Soc., Vol. 32, p. 268.

THE ELECTRON THEORY OF PHOSPHORESCENCE.[1]

BY CHESTER A. BUTMAN.

IT is the purpose of any theory of phosphorescence to explain the mechanism of the secondary phenomena which causes the luminosity to continue after the exciting light is shut off.

The theory proposed is that the incident light causes the electrons in the light-emitting atom of the complex phosphorescent molecule to be brought to a critical-energy-content causing the selective emissions characteristic of the substance; at the same time the light causes electrons to be ejected from the photoelectric atom; the return of the electrons to the molecule by collision causes the luminosity to continue.

The following is a list of most of the facts that any theory of phosphorescence must explain.

1. Luminosity after the exciting light is cut off.
2. Relation to Stokes' law.
3. Rate of phosphorescent decay.
4. The extinguishing action of red light.
5. The flashing up caused by heating.
6. Why a band spectrum is produced.
7. How two bands with different rates of decay can exist in the same material.
8. Why the generation-distribution curves of Lenard and Klatt are possible.
9. Relation to thermoluminescence.
10. Relation of triboluminescence.
11. Relation to gas phosphorescence.
12. Relation to the general production of luminosity.
13. Relation of the color of the material to the color of the bands.
14. Relation of the intensity of the phosphorescence to the intensity of the exciting source.
15. Why a maximum phosphorescent intensity exists.
16. Why a certain time is necessary to produce the duration bands.
17. Why infra-red phosphorescence can exist.

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[1] See also abstract of paper read before the American Physical Society, March 2, 1912; PHYS. REV., Vol. 34, p. 316, 1912.

18. Shift of the bands when exposed to long waves.
19. Relation to the absorption of light.
20. Why the brightening up is more prominent at low temperatures than the decay when exposed to red light.
21. Relation to fluorescence.
22. Excitation by alpha particles.
23. Excitation by beta particles.
24. Excitation by canal rays.
25. Excitation by Röntgen rays.
26. Relation of phosphorescent intensity to the amount of light-producing metal present.
27. Effect of the flux on the intensity.
28. Relation to the dielectric constant.
29. Why the alkaline earths and zinc occur so prominently in phosphorescent compounds rather than other elements.
30. Why sulphur is usually present in long duration phosphorescent compounds.
31. How it is possible to substitute selenium for sulphur and have a phosphorescent compound.
32. Relation to the photoelectric effect.
33. Relation to photoelectric fatigue and recovery.
34. Relation to the actinodielectric effect.
35. Relation between phosphorescent inorganic and organic bodies.
36. Relation to temperature changes.
37. Why phosphorescence is a property of the solid state.

The theory outlined here differs from that of Lenard in several particulars. In the case of the alkaline earth sulphides the photoelectric atom is held to be sulphur, while Lenard's theory indicates that it is the heavy metal. Recent experiments seem to support the view taken here. For instance, it has been shown by Pohl and Pringsheim[1] that the photoelectric effect of Bi stops at about 3,100 Ang. This means that Bi would not be photoelectrically active if the light had first passed through glass. Again Pohl and Pringsheim[2] have shown that when two photoelectric components are in combination that the combination is not excited by wave-lengths shorter than will excite either component not in combination. Hence it does not seem probable that in the phosphorescent combination Ca-S-Bi that the Bi is photoelectrically active after the light has passed through glass. Nevertheless Ca-S-Bi is strongly phosphorescent and gives a photoelectric effect with light that has passed through glass. However, it has been shown by the author[1] that sulphur is photoelectric with light through glass. It has been shown by Lenard and Saeland,[2] and confirmed by the author,[3] that those rays which cause Ca-S-Bi to phosphoresce, cause it to be photoelectric. Also those rays which do not cause it to phosphoresce do not give a photoelectric effect. Hence, it would seem that it is necessary to have a photoelectric component in order to have a phosphorescent compound. This last being the case, it is seen why it is that sulphur appears so prominently in phosphorescent compounds of long duration. The function of the sulphur then is to supply a photoelectric component having high insulating properties. It is necessary that phosphorescent substances should be good insulators so that the charges will recombine slowly, making a phosphorescent compound of long duration.

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[1] R. Pohl and P. Pringsheim, Verh. d. D. Phys. Ges., 13, p. 480, 1911.

[2] R. Pohl and P. Pringsheim, Verh. d. D. Phys. Ges., 12, pp. 215–228, 349–360, 1912.

Another fact which supports the theory that sulphur is responsible for the electrical behavior of the alkaline earth sulphides, is that sulphur is actinodielectric, as has been shown by the author.[4] It was discovered by Lenard and Saeland,[5] and the work extended by Oeder, Ramsauer and Hausser,[6] that Ca-S-Bi was actinodielectric. It was shown that if Ca-S-Bi be placed between a net and a plate (the plate being connected to an electrometer, the net to a source of potential) that if the substance was exposed to light a deflection of the electrometer would take place. This effect differs from the photoelectric effect in that the deflection takes place in the direction of the applied field, whereas the photoelectric effect goes in one direction. Another difference is that Ca-S-Bi gives no photoelectric effect with red light but does give an actinodielectric effect with it. It was shown by the author[7] that sulphur gave an actinodielectric effect with red light.

Another confirmation of the theory would seem to be found in the case of quinine sulphate. Miss Gates[8] found that this substance was phosphorescent while cooling after heating to about 100° C. She also found that the radiations from this substance ionized the air better with a *positive* field than a negative. It would seem, then, that the heating sets some of the electrons free from the sulphur so that the same condition obtains as that of the alkaline earth sulphides after having been exposed to light.

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[1] C. A. Butman, Amer. Jour. Sci., Vol. 34, pp. 133–139, 1912. "The Photoelectric Effect of Phosphorescent Material," also PHYS. REV., Vol. 34, p. 158, 1912.

[2] P. Lenard and Sem. Saeland, Ann. der Phys., Vol. 27, pp. 476–502, 1909.

[3] C. A. Butman, Amer. Jour. Sci., Vol. 34, pp. 133–139, 1912.

[4] C. A. Butman, Amer. Jour. Sci., Vol. 34, p. 139, 1912.

[5] Loc. cit.

[6] Rob. Oeder, C. Ramsauer, W. Hausser, Ann. der Phys., pp. 445–454, 1911.

[7] C. A. Butman, Amer. Jour. Sci., Vol. 34, pp. 133–139, 1912.

[8] J. J. Thomson, "Conduction through Gases," p. 425.

Again it is stated by Lenard[1] and confirmed by the author, that it is impossible to make a phosphorescent compound out of such elements as Ca and Bi without sulphur. However, it may be predicted from the theory that if a photoelectric component with good insulating properties, giving a photoelectric effect through glass could be found, it could be substituted in place of sulphur and a phosphorescent compound formed. This prediction would seem to be fulfilled in the recent work of Pauli[2] who has made a phosphorescent compound by substituting selenium for sulphur.

MECHANISM OF LIGHT PRODUCTION.

The rate of decay of the phosphorescent light would seem to indicate that the phenomenon was a kinetic phenomenon. The rate of decay of Sidot blende has been studied in great detail by Nichols and Merritt.[3] Practically no change is necessary in the considerations advanced by them to show that a recombination of the electrons by collision, as proposed here, would give the laws of decay.

According to this theory, the flashing up of the substance when exposed to heat can be attributed to increasing the rate of collision.

The phenomena of the photoelectric fatigue and recovery worked upon by the author[4] are best understood by assuming that the electrons set free get entangled in the substance and build up fields in it. "Fatigue would then be due to this accumulated field building up." "Recovery would seem to be due to the recombination of the electrons, which probably takes place quite slowly." The foregoing seems to offer an explanation of why a certain time is necessary to produce the duration bands. For it would seem that it would be necessary to set free a large number of electrons before luminosity would follow on their recombination. It also seems to show why a maximum phosphorescent intensity exists, as the rate of recombination would not probably exceed a given value.

That luminosity can be maintained by the recombination of the electrons would seem to be well supported from the phenomena of gas phosphorescence. The theory can be applied with success to the experiments by C. C. Trowbridge[5] on gas phosphorescence. He found that it took some time to form the phosphorescence (that is, for the gas to get into the proper electrical condition).

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[1] P. Lenard, Ann. der Phys., 31, p. 668, 1910.

[2] W. E. Pauli, Ann. der Phys., 38 (4), pp. 870–880, 1912.

[3] Nichols and Merritt, PHYS. REV., 27, p. 367, 1908.

[4] C. A. Butman, Amer. Jour. Sci., Vol. 34, pp. 135–139, 1912.

[5] C. C. Trowbridge, PHYS. REV., Vol. 32, pp. 129–151, 1911.

Another fact found was that "The intensity of luminosity of the phosphorescing gas depended in a marked manner on the pressure of the gas, and that the intensity was very much less when the gas pressure was low than when it was high." This last would seem to be explained as being due to the rate of recombination being more rapid at a higher pressure.

The theory would indicate that the rate of decay of the phosphorescence would vary according to the electrical conditions of the gas. This is seen to be the case from the fact that "The rate of decay may vary considerably while the gas pressure remains constant." However, one would expect that the rate of decay would depend somewhat on the pressure and it was found that there was a given pressure which gave the longest decay. Nevertheless, as the theory would indicate, the phosphorescence was not as bright as at higher pressures. He says further: "There is also a hysteresis effect which makes the behavior of the phosphorescence depend on the previous history of the gas, although the electrodeless discharge itself appears to suffer no alteration." This seems to be the same as the hysteresis effects observed in solid phosphorescent substances. The gas molecules appear to become charged and luminosity is continued, due to the recombination of the electrons at the time of collision. As Sir J. J. Thomson[1] has already assumed, the molecules must have acquired a certain amount of energy in order that luminosity be produced.

THE CAUSE OF THE SPECTRA OF PHOSPHORESCENT MATERIAL.

The light-producing element in the phosphorescent alkaline earth sulphides is held by everybody to be mainly due to the heavy metal. However, the spectra are somewhat modified by being in combination with the alkaline earth element and the sulphur. It is the idea of the author that the light from the heavy metal is a selective emission from that element. This explains why the color of the phosphorescent light is the same as the color of the substance by reflection, also why a banded spectrum is given out and why different bands can be generated by choosing incident light of the proper frequency. As is to be expected there is a direct relation between the regions where the light is absorbed and where it is emitted. Again Stokes's law should hold in some degree. These last have been verified by Bachem[2] recently. The work of Brunninghaus[3] also supports the view taken here.

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[1] J. J. Thomson, Nature, 73, pp. 495–499, 1906.

[2] Alb. Bachem, Ann. der Phys., Vol. 38, pp. 697–720, 1912.

[3] Bruninghaus, Le Radium, April, 1911.

The composition of an inorganic phosphorescent compound then is an alkaline earth metal which is transparent to the rays sent out from the heavy metal and which acts as a dilutant, a trace of a heavy metal giving a selective emission in the visible region, and a photoelectric component active near the visible region and also being a good insulator. These three elements to be in chemical combination. Only a trace of the heavy metal can be present in order that the insulating properties may not be impaired. Hence, infra-red phosphorescence is seen to be possible from the theory in that all that is necessary is to find a metal having a selective emission in the infra-red.

The changes produced by changing the temperature are hard to explain at present, but no doubt the change in the rate of collision and the mean free path changes the critical energy-content and also in some cases the period of the emitting electrons.

The extinguishing action of red light is another hard problem requiring explanation. This may be explained in the following manner. According to the theory it is necessary for the emitting electronic systems to have a certain amount of oscillating energy, in order to produce luminosity. Now the red light has a somewhat less frequency than the emitted light and it may be assumed it dampens the vibrations of the oscillating system so that they fall below their critical value and the luminosity ceases. An experiment which supports this view, is that if a phosphorescent substance be exposed to red light for a sufficient length of time it will not generate luminosity when exposed to the usual exciting sources. This may be interpreted that the red light caused the electronic systems to take up forced vibrations which were maintained for some time and that the usual exciting light could not bring the oscillations up to their critical value in a brief time. It seems quite certain that the oscillations are continued for a considerable time from the large number of interference fringes observed with light from a phosphorescent substance by Voigt.[1] Another experiment explained by this theory is the shift of the bands toward the red end of the spectrum when the phosphorescent substance is exposed to long light waves. It would appear that the long waves dampened the frequency causing the shift in the position of the band.

The foregoing discussion has dealt with the inorganic phosphorescent substances, but inasmuch as the organic substances examined by Kowalski[2] have practically the same properties as the inorganic, it would seem to hold in the case of organic substances also.

It is to be noticed that the theory outlined here gives a theory of

[1] Voigt, Arch. Neerl (2), 6, pp. 352-366, 1901.

[2] Kowalski, Phys. Zeit., 12, pp. 956–969, 1912.

fluorescence as well as phosphorescence. If the incident light causes the oscillating systems to come to their critical energy-content, and they emit light, this is a case of fluorescence. However, as soon as the exciting light was removed, the oscillating systems would fall below their critical value and cease to emit light. It would also seem from the theory that there would be no observable lag in the production of the emitted light, which appears experimentally to be the case. Many fluorescent substances are probably photoelectric, but being in the liquid state, the recombination takes place quickly. Nevertheless if these substances are brought into the solid state in the presence of an insulator, they become phosphorescent, as has been shown by Kowalski.[1]

AMHERST, MASS.,
August 14, 1912.

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[1] Loc. cit.

ON THE THEORY OF RELATIVITY: MASS, FORCE AND ENERGY.

By R. D. Carmichael.

Introduction.

In a previous paper[1] I have given an analysis of the characteristic postulates on which the theory of relativity depends and have developed in a general way some of the fundamental conclusions of this theory. The plan of treatment adopted makes it possible to arrive at these results by arguments which are not complicated in form; and the notions which enter into them are of simpler character than those usually to be found in memoirs on this subject.

It seems desirable that the general theory should be further worked out along the same lines, even though this should require in part a repetition of some things already in the memoirs. Accordingly, in the present paper I develop the fundamental properties[2] of mass, force and energy, using as a basis the theorems of my previous paper. *In the derivation of these results the theory of electricity is not employed.* Consequently the conclusions are of wider applicability than when they are proved by means of electrical theory; in particular, they may be applied to the derivation of results in the theory of electricity itself.

In § 1, after giving some definitions, I state the laws of conservation of momentum and energy and electricity and the principle of least action in the form in which I shall have occasion to use them. The law of conservation of electricity is employed in this paper only in the applications of the results concerning mass and energy.

In § 2 the question of the dependence of mass on velocity is treated. First the transverse mass of a moving body is determined by the elegant method of Lewis and Tolman. The relation between transverse mass and longitudinal mass is found by the method of Bumstead, and thus the longitudinal mass of a moving body is obtained.

In § 3 dimensional equations are employed to derive the relations of acceleration and force in two systems of reference.

In § 4 from considerations concerning the mass of a moving body two essential equivalents of postulate R are determined, each of which furnishes a possible means for the experimental proof or disproof of the theory of relativity. The researches of Bucherer have been thought to afford the requisite experimental confirmation in the first case; this matter is treated in § 5. In § 6 suggestions are given for a new *crucial* experiment for testing the theory of relativity, this being associated with the second essential equivalent of R in § 4. The writer desires to call especial attention to this proposed experiment.

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[1] Physical Review, Vol. 35 (1912), pp. 153–176.

[2] Several applications of the principal conclusions are also given.

In § 7 the intimate relation of the mass and the total energy of a body is pointed out and two theoretical means are suggested for determining the velocity of light indirectly, that is, without direct measurement of this velocity. These experiments, if they could be performed with sufficient accuracy, would afford an interesting and striking confirmation of the theory of relativity, provided of course that they turn out according to the predictions of this theory.

A few remarks on the principle of least action are found in § 8, and § 9 is given to some speculative considerations which are intended as brief suggestions of means by which one may represent to himself the conclusions of relativity as natural parts of a consistent view of physical phenomena.

§ 1. FUNDAMENTAL DEFINITIONS AND POSTULATES.

If m, M and v are respectively the mass, momentum and velocity of a body we shall assume (as in the classical mechanics) that they are connected by a relation of the form

$$M = mv;$$

and hence any one of these quantities is determined in terms of the other two (except that mass is not thus determined when velocity and momentum are zero). We shall take mass and velocity to be the two fundamental quantities, so that momentum is *defined* in terms of them.

Likewise we shall define the kinetic energy E of a moving body by means of the usual relation

$$E = \int_0^v M dv = \int_0^v mv dv.$$

Later we shall see that "mass" is variable and is not in general independent of the direction in which it is measured; consequently, we must take for m in the above formulæ the mass of the body in the direction of its motion.

We shall take for granted the following laws of conservation of momentum and energy and electricity:

POSTULATE C_1. *The sum total of momentum in any isolated system remains unaltered, whatever changes may take place in the system, provided that it is not affected by any forces from without.*

POSTULATE C_2. *The sum total of energy in any isolated system remains unaltered, whatever changes may take place in the system, provided that it is not affected by any forces from without.*

POSTULATE C_3. *The sum total of electricity in any isolated system remains unaltered, whatever changes may take place in the system, provided that the system as a whole neither receives electricity from nor gives out electricity to bodies not belonging to the system.*

The "action" of a moving body in passing from one position to another may be defined as the space integral of the momentum taken over the path of motion. If we denote this action by A we have therefore

$$A = \int Mds = \int mvds.$$

Now $ds = vdt$, so that we have also

$$A = \int mv^2dt.$$

If several bodies are involved we have

$$A = \Sigma\int mvds = \Sigma\int mv^2dt$$

where the summation is for the various bodies in the system.

We may state the fundamental principle of least action in the following form:

PRINCIPLE OF LEAST ACTION. *The free motion of a conservative system between any two given configurations has the property that the action A is a minimum, the admissible values $\bar{A}$ of the action with which A is compared being obtained from varied motions in which the total energy has the same constant value as in the actual free motion.*

§ 2. DEPENDENCE OF MASS ON VELOCITY.

Suppose that we have two systems of reference S_1 and S_2 moving with a relative velocity v. We inquire as to whether, and in what way, the mass of a body as measured on the two systems depends on v. Will a given body have the same measure of mass when that mass is estimated in units of S_1 and in units of S_2? And will the mass of a body depend on the direction of its motion by means of which that mass is measured? Our purpose in this section is to answer these two questions.

The two most important directions in which to measure the mass of a body are, first, that perpendicular to the line of relative motion of S_1 and S_2, and, secondly, that parallel to this line of motion. For convenience in distinguishing these we shall speak of the "transverse mass" of a body as that with which we have to deal when we are concerned with the motion of the body in a direction perpendicular to the line of relative motion of S_1 and S_2; when the motion is parallel to this line we shall speak of the "longitudinal mass" of the body.

Lewis and Tolman[1] determine what they call the "mass of a body in motion," employing for this purpose a very simple and elegant method. This "mass" is what we have just defined as the transverse mass of the body. We employ the excellent method of these authors in deriving the formula for transverse mass.

Suppose that an experimenter A on the system S_1 constructs a ball B_1 of some rigid elastic material, with unit volume, and puts it in motion with unit velocity in a direction perpendicular to the line of relative motion of S_1 and S_2, the units of measurement employed being those belonging to S_1. Likewise suppose that an experimenter C on S_2 constructs a ball B_2 of the same material, also of unit volume, and puts it in motion with unit velocity in a direction perpendicular to the line of relative motion of S_1 and S_2; we suppose that the measurements made by C are with units belonging to S_2. Assume that the experiment has been so planned that the balls will collide and rebound, over their original paths, the path of each ball being thought of as relative to the system to which it belongs.

Now the relation of the ball B_2 to the system S_1 is the same as that of the ball B_1 to the system S_2, on account of the perfect symmetry which exists between the two systems of reference in accordance with the results of the previous paper (already referred to). Therefore the change of velocity of B_2 relative to its starting point on S_2 as measured by A is equal to the change of velocity of B_1 relative to its starting point on S_1 as measured by C. Now velocity is equal to the ratio of distance to time: and in the direction perpendicular to the line of relative motion of the two systems the units of length are equal; but the units of time are unequal. Hence to either of the observers the change of velocity in the two balls, each with respect to its starting point on its own system, will appear to be unequal.

To A the time unit on S_2 appears to be longer than his own in the ratio $\sqrt{1-\beta^2} : 1$ (see previous paper, theorem IV., p. 167). Hence to A it must appear that the change in velocity of B_2 relative to its starting point is smaller than that of B_1 relative to its starting point in the ratio $\sqrt{1-\beta^2} : 1$. But the change in velocity of each ball multiplied by its mass gives its change in momentum. From postulate C_1 it follows that these two changes of momentum are equal. Hence to A it appears that the mass of the ball B_1 is smaller than that of the ball B_2 in the ratio $\sqrt{1 - \beta^2} : 1$.

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[1] Phil. Mag., 18 (1909), 510–523. See especially pp. 517–518.

Similarly, it may be shown that to C it appears that the mass of the ball B_2 is smaller than that of B_1 in the ratio $\sqrt{1 - \beta^2} : 1$.

From the general theorems concerning the measurement of length (in the theory of relativity) it follows that if the ball which has been constructed by A were transferred to C's system it would be impossible for C to distinguish A's ball from his own by any considerations of shape and size. Likewise, as A looks at them from his own system he is similarly unable to distinguish them. It is therefore natural to take the mass of C's ball as that which A's would have if it had the velocity v with respect to S_1 of the system S_2. Thus we obtain a relation existing between the mass of a body in motion and at rest.

Now, "mass" as we have measured it above is the transverse mass of our definition. From the argument just carried out we are forced to conclude that the transverse mass of a body in motion depends (in a certain definite way) on the velocity of that motion. The result may be formulated as follows:

Theorem I. *Let m_0 denote the mass of a body when at rest relative to a system of reference S. When it is moving with a velocity v relative to S denote by $t(m_v)$ its transverse mass, that is, its mass in a direction perpendicular to its line of motion. Then we have*

$$t(m_v) = \frac{m_0}{\sqrt{1 - \beta^2}},$$

where $\beta = v/c$ and c is the velocity of light $(MVLRC_1)$.[1]

In the statement of this theorem we have tacitly assumed that the mass of a body at rest relative to S, when measured by means of units belonging to S, is independent of the direction in which it is measured. If this assumption were not true we should have a means of detecting the motion of S, a conclusion which is in contradiction to postulate M.

In order to find the longitudinal mass of a moving body we first find the relation which exists between longitudinal mass and transverse mass. We employ for this purpose the elegant method of Bumstead.[2]

Let us as usual consider two systems of reference S_1 and S_2 moving with a relative velocity v, observers A and B being stationed on S_1 and S_2 respectively. Suppose that B performs the following experiment:

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[1] These letters attached to the theorem indicate that in its proof we have employed postulates M, V, L, R, C_1. Compare a similar usage in my previous paper.

[2] American Journal of Science (4) 26 (1908), pp. 498–500.

He takes a rod of two units length, whose mass is so small as to be negligible, and attaches to its ends two balls of equal mass. Then he suspends this rod by a wire so as to form a torsion pendulum. We assume that the line of relative motion of the two systems is perpendicular to the line of this wire.

Let us consider the period of this torsion pendulum in the two cases when the rod is clamped to the wire so as to be in equilibrium in each of the following two positions: (1) With its length perpendicular to the line of relative motion of S_1 and S_2; (2) with its length parallel to this line of motion.

As B observes it the period must be the same in the two cases; for, otherwise, he would have a means of detecting his motion by observations made on his system alone, contrary to postulate M. Then from the relation of time units on S_1 and S_2 it follows that the two periods will also appear the same to A. As observed by B the apparent mass of the balls is the same in both cases. We inquire as to how they appear to A. Let m_1 and m_2 be the apparent masses, as observed by A, in the first and second cases respectively. It is obvious that m_1 is the longitudinal mass and m_2 the transverse mass of the balls in question.

When the pendulum is in motion it appears to B that each ball traces a circular arc. From the relations between units of length in the two systems it follows that to A it appears that the balls trace arcs of an ellipse whose semi-axes are 1 and $\sqrt{1 - \beta^2}$ and lie perpendicular and parallel, respectively, to the line of relative motion of the two systems.

Let us now determine the period of each of these two pendulums as they are observed by A. By equating the expressions for these periods we shall find the relation which exists between m_1 and m_2.

Let x and y be the cartesian coördinates of a point as determined by A, the axes of reference being the major and minor axes of the ellipse in which the balls move. Let x' and y' be the coördinates of the same point as determined by B. Then the circular path of motion, as determined by B, has the equations

$$x' = \cos\theta, \quad y' = \sin\theta,$$

the angle θ being measured from the major axis of the ellipse. The equations of the ellipse, as determined by A, are

$$x = \cos\theta, \quad y = \sqrt{1 - \beta^2}\sin\theta.$$

In the first case—when the rod is perpendicular to the line of relative motion of S_1 and S_2—the amount of twisting in the wire when the ball is in a given position is the absolute value of the corresponding angle θ;

and therefore the potential energy[1] is proportional to θ^2, say that it is $\frac{1}{2}k\theta^2$. Now from the values of y and x above we have

$$y = x\sqrt{1 - \beta^2}\tan\theta.$$

For small oscillations we have $x = 1$ and $\tan\theta = \theta$; and therefore

$$y = \sqrt{1 - \beta^2}\cdot\theta.$$

Hence the potential energy is

$$\frac{1}{2}\frac{k}{1 - \beta^2}y^2;$$

and the equation of motion of the particle becomes

$$m_1\frac{d^2y}{dt^2} = -\frac{k}{1 - \beta^2}y.$$

Hence the period T_1 of oscillation is

$$T_1 = 2\pi\sqrt{\frac{m_1(1 - \beta^2)}{k}}.$$

In the second case—when the rod is parallel to the line of relative motion of S_1 and S_2—the amount of twisting in the wire for a given position of the balls is the absolute value of $(\pi/2) - \theta$. The potential energy is $\frac{1}{2}k[(\pi/2) - \theta]^2$. We have

$$x = \frac{y}{\sqrt{1 - \beta^2}}\cot\theta.$$

For small oscillations we have

$$y = \sqrt{1 - \beta^2},\ \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right) = \frac{\pi}{2} - \theta.$$

Hence the potential energy is $\frac{1}{2}kx^2$, and the period T_2 of oscillation is therefore

$$T_2 = 2\pi\sqrt{\frac{m_2}{k}}.$$

Equating the two periods of oscillation found above we have

$$m_2 = (1 - \beta^2)m_1.]$$

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[1] That the potential energy is proportional to θ^2 when measured by B is obvious. Since A observes a different apparent angle θ' (say) corresponding to B's observed angle θ it might at first sight appear that the potential energy as observed by A is proportional to θ'^2. That this is not the case is seen from the fact that for a given twist in the wire θ' depends on the direction of equilibrium of the bar, that is, it depends on the way in which the bar is attached to the wire; hence, if the potential energy as observed by A were proportional to θ'^2 it would depend on the way in which the bar is attached. Since this is obviously not the case we conclude that the potential energy is proportional to θ^2.

Remembering that m_1 and m_2 are the longitudinal mass and transverse mass, respectively, and making use of theorem I., we have the following result:

THEOREM II. *Let m_0 denote the mass of a body when at rest relative to a system of reference S. When it is moving with a velocity v relative to S denote by $l(m_v)$ its longitudinal mass, that is, its mass in a direction parallel to its line of motion. Then we have*

$$l(m_v) = \frac{m_0}{(1 - \beta^2)^{\frac{3}{2}}},$$

where $\beta = v/c$ and c is the velocity of light ($MVLRC_1C_2$).

§ 3. ON THE DIMENSIONS OF UNITS.

Denote the fundamental measurable physical entities mass, length and time by M, L and T respectively. Then the definition of derived entities gives rise to the so-called dimensional equations. Thus if B denote velocity, then from the definition of velocity we have the dimensional equation

$$V = \frac{L}{T}.$$

That such equations must be useful in obtaining the relations of a derived unit in two systems of reference is obvious. Thus from the above dimensional equation for V we may at once derive the fundamental result (see previous paper, theorem VI., p. 170) concerning the relation of units of length in the line of relative motion of two systems not at rest relatively to each other. For this purpose it is sufficient to employ postulate V. and theorem IV. of the previous paper. The reader can easily supply the argument. Or, conversely, if one knows the relations which exist between units of length and units of time in two systems one concludes readily to the truth of postulate V.

Likewise, from the dimensional equation

$$\text{acceleration} = \frac{L}{T^2}$$

one may readily determine the relations which exist between units of acceleration on two systems, it being assumed that the relations of time units and length units are known. Making this assumption, then, the two dimensional equations above give us the following theorem:

THEOREM III. *Let two systems S_1 and S_2 move with a relative velocity v in the direction of a line l, and let $\beta = v/c$ where c is the velocity of light. Then to an observer on S_1 it appears that the unit of velocity [acceleration] on*

S_1 bears to the unit of velocity [acceleration] on S_2 the ratio $1 : 1$ $[1 : \sqrt{1 - \beta^2}]$ *or* $1 : \sqrt{1 - \beta^2}$ $[1 : 1 - \beta^2]$ *according as the motion is parallel to l or perpendicular to l* ($MVLR$).

Let us use F to denote force. Then from the dimensional equation

$$F = \frac{ML}{T^2},$$

we shall be able to draw an interesting conclusion concerning the measurement of force.

Suppose that an observer B on a system S_2 carries out some observations concerning a certain rectilinear motion, measuring the quantities M', L', T', so that he has the equation

$$F' = \frac{M'L'}{T'^2}.$$

Another observer A on a system S_1 (having with respect to S_2 the velocity v in the line l) measures the same force, calling it F. Required the value of F in terms of F', when the motion is parallel to l and when it is perpendicular to l, the estimate being made by A.

When the motion is perpendicular to l—that is, when the force acts in a line perpendicular to l—we have

$$F_1 = \frac{ML}{T^2} = \frac{M'\sqrt{1 - \beta^2} \cdot L'}{T'^2(1 - \beta^2)} = \frac{F'}{\sqrt{1 - \beta^2}}.$$

When the motion is parallel to l we have

$$F_2 = \frac{ML}{T^2} = \frac{M'(1 - \beta^2)^{\frac{3}{2}} \cdot L'\sqrt{1 - \beta^2}}{T'^2(1 - \beta^2)} = (1 - \beta^2)F'.$$

These results may be stated in the following theorem:

Theorem IV. *In the same systems of reference as in theorem III., let an observer on S_2 measure a given force F' in a direction perpendicular to l and in a direction parallel to l, and let F_1 and F_2 be the values of this force as measured in the first and second cases respectively by an observer on S_1. Then we have*

$$F_1 = \frac{F'}{\sqrt{1 - \beta^2}}, \quad F_2 = (1 - \beta^2)F' \qquad (MVLRC_1C_2).$$

It is obvious that a similar use may be made of the dimensional equation of any derived unit in determining the relation which exists between this unit in two relatively moving systems of reference.

§ 4. EQUIVALENTS OF THE POSTULATES.

The problem of determining sets of postulates logically equivalent to those which have been used as a basis of the theory of relativity is obviously important. So far as a justification of the theory is concerned it is, however, unnecessary to have complete logical equivalents; all that is essential is to find a set of postulates, experimentally demonstrable, by means of which it is possible to demonstrate the characteristic conclusions of relativity concerning the relations of units of time and units of length in two systems of reference.[1] Such a set of postulates we shall call *essential equivalents* of the postulates of relativity. The object of this section is to determine essential equivalents of postulate R, that is, such postulates as may be taken in connection with postulates M, V, L, so that the new set shall be essentially equivalent to M, V, L, R.

For this purpose let us first consider the relation between the transverse mass of a moving body and its mass at rest as given in theorem I. Let us suppose that this theorem is true[2] (whether proved experimentally or otherwise); and let us seek its consequences. Suppose that the experiment by means of which we proved theorem I. is now repeated. If we again assume the law of conservation of momentum and equate the two observed changes in momenta, it is clear that we shall have a relation between measurements of time as carried out in the two systems of reference, and that this relation will be precisely the same as in the usual theory of relativity. Having this relation concerning time units we can then proceed as in the first paragraph in § 3 to derive the usual relations between units of length. Hence we have the following result:

THEOREM V. *If* m_0 *and* $t(m_v)$ *have the same meaning as in theorem I. and if for any particular kind of matter whatever we have the relation*

$$t(m_v) = \frac{m_0}{\sqrt{1 - \beta^2}},$$

then this fact and postulates $(MVLC_1)$ *form an essential equivalent of postulates* $(MVLRC_1)$.

Next, let us suppose that for some particular kind of matter we have the relation

$$t(m_v) = (1 - \beta^2)l(m_v),$$

where $t(m_v)$ and $l(m_v)$ denote the transverse mass and the longitudinal mass, respectively, as above. Then repeat the experiments by means of which we proved theorem II. As before the balls will appear to B to move on arcs of the circle

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[1] It is obvious that we should then be able to demonstrate theorems I. and II. concerning the mass of a moving body.

[2] All that is essential to the argument is the truth of theorem I. for a particle of matter of some one kind; it need not be assumed to be true universally.

$$x' = \cos\theta, \quad y' = \sin\theta.$$

Suppose that to A they appear to move along arcs of the ellipse[1]

$$x = \cos\theta, \quad y = \rho\sin\theta,$$

where ρ is a constant to be determined. As before, without the use of postulate R, it may be shown that to A the periods will be the same in the two cases. Then determine the periods as in the preceding discussion. The expressions for the period will contain ρ; in fact on equating them we shall find

$$m_2 = \rho^2 m_1.$$

But $m_1 = l(m_v)$ and $m_2 = t(m_v)$ whence on account of the relations between $l(m_v)$ and $t(m_v)$, we have at once

$$\rho = \sqrt{1 - \beta^2}.$$

This, in connection with postulate L, leads readily to the usual relations concerning the units of length in two systems of reference. Having these relations of length units, the dimensional equation

$$V = \frac{L}{T}$$

taken in connection with postulate V leads at once to the usual relation of time units, provided we take the motion along the line of relative motion of the two systems. Hence we have the following theorem:

Theorem VI. *If $l(m_v)$ and $t(m_v)$ have the same meaning as in theorems I. and II. and if for any particular kind of matter whatever we have the relation*

$$t(m_v) = (1 - \beta^2)\cdot l(m_v),$$

then this fact and postulates $(MVLC_1C_2)$ are essential equivalents of postulates $(MVLRC_1C_2)$.

§ 5. THE BUCHERER EXPERIMENT.[2]

Our postulates V, L, C_1 have been universally accepted as part of the basis of the classical mechanics. Many persons have found no difficulty in accepting postulate M; certain it is at least that we have absolutely no evidence to contradict it. We have seen in theorem V. that these four postulates, taken in connection with the formula for transverse mass, form an essential equivalent of $(MVLRC_1)$; in other words, the experimental demonstration of the formula for transverse mass carries with it the experimental proof of the theory of relativity, provided that postulates $(MVLC_1)$ are accepted as experimentally proved.

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[1] Since we are assuming postulate L it is clear that the path must be of this form.

[2] Compare Tolman, Physical Review, 31 (1910), p. 36.

Bucherer[1] has carried out some investigations which have been supposed to furnish this experimental verification for the formula of transverse mass, and hence for the whole theory of relativity. In order to draw this conclusion from Bucherer's direct results it is necessary to make use of a law which we have not yet employed, namely, the law of conservation of electricity which we have stated as postulate C_2. Since this law has customarily been accepted and has not yet led to contradictions it should certainly still be supposed to hold. Accepting it, then, we have in Bucherer's results a partial experimental confirmation of the theory of relativity, as we now show.

Bucherer's investigations have to do with the mass of a moving electron. There seems to be no means at hand for a direct measurement of this mass, and Bucherer resorted to the expedient of determining the ratio of charge to mass. Let us denote the charge by e, which we suppose to be constant, in accordance with postulate C_2. As before let m_0 and $t(m_v)$ denote the mass at rest and the mass when moving with velocity v, of the electron in consideration. Bucherer's experiments were carried out to determine the relation which exists between e/m_0 and $e/t(m_v)$. The measurements agreed in a remarkable way, not only as to general characteristics but also as to exact numerical results, with the formula[2]

$$\frac{e}{t(m_v)} = \frac{e\sqrt{1 - \beta^2}}{m^0}.$$

Taking this formula as thus experimentally demonstrated we have at once our fundamental relation for transverse mass: $\sqrt{1 - \beta^2}t(m_v) = m_0$.

From this it follows that the experimental demonstration of the theory of relativity is complete when we have proved M, V, L, C_1 and C_2, provided that one accepts Bucherer's proof of the above relation between e/m_0 and $e/t(m_v)$. That is, *the essentials of the theory of relativity flow from*

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[1] Annalen der Physik (4) 28 (1909), 513–536.

[2] As a matter of fact Bucherer did not measure the ratio e/m_0. Instead of this he considered the ratio $e/t(m_v)$ for a considerable range of values of v and noticed that its value always agreed with the formula $e/t(m_v) = k\sqrt{1 - \beta^2}$, where k is a constant. It appears natural, then, to assume that $m_0 = e/k$, whence one has the formula in the text. It should be emphasized that this assumption is necessary in order that the Bucherer results may be associated with our theorem as in the text, and consequently the conclusions there reached can be accepted with no stronger confidence than that which one has in the accuracy of the above assumption. See the next section where a means of experimental verification of the theory of relativity is suggested which does not depend on this assumption for its validity.

principles for each of which there is strong experimental confirmation. This important conclusion has often been pointed out.

To the present writer, however, it seems that one point especially should be subjected to further examination. Is it in fact true that the charge of a moving electron is independent of the velocity with which it moves? Let e_0 be the charge of the electron when at rest and denote by $t(e_v)$ its apparent charge when in motion with velocity v, the charge being measured by means of tests in which the line of action is perpendicular to the line of motion of the charge. In the above work we have assumed, in accordance with the usual practice, that $e_0 = t(e_v)$. Suppose however that the true relation were different from this, that, in fact, we have

$$t(e_v) = e_0\sqrt{1 - \beta^2};$$

then Bucherer's experiment would lead to the conclusion that $t(m_v) = m_0$, and thus the whole theory of relativity would be overturned. Furthermore, if any relation other than $e_0 = t(e_v)$ is the true one, some modification at least of the theory of relativity would have to be made or else one would have to give up postulate C_1 which asserts the law of conservation of momentum. This result brings to notice the great importance of the question of the constancy of electric charge on the electron. We shall treat this matter further in the next section.

§ 6. Another Means for the Experimental Verification of the Theory of Relativity.

Just as theorem V. was used for the theoretical basis of Bucherer's (partial) experimental demonstration of the theory of relativity so theorem VI. may be employed as the theoretical basis of a new experimental investigation which has not yet been carried out, one which bears the same essential relation as that of Bucherer to the confirmation or disproof of the entire theory of relativity. The object of this section is to indicate the nature of this experiment.

Let e_0 denote the charge of an electron when at rest with respect to a given system of reference. When it is in motion with a velocity v let $t(e_v)$ and $l(e_v)$ be the apparent charge when measured by means of tests whose lines of action are perpendicular and parallel, respectively, to the line of motion of the electron.

If we employ postulate C_3 we conclude that $e_0 = t(e_v) = l(e_v)$. We shall first assume the truth of one of these relations, namely, $t(e_v) = l(e_v)$, and we shall denote the common value of these two quantities by e. Now let us suppose that some means are found for measuring both the quantities $e/t(m_v)$ and $e/l(m_v)$, where $t(m_v)$ and $l(m_v)$ denote as usual the transverse mass and the longitudinal mass respectively of the moving electron, whose velocity is v. Bucherer's methods furnish the means of measuring the first of these ratios; it will be necessary to devise a way to determine the value of the second ratio.

Or, instead of finding a means of measuring the two quantities $e/t(m_v)$ and $e/l(m_v)$ it will be sufficient if one determines only their ratio, as will be obvious from the discussion following.

Suppose now that we find the relation predicted by the theory of relativity:

$$\frac{e}{t(m_v)} = \frac{e}{(1 - \beta^2)l(m_v)}.$$

This equation leads to the relation $t(m_v) = (1 - \beta^2)\cdot l(m_v)$. According to theorem VI. this would give a new experimental confirmation of the theory of relativity. The importance of such a result is apparent.

But we should also have more than this. Having now concluded that the theory of relativity is confirmed and this result having been reached without the use of a relation between e_0 and $t(e_v)$ we may now use the experiment of Bucherer to draw further conclusions concerning electric charges in motion. In particular, it is obvious that we should have a proof of the fundamental relation

$$e_0 = t(e_v).$$

That is to say, having assumed that $t(e_v)$ and $l(e_v)$ are equal we conclude further on experimental evidence that each of these is equal to e_0. Now it is difficult to conceive how $t(e_v)$ and $l(e_v)$ could be different, for this would imply that the notion of electric charge is in need of essential modification. In fact, if the charged body is moving, the notion of charge would be indefinite in meaning until we had assigned the direction along which such charge is to be measured. Thus, if the experiment should turn out as surmised above, we should not only have the strongest sort of experimental confirmation of the theory of relativity but we should also have a valuable verification of the fact that an electric charge does not vary in amount with the velocity of the body which carries it.

Suppose, on the other hand, that we make no assumption concerning the relation of $t(e_v)$ and $l(e_v)$ or of $t(m_v)$ and $l(m_v)$. On carrying out the experiments a relation of the form

$$\frac{t(e_v)}{t(m_v)} = k\,\frac{l(e_v)}{l(m_v)}$$

will be obtained where k is a constant or a variable depending on v. If it is found that k is different from unity we shall be forced to the conclusion that either our conception of mass in the classical mechanics or our conception of charge in the classical electrical theory is in need of essential modification. Again, if $k = 1$ and if we assume, as is natural, that $t(e_v) = l(e_v)$ then we have an experimental disproof of the theory of relativity. In fact we have such a disproof unless $k = 1/(1 - \beta^2)$, provided of course that we assume $t(e_v) = l(e_v)$.

From these remarks it is obvious that, whatever may be the result of the experiments, they will certainly lead to important conclusions of a fundamental nature; that is, we have here a *crucial* experiment, one that cannot fail to lead somewhither. It is to be hoped that some laboratory worker will soon perform the requisite experiments; the writer, who is a mathematician, can only regret that he cannot conveniently carry out the work himself.

A variation of the experiment of Bucherer would seem to be sufficient for the purpose here. Bucherer's results were obtained by subjecting the moving electron to a magnetic field and also to an electric field each at right angles to the line of motion. A variation of the direction of these fields relative to the line of motion of the electron would probably afford a means of making the necessary measurements for the experimental proof of the relations requisite for use in the preceding discussion.

§ 7. Mass and Energy.

If, as is frequently done, we employ for the definition of the kinetic energy E the relation (compare § 1)

$$E = \int_0^v M dv = \int_0^v mv dv,$$

it is clear that for the mass m we should take the longitudinal mass $l(m_v)$. Then let m_0 denote the mass of the body at rest, E_0 its energy when at rest (that is, the energy due to its internal activity), and E_v its energy when moving at the velocity v. Then clearly $E = E_v - E_0$, so that in view of theorem II. we have

$$E = E_v - E_0 = \int_0^v \frac{m_0 v dv}{(1 - \beta^2)^{\frac{3}{2}}};$$

whence, on integration, we have

$$E = E_v - E_0 = m_0 c^2 \left(\frac{1}{\sqrt{1 - \beta^2}} - 1 \right). \tag{1}$$

Hence for the kinetic energy of a moving body we have

$$E = m_0 c^2 (\tfrac{1}{2}\beta^2 + \tfrac{3}{8}\beta^4 + \cdots);$$

or, to a first approximation only,

$$E = \tfrac{1}{2} m_0 v^2.$$

Therefore the usual formula for kinetic energy in the classical mechanics is only a first approximation.

Since relation (1) is to be true for all velocities v it is obvious that we have

$$E_v = \frac{m_0 c^2}{\sqrt{1 - \beta^2}} + k, \quad E_0 = m_0 c^2 + k,$$

where k is a constant, that is, a quantity independent of v. From the first of these equations we conclude further that

$$E_v = c^2 \cdot t(m_v) + k,$$

so that the total energy of a body, decreased by the constant k, is directly proportional to its transverse mass. In case the body is at rest its mass in one direction is the same as in another; hence $m_0 = t(m_0)$. Bearing this in mind we have the following theorem:

THEOREM VII. *Let m_0 be the mass of a body when at rest with respect to a given system of reference and let $t(m_v)$ denote its transverse mass when it is moving with the velocity v (the case $v = 0$ is not excluded). Then the total energy E_v which it possesses is $c^2 t(m_v) + k$, where k is a constant.*

The following relations are immediate consequences of equations written out above:

$$\frac{E_v - E_0}{t(m_v) - m_0} = c^2, \quad E_v - k = \frac{E_0 - k}{\sqrt{1 - \beta^2}}.$$

Now, suppose that an experimenter contributes to a body which is at rest a known amount of energy and determines the velocity which this causes the body to acquire. If the two measurements are made with sufficient accuracy one will be able, by substituting the results in the first of the above equations, to determine in this way the velocity of light. Actually to carry out this remarkable method for measuring c would doubtless be very difficult; but the obvious great importance of the result is certainly such as to justify a careful consideration of the problem. If the value of c determined in this way should agree well with its value as otherwise found, this would give us an interesting confirmation of the theory of relativity.

Let us consider the mass of a rotating top, the mass being measured along the axis of rotation. According to our results this mass should be different from that of the same top when at rest, and the difference should bear a definite relation to the amount of energy which is involved in the rotation. If the measurements here involved could be made with sufficient accuracy we would have another means, independent of light itself, for the measurement of the light-velocity c. Again, this experiment would afford us a measure of transverse mass and in that way could lead to a confirmation of the theory of relativity, provided that we assume c as known from independent measurements; and this confirmation, it is to be noticed, would be independent of electrical considerations.

It seems to be impossible to determine the constant k which enters into the above discussion. But in the absence of any evidence to the contrary it would appear natural tentatively to assume that k is zero. On the basis of this assumption we should have the following remarkable conclusions: The mass of a body at rest is simply the measure of its internal energy. The transverse mass of a body in motion is the measure of its internal energy and its kinetic energy taken together.[1] Its longitudinal mass is its total energy multiplied by a simple factor. (The longitudinal mass, therefore, bears a simple ratio to the total energy.) One can hardly resist the conclusion that the transverse mass of a body depends entirely on its energy, and therefore that matter is merely one manifestation of energy.

§ 8. Remarks on the Principle of Least Action.

In the preceding section we have seen that in the theory of relativity the classical formula $E = \frac{1}{2}mv^2$ for the measure of kinetic energy is true only as a first approximation.. This is due to the fact that mass is a variable quantity. But the conclusion does not appear to necessitate our surrender of the law of conservation of energy.

The same causes which lead to a modification in the formula for E will also require a corresponding modification in the value of the action A as defined in § 1. The question arises as to whether the principle of least action is left intact. I cannot enter upon the investigation here; but the problem seems to me to be of importance, and consequently I am stating it in the hope that some one will be led to consider its solution.

Undoubtedly the principle of least action is one which should be given up only when there are strong reasons for it. It is a mathematical formulation of the law that nature accomplishes her ends with the least possible expenditure of labor, so to speak. Certainly this law is one which appeals to our minds with strong force. There is something about it which is æsthetically satisfying in a high degree. It seems to me, however, that a fresh study of it should be made in the light of the theory of relativity.

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[1] Compare Lewis and Tolman, Phil. Mag. (6) 18 (1909), p. 521.

§ 9. SPECULATIVE CONSIDERATIONS.

From some results in the preceding discussion it has appeared that the transverse mass of a body is merely a manifestation of its total energy, that it is in fact a measure of that energy. It is then natural to suppose, on the other hand, that anything which possesses energy has mass; and we thus conceive of mass and energy as coextensive.

Now a beam of light possesses energy; whence we conclude naturally that it also has mass. But we have seen that no "material body" can have a velocity as great as that of light. How are these two facts to be reconciled? If we define "matter" as that which possesses mass (and this is probably the best definition) we shall perhaps best be able to represent to ourselves the nature of matter if we think of it as a strain in the ether. Then the two facts which we have to reconcile would be entirely consistent if we suppose that the beam of light sets up a strain in the ether (whence its mass) but that this strain as a whole is not propagated with the velocity of light. In fact, if it moves at all it is probably with a velocity relatively much smaller than that of light.

Again, if mass is merely a manifestation of energy in the form of a strain in the ether it would follow that gravitation is simply an interaction among these several strains. A strain principally localized in one place would have lines of strain going out from it in all directions, and the action of these lines of strain upon one another would afford the effective means by which gravitation acts.

Whether these strains should be thought of as static in the ether or as due to the relative motion of the parts of the ether would probably be determined differently by different minds. If one inclines to the latter form of representation, the necessary movement might be conceived of as vortex whorls in the ether, setting up strains and lines of strain by aid of which we are able to interpret observed phenomena.

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THE PHYSICAL REVIEW.

ON THE THEORY OF RELATIVITY: PHILOSOPHICAL ASPECTS.

By R. D. Carmichael.

I. Introduction.

THOSE who look on physics from the outside not infrequently have the feeling that it has forgotten some of its philosophical foundations. And among physicists themselves this condition of their science has not entirely escaped notice.[1]

The physicist who, above all other men, has to deal with space and time has fallen into conventions concerning them of which he is often not aware. It may be true that these conventions are exactly the ones which he should make. It is certain, however, that they should be made only by one who is fully conscious of their nature as conventions and not as absolute realities beyond the power of the investigator to modify.

Likewise, a question arises as to what element of convention is involved in our usual conceptions of mass, energy, etc.; that the question is not easily answered will become apparent on reflection.

These and many other considerations suggest the desirability of a fresh analysis of the foundations of physical science. Now it is a ground of gratulation for all those interested in this matter that there has arisen within modern physics itself a new movement capable of contributing most effectively to the construction of a more satisfactory foundation for its superstructure of theory. I refer to the recent widespread interest in the principle of relativity and the rapid developments to which it has led.

It is at once admitted that the theory of relativity is not yet established on an experimental basis which is satisfactory to all persons; in fact, some of those who dispute its claim to acceptance are among the most eminent men of science of the present time. On the other hand there is a large and growing body of workers who are rapidly pushing forward investigations the inspiration for which is afforded by the theory of relativity.

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[1] Compare the discussion given by Crew, Physical Review, 31, 79–92.

This state of affairs will probably give rise to a considerable controversial literature. If the outcome of this controversy is the acceptance in the main of the theory of relativity, then this theory will afford just the means needed to arouse in investigators in the field of physics a lively sense of the philosophical foundations of their science. If the conclusions of relativity are refuted this will probably be done by a careful study of the foundations of physical science and a penetrating analysis of the grounds of our confidence in the conclusions which it reaches. This of itself will be sufficient to correct the present tendency to forget the philosophical basis of the science.

It follows that in any event the theory of relativity is certain to force a fresh study of the foundations of physical theory. If it accomplishes no more than this it will have done well.

The object of this paper is to point out some of the more immediate and fundamental conclusions of a philosophical nature to which the theory of relativity gives rise, especially those related to the foundations of physics. An attempt is made to present the discussion in such form as to be intelligible to one who is unacquainted with the theory of relativity; this is done for the convenience of those who have not hitherto been interested in the matter. It will probably be found desirable, however, to have as much previous knowledge of the subject as may be obtained from Comstock's paper in *Science*, N. S., 31 (1910), 767–772.[1]

II. THE FUNDAMENTAL BASIS OF RELATIVITY.

The fundamental basis of the theory of relativity may be set forth in a few statements each of which is a generalization from experiment. In so far as these statements are generalizations they are of course unproved experimentally. But they are nevertheless supposed to be the natural teachings of experiment; and as such are to be accepted with a good degree of confidence unless it can be shown that they are in contradiction to other known experimental facts.

The first of these statements gives expression to a law which is suggested by the famous Michelson-Morley experiment,[2] the object of which was to try out a certain theory concerning the ether by ascertaining whether a result predicted by it would be found by actual test. For a long time it had been supposed that the known facts about light, electricity and magnetism required for their explanation the theory that the ether of space is stationary. Such a conclusion led to the belief that the velocity of the earth through the ether could be determined by optical experiments. Thus it was predicted that the time which would be required for a beam of light to pass a given distance and return would be different in the two cases when the path of light was parallel to the direction of the motion and when it was perpendicular to this direction. The object of the Michelson-Morley experiment, as we have said, was to put this prediction to a crucial test.

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[1] One who desires to go further into the subject might consult with profit the author's previous papers in the PHYSICAL REVIEW, Vol. 35, Series 1, pp. 153–176; Vol. I., Series 2, pp. 161–178.

[2] See American Journal of Science (3), 34 (1887), 333–345.

The experiment was a bold one, seeing that the velocities to be measured were so little different; and yet it was carried out in such a brilliant way as to permit no serious doubt of the accuracy of the results. The difference of velocity predicted by theory was found by experiment not to exist; there was not the slightest difference of time in the passage of light along two paths of equal length, one in a direction parallel to the earth's motion and the other in a direction perpendicular to it.

There are different points of view from which one may look at this experiment. In the theory of relativity it is taken in the light of an attempt to detect the earth's motion through space by means of the effect of this motion on terrestrial phenomena. So far as the experiment goes, it indicates that such motion cannot be detected in this way. Furthermore, no one has yet been able to devise an experiment by means of which the earth's motion through space can be detected by observations made on the earth alone. The question arises: Is it possible to have any such experiment at all? In the theory of relativity this question is answered in the negative. The Michelson-Morley experiment and other experiments are thus generalized into one of the fundamental laws or postulates of relativity, which may be stated as follows:

A. The uniform translatory motion of any system cannot be detected by an observer traveling with the system and making observations on it alone.

Another part of the fundamental basis of the theory of relativity is a principle which has long been familiar in the theory of light and has never been found in disagreement with experimental facts. It may be stated thus:

B. The velocity of light in free space is independent of the velocity of the source of light.

By means of laws A and B, taken in connection with certain principles universally accepted in the classical mechanics, it may be shown[1] that the velocity of light is independent of the direction of motion of the observer. Thus we have:

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[1] See a treatment by the author in the first paper referred to above.

C. THEOREM. ***The velocity of light in free space as measured by any observer is independent of the direction of motion of that observer.***

We are thus led to inquire further as to whether the velocity of light in free space is independent of the absolute value of the velocity of the observer. It appears to be impossible to prove that it is so independent; and yet one is unable to conceive of any way in which it could be dependent on the absolute value without at the same time depending on the direction of the velocity also. But, accepting *A* and *B*, we prove *C* which asserts that such dependence does not exist. There seems, then, nothing left to do but to *assume* that dependence on absolute value does not exist; and this is what is done in the theory of relativity. In the absence of experimental information in contradiction to this assumption it is undoubtedly the natural one to make. Any other procedure would be out of harmony with the usual methods of science. This assumption, therefore, is one which we make as the most natural teaching of experimental facts; and as such we treat it as a "law of nature" so long as fresh experiment does not invalidate it. This law may be stated as follows:

D. ***The velocity of light in free space as measured by any observer is independent of the absolute value of the velocity of the observer.***

The law which we have stated as *A* above is often referred to as the first postulate of relativity. *B*, *C* and *D* taken together constitute the second postulate of relativity. We have broken this postulate into parts in order to state clearly just how it depends on experiment. Combining the parts we may state its essential content as follows:

E. ***The velocity of light is independent of the relative velocity of the source of light and the observer.***

The theory of relativity consists of those conclusions which can be derived by logical process (that is, mathematically) from *A* and *E* in conjunction with certain principles which are universally accepted in the classical mechanics—at least, this may be taken as a tentative definition of the theory of relativity. If one agrees that experiment has been properly generalized in *A*, *B* and *D* one has therefore the alternative of accepting the conclusions of relativity or of giving up almost the whole of the usual system of mechanics. That one should take the first horn of the dilemma hardly admits question.

III. FUNDAMENTAL CONCLUSIONS OF THE THEORY OF RELATIVITY.

For the sake of convenience in stating some of the fundamental conclusions of the theory of relativity let us suppose that we have two observers A and B placed on platforms moving with respect to each other. Let us suppose that A is on a platform denoted by S_1 and that B is on a platform denoted by S_2. Suppose that to A the platform S_2 appears to move with the velocity v in the direction indicated by the arrow at S_2; then to B the platform S_1 will appear to move with velocity v in the direction indicated by the arrow at S_1. Suppose further that the units of length employed by A and B are such that they arrive at the same numerical results in measuring the length MN, MN being perpendicular to the line of relative motion of S_1 and S_2. The platform S_1 and the instruments employed by A for measuring time and length will be spoken of as the system of reference S_1. Similarly, we shall speak of the system of reference S_2. For convenience we shall use β to denote the ratio v/c, where c is the velocity of light.

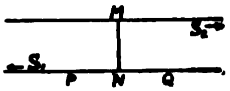

Fig. 1.

The two systems of reference S_1 and S_2 being thus defined, the question arises as to how the units of length and of time of S_1 are related to the corresponding units of S_2. If we accept as true the laws stated in A and E of the preceding section—as, in fact, is done in the theory of relativity—we are led to some very remarkable conclusions. We state first the relation of the time units:

To an observer A on S_1 the time unit of S_1 appears to be in the ratio $\sqrt{1-\beta^2} : 1$ to that of S_2, while to an observer B on S_2 the time unit of S_2 appears to be in the ratio $\sqrt{1-\beta^2} : 1$ to that of S_1.

Thus if A compares his clocks with those of B it will appear to A that A's clocks are running faster than B's clocks; on the other hand, if B compares his clocks with those of A it will appear to B that B's clocks are running faster than A's clocks. Thus the two observers are in hopeless disagreement as to the measurement of time. An analysis of this divergence is made below in section V.

Another matter of fundamental importance in the measurement of time will be brought to attention by the question: When are two events which happen at different places to be considered simultaneous? The nature of the difficulty can be seen from the following result in the theory of relativity, an analysis of which is to be found below in section VI.:

If an observer A on S_1 places two clocks at a distance d apart in a line parallel to the line of relative motion of S_1 and S_2 (say at P and Q respectively) and adjusts them so that they appear to him to mark the same time: then to an observer B on S_2 the clock on S_1 which is forward in point of motion appears to be behind in point of time by the amount

$$\frac{v}{c^2}\frac{d}{\sqrt{1-\beta^2}}.$$

Let us consider the units of length in S_1 and S_2. If A and B make measurements of length in the direction MN their results are in perfect agreement in the theory of relativity as in the classical mechanics; but the state of matters is very different when measurement is made in a line parallel to the line of relative motion of S_1 and S_2, as one sees from the following theorem:

Let l denote a line parallel to the line of relative motion of S_1 and S_2. Then to an observer A on S_1 the unit of length of S_1 along l appears to be in the ratio $\sqrt{1-\beta^2} : 1$ to that of S_2 while to an observer on S_2 the unit of length of S_2 along l appears to be in the ratio $\sqrt{1-\beta^2} : 1$ to that of S_1.

Thus it appears that when A and B are measuring length in a line parallel to their line of relative motion they are in hopeless disagreement. What this result signifies we shall attempt to explain in section IV.

From the results stated above it may be shown that:

The velocity of light is a maximum which the velocity of a material body may approach but can never reach.

If one brings into consideration the mass of a moving body another result, essentially equivalent to that just given, may also be obtained; this may be stated as follows (see further discussion in section VIII. below):

No finite force is sufficient to give a material particle a velocity as great as that of light.

In classical mechanics it is customary to assume that the mass of a given body is constant and that it is independent of the direction in which the mass is measured. But if the principle of relativity is accepted it follows that neither of these conclusions is valid. It turns out that the mass of a body depends on its velocity and also on the direction, relative to the line of motion of the body, along which that mass is measured. For convenience in distinguishing these measurements of mass we shall speak of the "transverse mass" as that with which we must reckon when we consider motion in a line perpendicular to the line of relative motion of the systems S_1 and S_2; when the motion is parallel to this line we shall speak of the "longitudinal mass" of the body.

The general conclusions of the theory of relativity concerning mass may now be stated as follows:

Let m_0 denote the mass of a body when at rest relative to a system of reference S. When it is moving with a velocity v relative to S, denote by t_v its transverse mass, that is, its mass in a line perpendicular to its line of motion. Similarly, denote by l_v its longitudinal mass. Then we have

$$t_v = \frac{m_0}{(1 - \beta^2)^{\frac{1}{2}}},$$

$$l_v = \frac{m_0}{(1 - \beta^2)^{\frac{3}{2}}}.$$

IV. THE NOTION OF LENGTH.

In the preceding section we saw that two observers A and B on relatively moving systems of reference S_1 and S_2 respectively are in disagreement as to units of length along a line l parallel to their line of relative motion. This disagreement is of a very peculiar character. To A it appears that B's units are longer than his own. On the other hand, it seems to B that his units are shorter than A's. In the two cases the apparent ratio is the same; more precisely, the unit which appears to either observer to be the shorter seems to him to have the ratio $\sqrt{1 - \beta^2} : 1$ to that which appears to him to be the longer. Although they are thus in disagreement there is yet a certain symmetry in the way in which their opinions diverge.

Let us suppose that these two observers now undertake to bring themselves into a closer agreement in measurements of length along the line l. Suppose that B agrees arbitrarily to shorten his unit so that it will appear to A that the units of A and B are of the same length. Then, so far as A is concerned, all difficulty has disappeared. How is B affected by this change? We see that the difficulty which he experienced is not disposed of; on the other hand it is greater than before. Already, it seemed to him that his unit was shorter than A's. Now, since he has shortened his unit, the divergence appears to him to be increased. Moreover, the symmetry which we found in the former case is now absent.

Furthermore, if any other changes in the units of A and B are made we shall always find difficulties as great as or greater than those which we encountered in the initial case. There is no other conclusion than this: We are face to face with an essential difficulty—one that is not to be removed by any mere artifice. What account of it shall we render to ourselves?

This much is already obvious: The length of an object is not an absolute something; it depends upon the measurer in an essential way.

We have just spoken of the length of an object, *a material object*. One can hardly refrain from raising the question of the *abstract* notion of length as apart from any material thing having length. But this problem has new difficulties of its own. All lengths of which I have experience are lengths of material objects or lengths between material objects. Can one formulate in the mind the notion of length as apart from these material things which are measured; and, if so, how can such a notion of length be applied experimentally to the measurement of material things?

Questions of this nature we shall lay aside and shall return to the consideration of the length of material objects.

We have certain intuitive notions concerning the nature of matter which it is necessary for us to examine now. We have usually supposed that to revolve a steel bar, for instance, through an angle of ninety degrees has no effect upon its length. Let us suppose for the moment that this is not so; but that the bar is shorter when pointing in some directions than in others, so that its length is the product of two factors one of which is its length in a certain initial position and the other of which is a function of the direction in which the body points relative to that in the initial position. Suppose that at the same time all other objects experience precisely the same change for varying directions. It is obvious that in this case we should have no means of ascertaining this dependence of length upon the direction in which the body points.

To an observer placed in a situation like this it would be natural to assume that the length of the steel bar is the same in all directions. In other words, in arriving at his definition of length he would make certain conventions to suit his convenience.

Now suppose that the system of such an observer is set in motion with a uniform velocity v relative to the previous state of the system; and that at the same time all bodies on his system undergo simultaneously a continuous dilatation or contraction. This observer would have no means of ascertaining that fact; and accordingly he would suppose that his steel bar had the same length as before. In other words, he would unconsciously introduce a new convention concerning his measurement of length.

There is no *a priori* reason why our actual universe should not be such as the hypothetical one just described. To suppose it so unless our experience demands such a supposition would be unnatural; because it would introduce an unnecessary inconvenience. But suppose that in our growing knowledge of the universe there should come a time when we could more conveniently represent to ourselves the actual facts of our experience by supposing that all material things are subject to some such deformations as those which we have indicated above; there is certainly no *a priori* reason why we should not conclude that such is the essential nature of the structure of the universe.

Naturally we would not come to this conclusion without due consideration. We would first enquire earnestly if there is not some more convenient way by which we can reconcile all experimental facts; and only in the event of a failure to find such a way would we be willing to so profoundly modify our views of the material world.

Now, if we agree to suppose that our actual universe is subject to a certain (appropriately defined) deformation of the general type discussed above it would follow that observers A and B on the respective systems S_1 and S_2 would be in just such disagreement as to units of length as that which exists, according to the theory of relativity. Therefore, that which at the outset seemed to be of such essential difficulty is easily enough explained, *if we are willing to modify so profoundly our conception of the nature of material bodies.*

Whether in the present state of science experimental facts demand such a radical procedure is a question which will be answered differently by different minds. To one who accepts the postulates of relativity there is indeed no other recourse; one who refuses to accept them must find some other satisfactory way to account for experimental facts. The Lorentz theory of electrons gives striking evidence in favor of supposing that matter is subject to some such deformations as those mentioned above; and this evidence is the more important and interesting in that the deformations (as conceived in this theory) were assumed to exist simply in order to be able to account directly for experimental facts.

V. The Measurement of Time.[1]

That two observers in relative motion are in hopeless disagreement as to the measurement of length in their line of relative motion is a conclusion which is probably (at first) sufficiently disconcerting to most of us; and some no doubt have the feeling that any explanation of it so far offered is at best artificial and does not reach the root of the matter. But it is an even greater shock to intuition to conclude, as we are forced to do according to the theory of relativity, that there is a like ineradicable disagreement in the measurement of time. A discussion similar to that in the preceding section brings out the fact that our observers A and B cannot possibly arrive at consistent means of measuring intervals of time. The treatment is so far similar to the preceding discussion for length that we need not repeat it; we shall content ourselves with a brief discussion of conclusions to be drawn from the matter.

Why is this inability of A and B to agree in measuring time received in our minds with such a distinct feeling of surprise and shock? It is doubtless because we have such a lively sense of the passage of time. It seems to be a thing which we know directly, and the conclusion in question is contrary to our unsophisticated intuition concerning the nature of time.

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[1] In connection with this section and the following one the reader should compare the excellent and interesting treatment of the problem of measuring time to be found in Chapter II. of Poincaré's Value of Science (translated into English by Halsted).

But what is it that we know directly? We have an immediate perception of what it is for two conscious phenomena to coexist in our mind; and consequently we perceive immediately the simultaneity of events in our minds. Further, we have a perfectly clear sense of the order of succession of events in our own consciousness. Is that not all that we know directly?

The difficulties which *A* and *B* experience in correlating their measurements of time grow out of two things, of neither of which we have direct perception.

In the first place there are two consciousnesses involved; and what reason have we to suppose that succession of events is the same for these two? This question we shall not treat, assuming that the principal matter can be put into such impersonal form as to obviate this difficulty altogether. (As a matter of fact, so far as anything characteristic of the theory of relativity is concerned this can be done.)

The other difficulty has to do with the measurement of time as opposed to the mere psychological experience of its passage. In this matter we are absolutely without any direct intuition to guide us. We have no immediate sense of the equality of two intervals of time. Therefore, whatever definition we employ for such equality will necessarily have in it an important element of convention. To keep this well in mind will facilitate our discussion.

Our problem is this: How shall we assign a numerical measure of length to a given time-interval; say to an interval in which a given physical phenomenon takes place? We shall arrive at the answer by asking another question: Why should we seek to measure time-intervals at all, seeing that we have no immediate consciousness of the equality of such intervals? There can be only one answer: we seek to measure time as a matter of convenience to us in representing to ourselves our experiences and the phenomena of which we are witnesses. In such a way we can render to ourselves a better account of the world in which we live and of our relation to it.

Now, since our only reason for attempting to measure time is in a matter of convenience, the way in which we measure it will be determined by the dictates of that convenience. The system of time measurement which we shall adopt is just that system by means of which the laws of nature may be stated in the simplest form for our comprehension.

Let us return to the case of the two observers *A* and *B* of section III. Suppose that each of them has chosen a system of measuring time that suits his convenience in the interpretation of the laws of nature on his system. There is no *a priori* reason why the two observers should measure time-intervals in the same way. In fact, since there is an arbitrary element in the case of each method of measurement and since the two systems are in a state of relative motion, it is not at all unnatural that the units of *A* and *B* should differ.

Now it is to be noticed that each of the observers *A* and *B* is in just the situation in which we find ourselves. We have chosen a method of measuring time which seems to us convenient. In so far as that method depends on convenience it is relative to us who are observers, and therefore it has in it something which is arbitrary. There is no doubt that it would be desirable for us to know what it is which is arbitrary, which is relative to us who observe; but it is equally obvious that it must be difficult for us to determine what this arbitrary element is.

The theory of relativity makes a contribution to the solution of this problem. We suppose that two observers on different systems find the laws of nature the same as we find them; or, more exactly, we suppose that they find certain specific laws the same as we find them. Then we inquire as to their agreement in measuring time and see that they differ in a certain definite way. This difference is due to things which are relative to the two observers; and thus we begin to get some insight into the ultimate basis of our own method of measurement.

The matter will become clearer if we speak of the simultaneity of events which happen at different places; and therefore we turn to a discussion of this topic.

VI. Simultaneity of Events Happening at Different Places.

What shall we mean by saying that two events which happen at different places are simultaneous?

First of all it should be noticed that we have no direct sense of what this statement should mean. I have a direct perception of the simultaneity of two events in my own consciousness. I consider them simultaneous because they are so interlocked that I cannot separate them without mutilating them. If two things happen which are far removed from each other I do not have a direct perception of both of them in such way that I perceive them as simultaneous. When should I consider such events to be simultaneous?

To answer this question we are forced to the same considerations as those which we met in the preceding section. There can be no absolute criterion by which we shall be able to fix upon a definition as the only appropriate one. We must be guided by the demands of convenience, and by this alone.

In view of these considerations there is nothing unthinkable about the conclusion (in the theory of relativity) concerning simultaneity which we have given in section III. An observer A on one system of reference regulates clocks so that they appear to him to be simultaneous. It is apparent that to him the notion of simultaneity appears to be entirely independent of position in space. His clocks, even though they are separated by space, appear to him to be running together, that is, to be together in a sense which is entirely independent of all considerations of space.

But when B from another system of reference observes the clocks of A's system they do not appear to him to be marking simultaneously the same hour; and their lack of agreement is proportional to their distance apart, the factor of proportionality being a function of the relative velocity of the two systems.

Thus instants of time at different places which appear to A to be simultaneous in a sense which is entirely independent of all considerations of space appear to B in a very different light; namely, as if they were different instants of time, one preceding the other by an amount directly proportional to the distance between the points in space at which events occur which mark these instants. Even the order of succession of events is in certain cases different for the two observers, as one can readily verify.

It thus appears that the notion of simultaneity is relative to the system on which it is determined. In other words, *there is no such thing as the absolute simultaneity of events which happen at different places.* The only meaning which simultaneity can have is that which is given to it by convention.

Remark.—The difficulties which we have investigated in this and the preceding section are not peculiar to the theory of relativity. The fact that the quantitative measurement of time is relative to the observer who measures it has been insisted upon by philosophers, notably by Poincaré in his Value of Science already referred to. The considerations on which these conclusions have been based have been largely of a speculative character.

The interest of the subject from the point of view of the theory of relativity is that here we have an experimental basis for the conclusions which were previously reached by speculative considerations. Starting out from certain laws which we have (tentatively) accepted as demonstrated by experiment we have by logical processes alone reached these remarkable conclusions concerning the nature of time as conceived by us. Thus we have experimental demonstration of important results previously reached only by speculative considerations.

In the very nature of things speculation must often outrun experiment. The philosopher vaguely and boldly conceives a truth which it requires years of patient labor to establish on a firm and satisfactory foundation.

It is one of the glories of modern science that things hitherto of a speculative nature are being brought under the domain of experimental fact. In the process many speculations are overturned and thrown to the winds; but that which is really of value we may believe will always be safe and will some day find its justification in results achieved in the laboratory.

VII. Time as a Fourth Dimension.

I have no intention of asserting that time is a fourth dimension of space in the sense in which we ordinarily employ the word "dimension"; such a statement would have no meaning. I wish to point out rather that it is in some measure connected with space, and that in many formulæ it must enter as it would if it were essentially and only a fourth dimension.

This will come out clearly if we consider the Einstein formulæ of transformation from one system of reference to another. These have been worked out in detail in several different places,[1] and the result will be assumed here. It may be stated as follows:

Let us consider three mutually perpendicular axes Ox, Oy, Oz of which Ox is in the line of relative motion of two systems of reference S and S'. Likewise let $O'x'$, $O'y'$, $O'z'$ be three mutually perpendicular axes parallel respectively to Ox, Oy, Oz. Let the first be fixed to S and the second to S'. At time $t = 0$ let O and O' coincide. Then if $t, x, y, z;\ t', x', y', z'$ are the time and space coördinates on S and S' respectively, we have

$$t' = \frac{1}{\sqrt{1-\beta^2}}\left(t - \frac{v}{c^2}x\right),$$

$$x' = \frac{1}{\sqrt{1-\beta^2}}(x - vt),$$

$$y' = y,$$
$$z' = z,$$

where c is the velocity of light, v the relative velocity of S and S', and $\beta = v/c$.

In these formulæ the time variable t enters in a way precisely analogous to that in which the space variables x, y, z enter.

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[1] Compare Physical Review, Vol. 35, pp. 171–173.

Suppose now that the law of some phenomenon as observed on S' is given by the equation

$$F(x', y', z', t') = 0$$

and we desire to know the expression of this law on S. We substitute for x', y', z', t', their values in terms of x, y, z, t given above; and thus obtain an equation stating the law in question.

From these considerations it appears that in many of our problems, namely, in those which have to do at once with two or more systems of reference, the time and space variables taken together play the rôle of four variables each having to do with one dimension of a four-dimensional continuum.

This conclusion alone raises philosophical questions of profound importance concerning the nature of space and time; but into these we cannot enter here.

VIII. A Maximum Velocity for Material Bodies.

There are several ways by which it may be shown that a material body cannot have a velocity as great as that of light. One of the simplest is that which comes from a consideration of mass. Let us consider the equation

$$l_v = \frac{m_0}{(1 - \beta^2)^{\frac{3}{2}}},$$

where m_0 is the mass of a body at rest relative to a given system of reference S, l_v is the longitudinal mass of the body moving with a velocity v with respect to S. If we consider larger and larger values of the velocity v we see that l_v increases and becomes infinite as v approaches c. This is equivalent to saying that the longitudinal mass of any material body becomes infinite as the velocity of that body approaches c. Therefore it would require an infinite force to give to a material body the velocity c; that is, c is a maximum velocity which the velocity of a material body may approach but can never reach.

We may obtain the same result in a different way, and thus arrive at a deeper understanding of the matter. From the first formula of transformation given in the preceding section we have

$$t'\sqrt{1 - \beta^2} = t - \frac{v}{c^2}x.$$

Now suppose that $v/c > 1$ and that t and x are real. Then t' is imaginary. Hence, if two systems have a relative velocity greater than that of light, the time measurement in one of them is expressed as an imaginary number; which is clearly absurd. Hence the relative velocity of the two systems is less than c, since $v = c$ obviously leads to an absurdity.

This conclusion concerning the maximum velocity of a material body brings up important considerations concerning the essential nature of mass and material things. How shall we conceive of matter so that it should have this astonishing property?

In the present state of science any answer to this question must necessarily be of a speculative nature; but it is probably worth while to mention briefly a theory of mass which is consistent with the existence of a maximum velocity for a material body.

Let us suppose that the mass of a piece of matter is due to a kind of strain in the ether, and that this strain is principally localized in a relatively small portion of space, but that from this center of localization there go out to infinity in all directions lines of strain which belong essentially to the piece of matter. (We make no assumption as to how this strain is set up; it may be due largely or entirely to the motion of electrons in the molecules of the matter.) Suppose that these lines of strain, except in the immediate neighborhood of the center of localization, are of such nature as to escape detection by our usual methods. Suppose further that when the piece of matter is moved, that is, when the center of localization is displaced, these lines of strain have a corresponding displacement, but that the ether of space resists this displacement, the degree of resistance depending on the velocity.

If the mass of matter is due to such a strain in the ether it is natural to suppose that mass is a measure of the amount of that strain. But, on our present hypothesis, we see that when matter is moved through space there is an increase of the strain on the ether due to such motion. This manifests itself to us in the way of an increase in the mass of the given piece of matter.

Moreover, when the body is in motion it is natural to suppose that these lines of strain are not distributed evenly in all directions. On account of this fact it would not be a matter for surprise if the mass of a moving body were different in different directions.

It thus appears that appropriate hypotheses (which have nothing in them inherently unnatural) would lead us to expect the same descriptive properties of mass as those which are actually found to exist if one accepts the postulates of relativity. Hence we conclude that there is nothing *a priori* improbable in the conclusions of relativity concerning the nature of mass. Therefore if we find satisfactory grounds for accepting the initial postulates of relativity, we shall not throw them overboard because of the strange conclusions concerning mass to which they have led us.

IX. ABSOLUTE REST. ABSOLUTE POSITION IN SPACE.

To any one who has examined critically the foundations of the theory of relativity it has certainly become apparent that the unexpected conclusions to which one is led concerning the relations of systems of reference is intimately connected with the point of view which the observer on each system takes in making his measurements. Each observer assumes his system to be at rest; and there seems to be no reason for preferring one assumption to the other or for replacing both of them by a third.

If, however, there were such a thing as absolute rest, or absolute position in space, the matter would be different. Each observer should reckon his velocity relative to that absolute; and the reason for divergence which existed before would no longer be found. We should, however, find it necessary to revise many of our "laws of nature," among them the postulates of relativity, if these laws are to be stated with reference to an absolute.

Since it appears to the present writer that absolute rest and absolute position are indefinable, this matter will be dismissed without further consideration.

X. RELATION OF THE THEORY OF RELATIVITY TO THE PHILOSOPHICAL CONTROVERSY CONCERNING THE ONE AND THE MANY.

The author is probably due the reader an apology for injecting into this paper any remarks concerning the abstruse and difficult question as to whether the universe is monistic or pluralistic—a question which has engaged philosophers from the time of the earliest Greek thinkers down to our own day. But if any one of the special sciences has some light to throw on a question of such profound general importance it seems desirable that those engaged in the study of that science should make it known.

If we accept the statement of one of the most eminent philosophers of our day that the universe is one to us in so far (and only in so far) as we know the connections by which it is bound into a One, it will doubtless be interesting to us to examine in what way the theory of relativity leads us to observe new connections. To speak briefly of these is my purpose in the present section.

1. We have already seen that time is not something apart from one who measures it and the system to which he belongs, and that (so far as we are concerned, at least) there is an intimate interlocking of time and space in an essential manner so that we are not able absolutely to extricate the one from the other. That is to say, there is a mutual interdependence of space and time for us who measure them so that we have

to consider them not as mutually exclusive "forms" of thought but as parts of a larger matrix, a sort of four-dimensional continuum, which at once includes both of them and binds them into a unity higher than that of either of them.

2. The theorem which we stated in section III. concerning units of length in the line of relative motion of two systems of reference not at rest relative to each other leads to the conclusion that when a material body is set in motion it undergoes a shortening in the direction of the line of motion and that this deformation takes place as it were automatically without the expenditure of work. This brings to light a very peculiar connection between material bodies and the "empty space" in which they lie or through which they move. Compare the discussion of mass in section VIII.

3. In the classical mechanics the mass of a body is an invariable quantity. In particular, it is independent of the velocity with which the body moves. Its measure is the same in whatever direction it is measured. In section III. we have already seen that none of these statements are true in the theory of relativity. The mass of a body is intimately connected with the absolute value of the velocity with which it moves relative to a system of reference and also (what is stranger still) with the direction in which the mass is measured relative to the direction of motion of the body.

4. It may also be shown[1] that the measure of gravitation depends in a similar way on the motion of the system on which it is measured.

5. Again, the total amount of energy[2] which a body possesses is a simple linear function of its transverse mass, so that energy and mass are connected in a most intimate manner. In fact, changes in energy and changes in transverse mass are proportional, the factor of proportionality being the square of the velocity of light.

Thus we have enumerated several important and striking connections, brought to light by the theory of relativity, where we have heretofore supposed that there were no connections whatever. If we look at these a little more closely we shall be able to perceive a still more profound connection underlying those which we have already mentioned. A material body in motion has kinetic energy. The connections which we treated depend on relative motion. It is then not far to the conclusion that these connections depend essentially on the energy which is involved in this relative motion. Thus all the interlocking relations mentioned might be thought of as due to the energy states involved in the several cases.

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[1] See Bumstead, American Journal of Science (4), 26 (1908), 501.

[2] See Physical Review, Vol. I., Series 2, p. 176.

This idea seems to me to be one fruitful of important conclusions; but it cannot be taken up in detail at this place.

Less abstruse connections than those mentioned above are also being brought to light. Of these one only will be considered. Page[1] has shown how, on the basis of the theory of relativity, the fundamental relations of electrodynamics may be derived from those of electrostatics. That is to say, where in the classical theory we have the *two* subjects of electrostatics and electrodynamics to be developed separately we have in the theory of relativity the *single* subject which is the synthesis of these two. This synthesis is of an essential nature; it is not a juxtaposition of things which belong apart, but a real unifying of two formerly distinct subjects into one larger and unseparated whole.

This may be taken as a special case of the way in which the theory of relativity enables us to obtain the laws of a moving system from those of a system at rest. The further development of this branch of the subject, one would believe, will lead to important results.

In this way it is obvious that the theory of relativity will introduce a much greater unity into physical science; the value of such a unifying factor can hardly be overestimated.

From the foregoing considerations we conclude finally that the theory of relativity has already made and is yet making important contributions toward the great philosophical problem concerning the one and the many.

XI. ON THE NATURE AND POSSIBILITY OF EXPERIMENTAL PROOF OF THE THEORY OF RELATIVITY.

There are at least two ways in which it may be possible to demonstrate experimentally the accuracy of the theory of relativity.

The first one consists in the direct proof by experiment of the postulates on which the theory is based. These proved, the whole theory then follows by logical processes alone. In my first paper referred to above I have already given a sufficient discussion of this method.

A second method would be as follows: Among the consequences of the theory of relativity seek out one which has the property that if it is *assumed* the postulates of relativity may themselves be then deduced by logical processes alone. If then this assumption is proved experimentally this is sufficient to establish the postulates of relativity, and hence the whole theory. Or, one may find such experimental results as lead to all the essential conclusions of relativity, whence one naturally concludes to the accuracy of the whole theory. A discussion of proofs of this kind is to be found in my second paper referred to above.

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[1] American Journal of Science (4), 34 (1912), 57–68.

This indirect method of proof is open to an objection of a kind which does not obtain in the case of the direct method previously mentioned. In the indirect method some auxiliary law, as for instance the law of conservation of electricity, must usually be employed in deducing the relativity postulates or essential conclusions from the new assumption which one attempts to justify by experiment. There is always the possibility that the auxiliary law is itself wrong; and consequently one's confidence in the accuracy of the relativity postulates as thus deduced can be no stronger than that in the truth of the auxiliary law. The same objection can also be raised against many conclusions which we are accustomed to accept with confidence.

To many persons it appears that the first method of proof mentioned above has been carried out successfully and satisfactorily. But if one does not share this opinion it is still legitimate to accept the theory of relativity as a working hypothesis, to be proved or disproved by future experiment. It is an historical fact, patent to every student of scientific progress, that many of our fundamental laws have been accepted just in this way. Take, for instance, the law of conservation of energy. There is no experimental demonstration of this law; and in the very nature of things it is hard to see how there could be. On the other hand, it is at variance with no known experimental fact. Moreover, it furnishes us a very valuable means of systematizing our known facts and representing them to our minds as an ordered whole. In other words, it is the most *convenient* hypothesis to make in the face of the phenomena which we have observed. Similarly, even if one does not believe that the theory of relativity has been conclusively demonstrated, should he not accept that theory (tentatively at least) provided it furnishes him with the most convenient means of representing external phenomena to his mind?

Finally, it should be said that every supposed proof of the theory of relativity is of such character that objections can be raised to it; likewise, every supposed disproof of the theory is in the same state. In the mean time, though we cannot accept the theory with all confidence, we can at least use its conclusions to suggest experiments which otherwise would not have been conceived. Therefore, whether true or false, the theory will be useful in the advancement of science. On this account, if on no other, it should appeal to every person interested in scientific progress.

INDIANA UNIVERSITY.
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CHANGE OF INDEX OF REFRACTION OF WATER WITH CHANGE OF TEMPERATURE.

By Frederick A. Osborn.

Introduction.

THE change of the index of refraction of water with change of temperature has been studied by a number of investigators, among whom the following may be mentioned:

Jamin[1] made observations on the change of the index of refraction of water between 0° and 30°. He used an interference method, but apparently did not publish any values. He contented himself with a general statement of the change and gave the empirical formula

$$K_t = K_0 - (0.000012573t) - (0.000001929t^2)$$

as expressing his results.

Gladstone and Dale,[2] soon after the work of Jamin, studied the same problem, making use of a hollow prism and the method of minimum deviation. They worked between the temperatures of 0° and 40° and remark "our determinations were performed repeatedly and most carefully on water near the freezing point." They call attention to the fact that the change of index between 0° and 5° is much less than that between 5° and 10°, finding from their work that the change is .0001 and .0002 respectively. Their method enabled them to express the change of index to the fourth decimal place.

Ruhlman,[3] by a method similar to that of Gladstone and Dale, obtained the index of refraction of water between 0° and 100° for the lithium, sodium and thallium lines. His values are given to five decimal places.

Lorenz[4] used an interference method and worked between 0° and 34°. $d\mu/dt$ was found from the shift of the fringes with change of the temperature and an accuracy of 3 units in the seventh decimal place was claimed. No experimental data were given but his results are expressed by the equation

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[1] Comptes Rendus, 43, p. 1191, 1856.

[2] Phil. Trans., p. 887, 1885.

[3] Pogg. Ann., 132, p. 1177, 1867.

[4] Wied. Ann., 11, p. 70, 1880.

$$\frac{d\mu}{dt} = -10^{-6}(0.076 - 5.606t + 0.06403t^2) \text{ for the Na line,}$$

$$= -10^{-6}(0.952 - 5.586t + 0.06402t^2) \text{ for the Li line.}$$

Dufet[1] used two different methods. In the prism method, the prisms filled with water at a given temperature, were adjusted to minimum deviation. The water was then exchanged for water at a higher temperature, and the displacement of the lines was observed as the temperature fell. From this shift of the lines, and the other measurable quantities, he computed $d\mu/dt$. In the interference method he used Talbot's fringes, and observed the passage of about 180 fringes between 0° and 50°. In part of this work a quartz plate was immersed in the water, and for part a crown glass plate was substituted. The values for $d\mu/dt$ obtained by his different methods sometimes differ from each other by as much as 45×10^{-7}. Dufet expresses his results by the equation

$$\frac{d\mu}{dt} = -10^{-7}(125.46 + 41.285t - 0.013304t^2 - 0.0046t^3).$$

Values for $d\mu/dt$ obtained from this equation occasionally differ from his mean experimental values by as much as 7×10^{-7}.

Kettler[2] repeated Ruhlman's work for temperatures above 20° using a total reflection refractometer.

Walter,[3] using the minimum deviation method, has done some careful work for the range 0° to 30°. His results are given to the fifth decimal place.

Conray,[4] by the same method, but with a spectrometer graduated to 10″, has made a study of the change of index between 0° and 10°. He claims an accuracy in the sixth decimal place, although greater differences between successive determinations for the same temperature range are found.

Bender,[5] with the Pulfrich refractometer, has studied the change of the index of refraction of water between 0° and 40° using the H_α, H_β, H_γ lines. His values are expressed to five decimal places.

More recently Gifford,[6] using a new prism method, has determined the index at 15° for a large number of wave-lengths. The values obtained are considered correct to within two units in the fifth decimal place. A still later determination by Baxter, Burgess, and Daudt[1] gives the value of the index for Na light at the temperatures 20°, 25°, and 30°. These investigators used the prism method also and consider their results correct to three units in the fifth decimal place.

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[1] J. de Phy., p. 401, 1885.

[2] Wied. Ann., 33, p. 353, 506, 1888.

[3] Wied. Ann., 46, p. 422, 1892.

[4] Proc. Royal Soc., 58, p. 228–234, 1895.

[5] Ann. d. Phy., p. 343, 1899.

[6] Roy. Soc. Proc., 70, p. 329, 1902; 78, p. 406, 1906–7.

While the change of the index of refraction of water with change of temperature has been studied by these and many others, the methods used and the results obtained are such as to make it seem desirable to repeat the work.

It was thought that the Abbe-Pulfrich interferometer could be made to give more accurate results than those thus far obtained and the present paper has for its object the application of the interferometer to the determination of the change of the index of refraction of water with change of temperature.

METHOD.

The method used in this piece of work is that first used by Reimerdes[2] in his work on the change of the index of refraction of quartz. The change in the index of the liquid is determined by the shift of interference fringes in a fused quartz system.

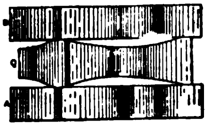

Fig. 1.

The arrangement of the system is shown in Fig. 1. A bed plate A, of fused quartz, 38.4 mm. in diameter and 10 mm. thick, with its upper face plane polished, is placed on a horizontal surface. Upon the quartz surface is placed a fused quartz ring Q of 31.8 mm. in external diameter, 24.4 mm. internal diameter, and approximately 9.9 mm. high. The cover plate B of fused quartz, is approximately 36 mm. in diameter and 8 mm. thick. Its surfaces are plane polished and inclined to each other at an angle of about 20″, so as to deflect from the field the light reflected from its upper surface.

The quartz ring was cut from a plate of quartz whose surfaces were plane polished and slightly inclined to each other. The ends of this ring were ground away in such a manner as to leave on each end three equidistant feet, whose surfaces are portions of the original plane polished surfaces of the plate from which the ring was cut. The inclination of the faces of this plate to one another was such that when the ring is placed, as in the figure, between the bed and cover plates the opposite reflecting surfaces, *i. e.*, the upper surfaces of the bed plate and the lower surface of the cover plate, are properly inclined to produce suitable interference fringes.

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[1] Am. Chem. Soc. Jour., 33, p. 893, 1911.

[2] Inaugural Dissertation, Jena, 1896.

The cover plate B has on its lower surface at the center, a small circle which serves in the field of view as the reference point from which the measurements are made.

When the apparatus is in adjustment and the field illuminated by monochromatic light, the field is seen to be crossed by parallel interference fringes. Each dark band represents a definite thickness of the air between A and B. If λ be the wave-length of the light used, then the difference in the thickness of the air layer for any two adjacent dark bands is $\lambda/2$. If the space between A and B be filled with a liquid, the effect is to increase the optical thickness between A and B from d to μd and the distance between any two fringes, the band width, is less than the distance between these same fringes in air.

When the fused quartz system and the liquid are heated, the fringes move across the field toward the thick edge of the wedge. The index of refraction of the water decreases with rise of temperature, hence the wave-length in the water becomes longer. There will be fewer wave-lengths between any two opposite points of the wedge. But a given fringe is due to the retardation of a definite number of wave-lengths of the light reflected from the upper surface of the bed plate over that reflected from the lower surface of the cover plate, so if the wave-lengths become longer, a thicker part of the wedge is needed to give the same number of wave-lengths retardation. Therefore, the bands move toward the thick edge of the wedge and the number passing the reference circle indicates the decrease in the number of waves between the surfaces at that point due to the lengthening of λ.

The expansion of the quartz ring increases the distance between the plates and decreases the number of fringes that would otherwise cross the reference circle toward the thick edge of the wedge, or increases the number of wave-lengths under the reference circle, which would cause a shift of the fringes toward the thin edge of the wedge. The resultant shift of fringes is the difference of these two effects and, owing to the small expansion of the fused quartz, is toward the thick edge of the wedge.

The change in the index of refraction for a given temperature change is given by the formula

$$\frac{d\mu}{dt} = -\frac{N\frac{\lambda}{2}}{L_1(t_2 - t_1)} - \alpha\mu_1.$$

That this is so appears from the following considerations.

Let n be the number of fringes shifted for the temperature interval $(t_2 - t_1)$.

L_1 and L_2 be the thicknesses of the liquid (length of ring) at t_1 and t_2.
t_1 and t_2 be the lower and higher temperatures.
μ_1 and μ_2 be the indices of refraction of the liquid at t_1 and t_2.
λ be wave-length in air of the light used.
λ_1 and λ_2 be corresponding wave-lengths in the liquid at t_1 and t_2.
α be the mean coefficient of expansion of the fused quartz ring for the given temperature interval $(t_2 - t_1)$.

$$\mu_1 = \frac{\lambda}{\lambda_1} \text{ and } \mu_2 = \frac{\lambda}{\lambda_2}, \quad \lambda_1 = \frac{\lambda}{\mu_1} \text{ and } \lambda_2 = \frac{\lambda}{\mu_2},$$

$$n = \frac{2L_1}{\lambda_1} - \frac{2L_2}{\lambda_2} = \frac{2L_1}{\frac{\lambda}{\mu_1}} - \frac{2L_2}{\frac{\lambda}{\mu_2}} = \frac{2L_1}{\frac{\lambda}{\mu_1}} - \frac{2L_1[1 + \alpha(t_2 - t_1)]}{\frac{\lambda}{\mu_2}}$$

$$= 2L_1\frac{(\mu_1 - \mu_2)}{\lambda} - \frac{2L_1}{\lambda}\mu_2\alpha(t_2 - t_1),$$

$$\mu_2 - \mu_1 = -\frac{n\frac{\lambda}{2}}{L_1} - \mu_2\alpha(t_2 - t_1),$$

$$\frac{d\mu}{dt} = \frac{\mu_2 - \mu_1}{t_2 - t_1} = -\frac{n\lambda}{2L_1(t_2 - t_1)} - \mu_2\alpha$$

$$= -\frac{n\lambda}{2L_1(t_2 - t_1)} - \mu_1\alpha,$$

as the difference between μ_1 and μ_2 is not sufficiently large to affect the value of the corrections.

APPARATUS.

The instrument used was an Abbe-Pulfrich interferometer such as described in detail by Pulfrich[1] and also by Randall.[2] The source of the light was a small mercury arc lamp, giving a very brilliant light in which the line in the green was especially prominent.

The quartz system was set up in a glass cell, 7 cm. in diameter and 4 cm. deep, which was placed in the double walled galvanized iron tank, *T*. The cover, *G*, closed the tank and made it water-tight by means of mercury between the double walls. The small opening in *G* at the top, 3 cm. in diameter, was closed by a glass plate. A Beckmann thermometer, *B*, and a stirrer, *S*, were introduced into the cell through the cover as shown in Fig. 2. The tank with its optical system was rigidly supported by the arm *L*, which was clamped to the stone pier on which the interferometer rested.

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[1] Zeitschr. fur Instrumentenkunde, 1898, p. 261.

[2] PHYS. REV., 20, 1905.

The heating was obtained by means of two coils of No. 28 iron wire wound on the insulated vessel *V*, which was 18 cm. in diameter and 25 cm. deep. The coils, shown by the small circles, were wound separately, the turns about 5 mm. apart, so that the coils could be used alone, in series, or in parallel. The coils were covered with a layer of asbestos and then with wool felt. This vessel was made to fit closely into another iron vessel which was set inside a larger tank 35 cm. in diameter and 40 cm. high. The space between the two could be filled with water or with ice.

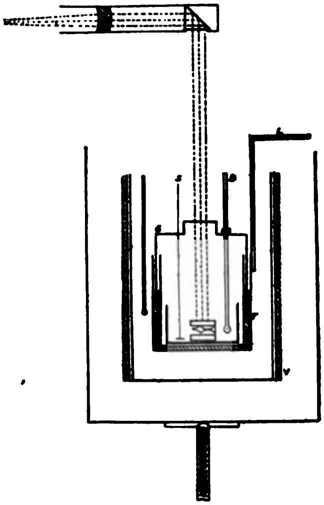

Fig. 2.

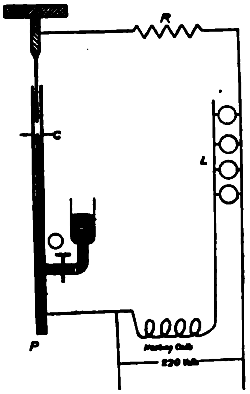

Fig. 3.

The thermostat used was a slight modification of a form which has already been described by the writer.[1] Around the inside of *V* was a liquid thermometer of 180 c.c. volume. This thermometer consisted of a ring of glass tubing, 17 cm. in diameter with six thin-walled vertical glass tubes, 20 cm. long, fused into it and hanging downward, all filled with toluol, except one which was filled with mercury and connected with the regulator at *P*, Fig. 3.

Fig. 3 shows the regulator and the electrical connections. The heating current was regulated by a lamp bank, *L*, not more than six 16 c.p. 220-volt lamps being required for the highest temperature. The thermostat was adjusted for a given temperature, by making the resistance in the rheostat, *R*, of such value that the current in the heating coils was insufficient to raise the temperature of the bath; the latter, cooling slightly, caused the mercury in the capillary, *C*, to fall, thus cutting out the rheostat circuit, and allowing more current to go through the heating coils. When the temperature of the bath rose, the mercury rose also in the capillary, cutting in the rheostat.

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[1] Jour. Phys. Chem., 9, 1905, p. 297.

Since the large volume of the liquid thermometer made it more sensitive than the thermometer in *V*, the bath was easily held at a temperature, constant to within 2 or 3 thousandths of a degree for half a day or more. The consequent variation of the temperature in the cell was not more than .002° in the same ime.

Stirrers in *T* and *V* kept the water thoroughly agitated.

ADJUSTMENT OF APPARATUS.

The preliminary adjustments of the instrument and the optical system for work with the fringes has been described by Pulfrich.[1]

When the system was in good adjustment, the liquid whose index of refraction was to be studied was poured into the glass cell, until, when the cover plate *B* was put on the quartz ring, the liquid came within a mm. of the top of *B*. The cover *G* was then put on and the entire bath raised by means of a screw until the water around the tank *T* came within a cm. of the glass top on *G*. The temperature of the bath was then raised to the desired point and the thermostat set in operation by closing the stop cock *O* and adjusting the platinum point in the capillary tube.

The temperature in the cell would gradually rise for two or three hours, after which the Beckmann thermometer showed no change of temperature greater than .002°. When the temperature had been constant for two hours longer, the position of the bands was read and also the temperature. One hour later both were read again. If no change in either was found, the temperature of the liquid was assumed to have become constant.

The temperature was then slowly raised 5° and the displacement of the fringes was counted. This displacement was, after the first 10 minutes, very uniform, until the temperature of the cell was within 0.5° of that of the surrounding water bath when the displacement became slow. The entire time for this displacement of the fringes was usually about two hours. But, as in getting the initial position of the bands, the final reading was not taken until the thermometer had been constant to within .002° for three hours. The number of whole bands shifted being known by actual count, the total shift was then given by the formula.

$$N = M + d_2 - d_1,$$

where M = number of whole bands counted, d_2 and d_1 the fractions of a band at the higher and lower temperatures respectively. These fractions were found from the readings of the micrometer with which the interferometer was provided.

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[1] Zeitsch. fur Instrumentenkunde, 13, Oct., Nov., Dec., 1893.

THERMOMETERS.

Two thermometers were used: For the water bath, thermometer No. 2,107, range − 10° to 50° by 1/10°, recently calibrated at the National Bureau of Standards; for the liquid in the cell a Beckmann thermometer, compared with No. 2,107, and, for the 5° interval, compared with one made by Haak, reading to .01° which had been calibrated at the Reichsanstalt.

The mean temperature of the water in the cell was obtained in each case by translating the readings of the Beckmann thermometer into degrees centigrade.

SPHEROMETER.

A Geneva Society spherometer was used to measure the quartz ring, and the readings could be made to .001 mm. A mean of thirty readings gave for the length of the ring 9.901 mm.

DEGREE OF ACCURACY.

From the formula

$$\frac{d\mu}{dt} = -\frac{n\lambda}{2L_1(t_2 - t_1)} - \alpha\mu_1$$

it is evident that the value of $d\mu/dt$ depends upon the three measurable quantities, n, L, and t.

In finding n from the formula

$$n = M + d_2 - d_1,$$

the d's can be found with an error affecting n by ± .02 of a fringe. The resulting error in $d\mu/dt$ is 10×10^{-8}. The difference in temperature could be determined with an error of ± .002. The error in $d\mu/dt$ from this source is 4×10^{-8}. The length of L could be found to ± .001 mm., resulting in an error in $d\mu/dt$ of 1×10^{-8}.

The total error in the final result due to these errors is

$$E = 10^{-8}\sqrt{10^2 + 4^2 + 1^2} = 10.8 \times 10^{-8},$$

or 1 unit in the seventh decimal place.

Scheel has found the length of a fused quartz ring similar to the one used here to be well represented by the expression

$$l_t = l_0(1 + 0.322 \times 10^{-6}t + 0.00147 \times 10^{-6}t^2).$$

The mean value of α computed from the above formula for each temperature range was used; and μ_1, as found by the prism method, using a Zeise spectrometer, is 1.3342. μ_1 need not be known with the greatest accuracy as an error of .001 is negligible as affecting the factor $\alpha\mu_1$.

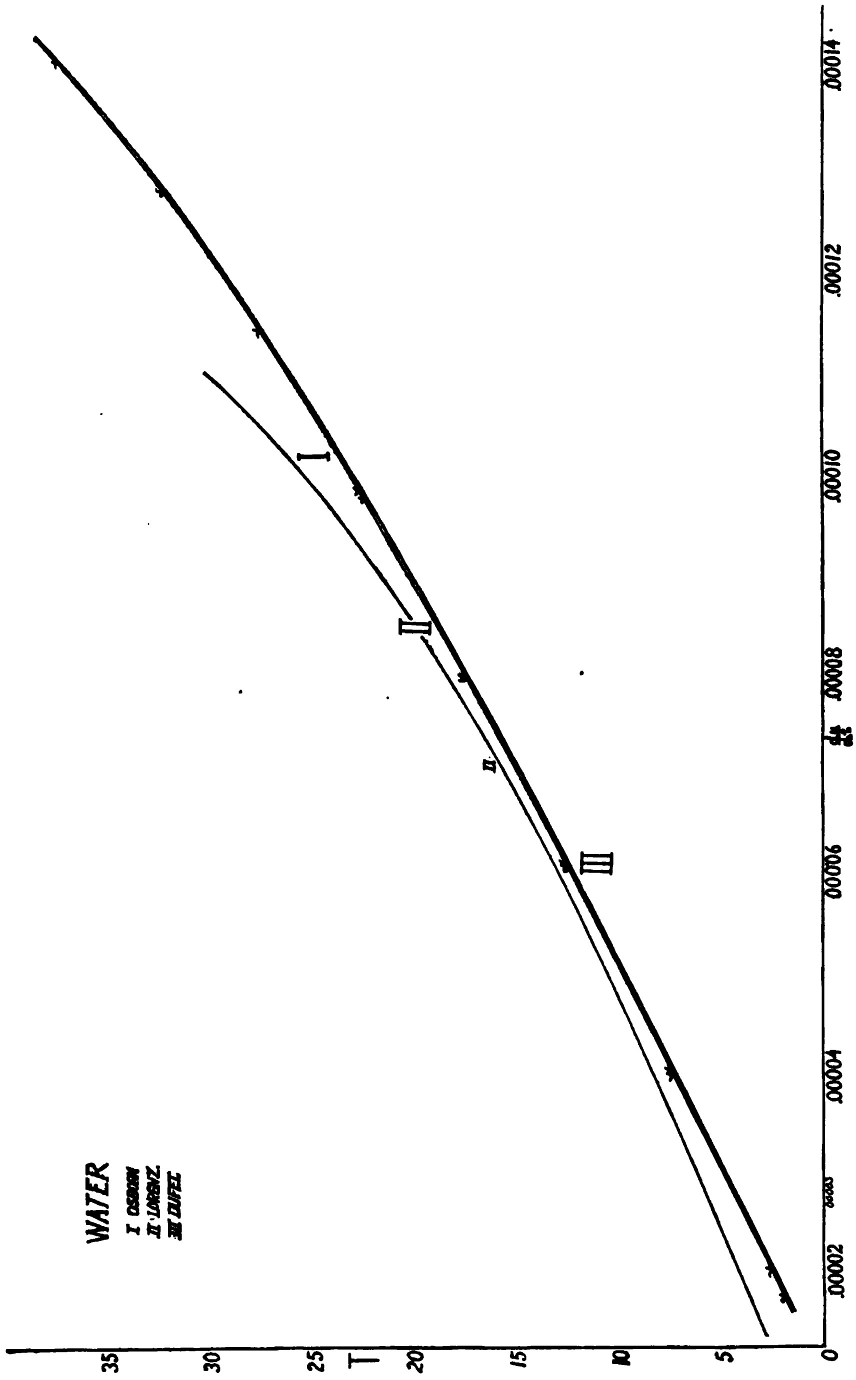
WATER
I OSBORN
II LORENZ
III DUFET
I
II
III
II
T
0
5
10
15
20
25
30
35
.00002
.00004
.00006
dn/dt
.00008
.00010
.00012
.00014

Fig. 4.

EXAMPLE OF A COMPLETE DETERMINATION.

Length of fused quartz ring . . . . 9.901 mm.
Thickness of water layer . . . . 9.901 mm.
Conductivity of water used (free from dissolved air) . . . . 1.7×10^{-6}

Time.	Temperature of Bath.	Temperature of Water in the Cell, Beckmann Thermometer.	Temperature of Emergent Column of Beckmann.	Shift of Fringes.	d's from Micrometer Readings.
9:30 A.M.	25.00° C.	1.150			12 noon
1:00		.170	23.5	0	
2:00		.190			$d_1 = .190$
1:00		.190			1:30 P. M.
1:30		.190			$d_2 = .186$
					Mean .188
1:45	25.5			1	
50		1.900		1	
54				1	
58				1	
2:02				1	
06				1	
10				1	
14				1	5:00 P. M.
18				1	
22				1	
26				1	$d_2 = -.047$
29				1	
33				1	
37				1	
40				1	6:30 P. M.
44				1	
48				1	$d_2 = -.043$
54				1	
59				1	
3:16	29.98			1	Mean $-.045$
4:00		6.120	25.5	1	
5:00		.130		21	
30		.130			
6:30		.130			

$$N = M + d_2 - d_1 = 21 + (-0.045) - 0.188 = 20.767,$$

$$t_2 - t_1 = 6.130 - 1.190 = 4.940°,$$

$t_2 - t_1$ corrected for emergent stem and mercury out of bulb $= 4.999°$,

$$\frac{d\mu}{dt} = -\frac{20.767 \times .0005460}{2 \times 9.901 \times 4.999} - .00000048 = -.0001150$$

at 27.49°.

Table I.

Water.

Conductivity of water, 1.7×10^{-6} (air-free).
L = 9.901 mm.
λ = .0005460 mm.

Date.	Series.	Temp. Range.	Temp. Diff. t_2-t_1	Total Shift of Fringe.	$\frac{d\mu}{dt}$	At
March 23	1	20–25	5.014	17.95	.0000992	22.58°
25	1	20–25	4.992	17.90	983	22.33
26	2	25–30	4.999	20.767	1150	27.49
27	2	30–25	4.981	20.684	1150	27.51
28	2	30–35	4.952	22.978	1284	32.25
28	2	35–30	4.860	22.540	1284	32.23
April 2	2	35–40	5.015	25.541	1409	37.50
3	2	40–35	5.013	25.528	1409	37.50
5	3	15–20	4.934	14.393	809	17.37
6	3	20–15	4.860	14.156	808	17.38
6	3	15–10	4.965	11.170	625	12.49
8	3	10–5	4.990	7.617	424	7.49
8	3	5–0	4.714	3.805	227	2.61
11	4	10–15	5.009	11.284	624	12.50
11	4	15–20	5.007	14.607	809	17.37
12	4	20–25	4.961	17.786	993	22.53
12	4	25–30	4.893	20.331	1150	27.46
13	4	30–35	5.052	23.429	1284	32.27
13	4	35–40	4.910	25.017	1410	37.48
14	4	5–10	4.730	7.271	420	7.34
14	4	5–0	4.495	3.632	227	2.59

Table I. gives the complete set of experimental data. Each series represents a new set-up of the apparatus and a new supply of water taken from a common source.

Fig. 4, Curve I., expresses the results graphically, and they may also be given by the equation

$$\frac{d\mu}{dt} = -10^{-7} \times (118.73 + 41.4184t - 0.02376t^2 - .0043757t^3).$$

This equation expresses correctly the experimental results to approximately one unit in the seventh decimal place.

Conclusion.

The results contained in Table I. show that the method here employed gives more concordant results than any method previously used and that in the determination of $d\mu/dt$ an accuracy of one unit in the seventh decimal place has very approximately been attained. This result has been achieved principally through the accuracy with which the shift of the

Table II.

Comparison of Results.[1]

Temp.	Lorenz.	Dufet.	Osborn.	Temp.	Lorenz.	Dufet.	Osborn.
38°		.0001423	.0001418	19°	.0000833	.0000874	.0000867
37		1402	1398	18	801	838	831
36		1380	1375	17	767	801	794
35		1357	1352	16	732	764	757
34		1333	1327	15	696	726	720
33		1308	1301	14	659	688	682
32		1282	1276	13	620	650	644
31		1256	1249	12	580	611	605
30	.0001105	1228	1221	11	538	572	565
29	1087	1200	1194	10	496	532	526
28	1067	1170	1163	9	452	493	486
27	1045	1140	1134	8	407	453	447
26	1024	1109	1103	7	360	412	406
25	1001	1078	1072	6	313	372	365
24	976	1045	1039	5	264	331	325
23	950	1012	1006	4	213	290	284
22	923	978	972	3	162	249	243
21	894	944	938	2	109	208	201
20	864	909	903	1	55	167	160

interference bands could be measured by the Abbe-Pulfrich interferometer and also by the extremely exact control and measurement of the temperature.

In comparing the results here obtained with those of other observers, it will be noticed from the results tabulated in Table II. and also from the Curves I. and III. that a constant difference exists between the results of Dufet and those of the writer. This difference is six or seven units in the seventh decimal place. The experiments of Lorenz furnish data for estimating the variation of $d\mu/dt$ with respect to λ. The change in $d\mu/dt$ to be expected for a change from the Na line, λ 5,893, used by Dufet to the Hg line, λ 5,460, used by the writer, is, according to Lorenz, 6.8 units in the seventh decimal place in the temperature region 10°–30°. The results of the two investigations appear therefore to be in complete accord. This is somewhat remarkable when one considers that each of Dufet's final results is the mean of four different determinations which often differ from one another by several units in the sixth decimal place.

The empirical equation here given expresses the experimental results to one unit in the seventh decimal place. It is given in preference to adopting the Dufet equation with a suitable alteration of its constant term as it agrees more closely with the experimental data.

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[1] Computed from the empirical equations.

Gifford's data on the change of the temperature refractive coefficients with wave-length are not sufficiently explicit to warrant an estimate of the difference which should exist between the values here presented and those of Dufet.

The changes in the index for the temperature interval 20° to 25° and 20° to 30° as computed by the formulæ of Dufet and the writer, the later values being corrected for wave-length, *i. e.*, to Na 5,893, agree with the recently measured changes according to Baxter, Burgess, Doudt, as the following table shows.

	B., B. & D. Observed.	Dufet Comp.	Osborn Comp.
20°–25°	.00051	.0004973	.0004975
20°–30°	.00109	.0010745	.0010748

The differences all lie within the estimated errors of observation.

To Professor Reed, under whose direction this work was done, the writer wishes to express his thanks.

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UNIVERSITY OF MICHIGAN.

ON THE ELECTRICAL NATURE OF COHESION.

By Fernando Sanford.

One of the most direct methods of measuring cohesion between a dissolved substance and its solvent is by means of volume changes in the solvent. This method has been used with great success by Heydweiller in a series of investigations, the results of which he has published in the Annalen der Physik.[1] In these papers Heydweiller has determined for a large number of the chemical elements a cohesion constant by means of which he is able to calculate the cohesion between water and compounds of these elements. This constant he calls the *Ionenmodulus*. It is calculated in various ways according to the assumptions made in regard to the relative change in the volume of the water and the dissolved substance. In the one here used, which he indicates by the symbol A_s, the modulus represents the percentage increase of the density of the water in which the ions are dissolved. The numbers are extrapolated for infinite dilution from curves plotted for various concentrations, so that complete dissociation may be assumed. It is also assumed that the ions, themselves, occupy the same volume which they occupied in the solid or liquid state before solution.

The values of A_s used in this paper are the final values adopted by Heydweiller, and are to be found in *Tabelle* 30^1, Ann. d. Phys., 37, p. 765, 1912.

I have already referred in several papers to an electrical constant which I have called the characteristic charge of the ion, and which is calculated by multiplying the mobility of the ion as determined in electrolysis by its ionic mass. I wish to show in the present paper that this electric constant of the ion is proportional to Heydweiller's *Ionenmodulus*.

If m = the mass of the ion and u = its mobility, Heydweiller's *Ionenmodulus* may be calculated from the equation $A_s = muk$, where k = .00112 for univalent ions and .00372 for divalent ions.

In the following table the mobility of the ions is taken from Kaye and Laby's Tables, p. 88. The velocities taken are those actually observed, *i. e.*, one half the numbers usually taken in the case of the divalent ions.

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[1] A. Heydweiller, Ann. d. Phys., 30, p. 873, 1909; 33, p. 145, 1910; 37, p. 739, 1912.

If the usual numbers for the mobility of divalent ions be used, k becomes .00186 for these ions.

TABLE I.

Ion.	Mobility.	Ionic Mass.	*muk.*	A_e
	$k = .00112$			
Li	34.6	7	.27	.35
Na	45.2	23	1.31	1.38
K	67	39.1	2.34	2.10
Rb	70.5	85.45	6.9	6.32
Cs	70.5	132.81	10.50	10.58
NH_4	66.3	18	1.33	.98
Cl	67.8	35.46	2.70	3.02
Br	70	79.92	6.28	6.68
I	68.8	126.92	9.80	10.27
OH	180	17	3.46	3.40
NO_3	64	62	4.68	4.54
$C_2H_3O_2$	42.1	59	2.78	3.04
	$k = .00372$			
H	330	1	1.22	1.05
F	48.3	19	3.42	3.16
Mg	23.8	24.32	2.17	2.66
Ca	26.8	40.09	4.00	4.04
Sr	26.8	87.62	8.74	8.76
Ba	28.7	137.37	14.66	13.08
Cd	24.6	112.40	10.30	10.86
Pb	31.7	207.1	24.4	20.68
Cu	24.5	63.57	5.79	7.26
Zn	24.2	65.37	5.88	7.22
Ag	56	107.88	22.46	10.02
SO_4	35.5	96	12.65	11.54

It will be seen that the values of A_e calculated in this way differ from the observed values by less than the probable error of determination except in a few cases. Hydrogen and fluorine are placed with the divalent ions, though they undoubtedly belong in the monovalent group, and silver, copper and zinc give too low values of A_e. Silver is placed with the divalent ions in the table, and it will be seen that with the divalent constant it gives A_e twice as high as it should be. On the other hand, it gives too small a value with the monovalent constant. For copper and zinc the values of *mu* should be multiplied by the sum of both constants to give Heydweiller's values of A_e.

Notwithstanding these discrepancies, which will probably be explained later, the evidence seems conclusive that the quantity *mu*, which I have called the characteristic charge of the ion, is a determining factor in cohesion.

The relations brought out in Table I. are shown graphically in Fig. 1, in which Heydweiller's values of A_s are plotted against the ionic charges, *mu*. Here the values of *u* for the divalent ions are those which are generally used, and are twice as great as those used in the table.

If the product *um* represents an electric charge, then in the case of the negative ions this charge must be negative, while in the case of the hydrogen and metallic ions it is positive. It will be seen from the above data that the attraction of the dissolved ions for water is practically independent of the character of the charge. In chemical combinations, however, the negative ion seems to play the principal part in determining cohesion. This is shown in Fig. 2, in which the specific cohesion of the sodium and potassium salts of chlorine, bromine and iodine are plotted against the charges of the negative ions as calculated from their mobility, assuming the hydrogen charge as + 4.84. The specific cohesion was determined by Motylewski by the drop method, using melted salts, and is quoted by Smiles (Chemical Constitution and Physical Properties, p. 40) from Zeit. Anorg. Chem., 38, 410, 1904.

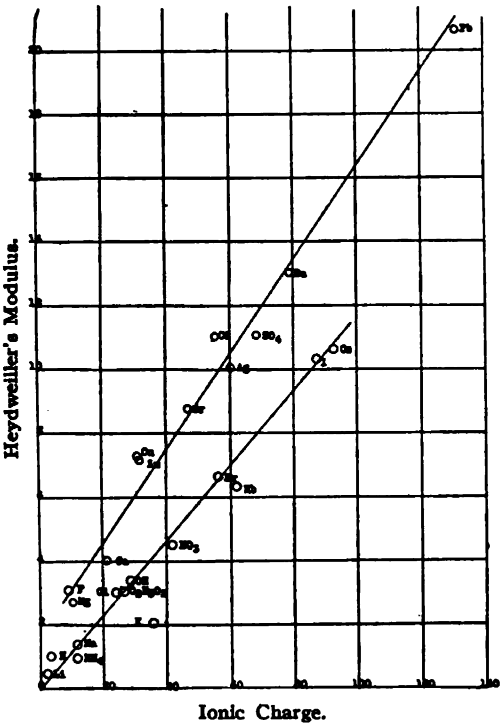

Fig. 1.

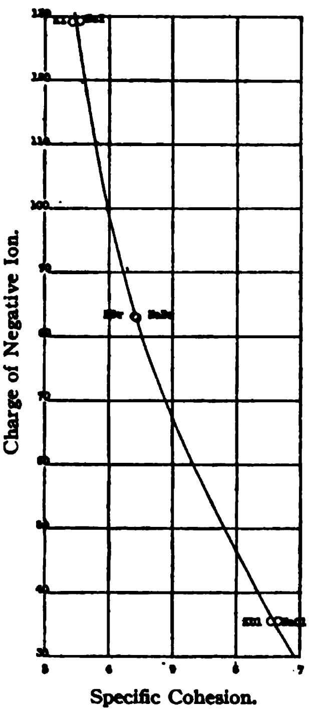

Fig. 2.

It will be seen from the curve that the specific cohesion of the corresponding sodium and potassium salts is nearly the same, the sodium salt regularly having the higher cohesion. If the mean cohesion of the corresponding sodium and potassium salts be taken, it may be calculated from the formula $C = 630/(e + 60)$, where C represents Motylewski's values for the specific cohesion and e = the charge of the negative ion. The values calculated in this way are compared with the observed values in the table below.

Table II.

Salt.	e	C Obs.	C Calc.
NaCl	35.2	6.63	6.59
KCl	35.2	6.55	
NaBr	82.8	4.44	4.43
KBr	82.8	4.41	
NaI	129	3.53	3.34
KI	129	3.44	

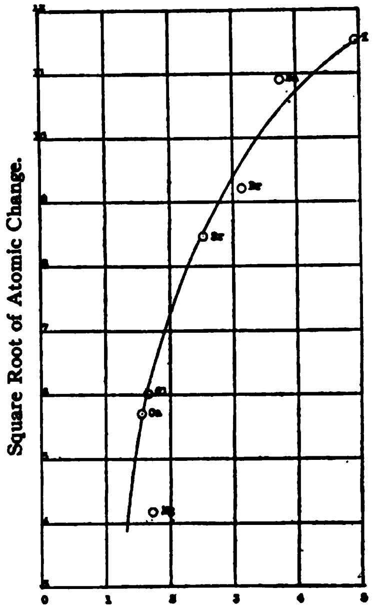

Fig. 3.

Another property which might reasonably be expected to vary with cohesion is density. When the density is compared with the atomic charge it is found that within a given group the density varies approximately as the square root of the atomic charge. If the usual numbers for the mobility of the ions of the barium group be used in calculating their atomic charges, the density of the elements of this group varies with the square root of the atomic charges at the same rate as does the density of the negative elements of the halogen group. This fact is shown graphically in the curve in Fig. 3, where the densities of the elements in the solid or liquid form are compared with the square roots of their atomic charges. Magnesium is the one element which is conspicuously out of place in this curve. Its density is proportionally greater in comparison with its other properties than that of any other element of its group.

The data from which this curve is drawn are given in Table III.

TABLE III.

Element.	Density.	$\sqrt{e}$.	Element.	Density.	$\sqrt{e}$.
F	——	3.8	Mg	1.74	4.16
Cl	1.66	6.04	Ca	1.58	5.7
Br	3.15	9.2	Sr	2.54	8.46
I	4.95	11.5	Ba	3.75	10.9

When two elements of the above groups combine to form salts, the specific gravities of the salts are approximately proportional to the products of the square roots of their atomic charges. Thus if e be the atomic charge of the positive ion and e' be the charge of a single atom in the negative ion, the specific gravity of these salts may be roughly calculated from the equation $d = k\sqrt{2ee'} + 1.5$, where $k = .022$. The observed values and the values calculated from this equation are given below.

TABLE IV.

Salt.	$\sqrt{2ee'}$.	d Calc.	d Obs.
$MgCl_2$	35.5	2.28	2.18
$CaCl_2$	48.7	2.57	2.26
$SrCl_2$	72.3	3.09	3.05
$BaCl_2$	93.2	3.55	3.85
$CaBr_2$	74	3.13	3.32
$SrBr_2$	109.7	3.90	3.96
$BaBr_2$	141.5	4.62	4.78
CaI_2[1]	92.8	3.54	4.9
SrI_2	137.4	4.51	4.41
BaI_2	177	5.40	4.92

The fact that the above values are only rough approximations to the true values is explained by the curve in Fig. 4, where it is seen that the relation between the density and the products of the square roots of the atomic charges is not accurately represented by a straight line.

The fact that the specific gravities of these, and other, salts are related to the charges of their ions may be shown in another way. Thus, if the algebraic sum of the positive and negative ions of a molecule be taken, it should give a resultant molecular charge which may be either positive or negative. When these resultant molecular charges with their appropriate signs are plotted against the specific gravities, the salts are found to be arranged in their appropriate groups with respect to both their positive and negative ions. In Fig. 5, the specific gravities of the chlorides,

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[1] The density of CaI_2 is clearly out of relation to the other salts in this table. It seems to have been determined but once, viz., by Ruff and Plato, Berliner Berichte, 35, and I will give further reasons in this paper for thinking that there is an error in the determination.

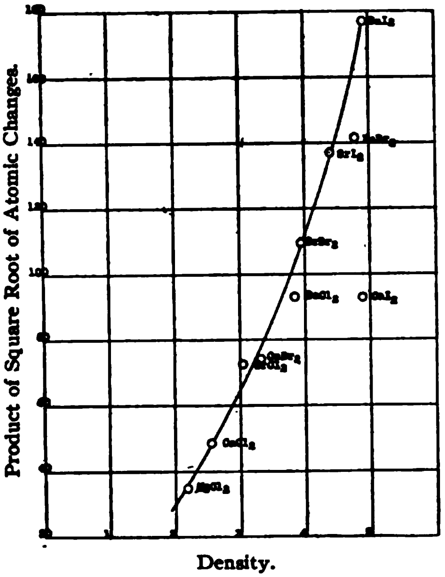


Fig. 4.

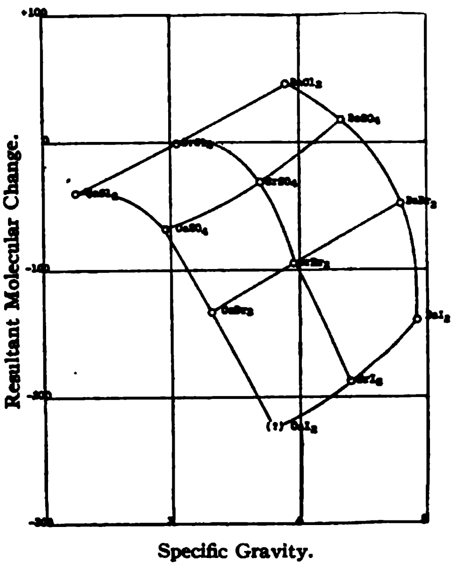
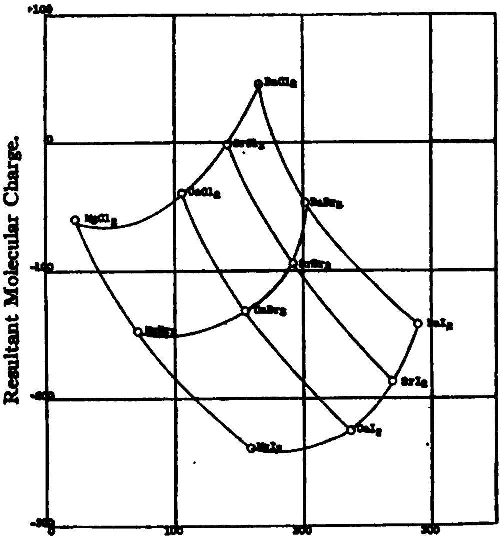


Fig. 5.

bromides, iodides and sulphates of calcium, strontium and barium are plotted as abscissas and their resultant molecular charges as ordinates.

It will be seen from this diagram that Ruff and Plato's value (4.9) for the specific gravity of CaI_2 is entirely out of relation to that of the other salts. I have accordingly ventured to give to this salt the provisional value 3.8, which it would seem to require from Fig. 4.

A closely analogous diagram to Fig. 5 is shown in Fig. 6. It seems to be a general law that those salts which produce the greatest volume contraction on solution in water give solutions which have the highest temperature expansion coefficients. The expansion coefficients used in Fig. 6 are given in the table below along with the specific gravities and the resultant molecular charges of the same salts. The expansion coefficient of the salt is determined by subtracting the expansion of water from that of the salt solution for the same temperature range. The increase in volume is for the interval between 19.5 degrees and 40 degrees, and is taken from Landolt and Boernstein's Tables. The molecular concentration is 20 gram equivalents of the salt in 10,000 grams of water. The increase in volume is given in hundred-thousandths of the volume taken at 19.5 degrees.



Expansion Coefficient.

Fig. 6.

Table V.

Salt.	Resultant Charge.	Specific Gravity.	Expansion Coefficient.
$MgCl_2$	− 54.2		22
$CaCl_2$	− 39.6	2.26	105
$SrCl_2$	− 2	3.05	142
$BaCl_2$	+ 44.8	3.88	165
$MgBr_2$	−147.6		70
$CaBr_2$	−133	3.32	153
$SrBr_2$	− 95.4	3.96	182
$BaBr_2$	− 48.6	4.78	201
MgI_2	−240		158
CaI_2	−225	3.8(?)	236
SrI_2	−187.8	4.41	268
BaI_2	−141	4.92	288
$MgSO_4$	− 83		
$CaSO_4$	− 68.4	2.96	
$SrSO_4$	− 30.8	3.70	
$BaSO_4$	+ 16	4.33	

Stanford University,
November 9, 1912.

BROWNIAN MOVEMENTS IN GASES AT LOW PRESSURES.

By R. A. Millikan.

THE essential improvements which the "oil drop method"[1] introduced into the study of Brownian movements consisted in

1. Enabling the observer to hold a given particle under observation as long as desired, even in a rarified gas, by counteracting the downward pull of gravity by the upward pull of an electrical field.

2. Eliminating completely all uncertainty as to the resistance offered by the medium to the motion of the drop through it.

This last result was accomplished by combining the Brownian movement equation

$$\bar{D} = \sqrt{\frac{4}{\pi}\frac{RT}{NK}t}, \qquad (1)^{2}$$

in which R is the gas constant, T absolute temperature, N the number of molecules in a gram molecule, and K the unknown resistance factor, with the characteristic equation of the "oil drop method," viz.,

$$e = \frac{K}{F}(v_1 + v_2)_1, \qquad (2)^{3}$$

in which F is the electrical field strength, e the elementary electrical charge and $(v + v_2)_1$ the greatest common divisor of the series of values assumed by the sum of speeds $(v_1 + v_2)$ as the drop changes charge through the capture of ions. The resulting equation which gives the average Brownian displacement is

$$\bar{D} = \sqrt{\frac{4}{\pi}\frac{RT(v_1 + v_2)_1 t}{F(Ne)}}, \qquad (3)$$

---

[1] R. A. Millikan, Phys. Rev., XXXII., p. 349, April, 1911.

[2] This is merely the Einstein equation $\overline{\Delta x^2} = 2RT/NK \cdot t$ modified so as to give the average displacement instead of the average of the squared displacements by substituting from the Maxwell distribution law wihch holds for Brownian displacements as well as for molecular velocities $\bar{D} = \overline{\Delta x} = \sqrt{2/\pi}\sqrt{\overline{\Delta x^2}}$.

[3] This equation is merely equation 1, p. 353 (Phys. Rev., XXXII.), viz. $e_n = \frac{mg}{Fv_1}(v_1 + v_2)$ after $\frac{mg}{v_1}$ has been replaced by K (which is constant for a given drop) and after the whole equation has been divided through by n, the number of elementary charges in e_n.

an equation which differs from Brownian movement equations which preceded it in being completely independent of all assumptions as to the density or shape of the Brownian particle, or as to the law of motion of the particle through the medium. It contains nothing but accurately measurable elements and is actually found to yield under all circumstances calculated values of $\bar{D}$ which agree with observed values within the limits of experimental error (about 4 per cent. for the individual drops worked with, though only 0.6 per cent. for the mean of all the drops used).[1]

Another forward step in the study of Brownian movements becomes possible as soon as we become able to determine the constant K. This constant is defined by the equation

$$X = Kv_1, \tag{4}$$

in which X is the force acting on the particle and v_1 the velocity produced by this force. This step has now been taken by working with oil drops at all pressures from 5 mm. up by the methods already referred to (*l. c.*, 1, p. 1). The details of this work are shortly to be published in full, but the result which gives the law of motion of a drop of radius a through a gas of viscosity μ, and mean free path l^1 under the influence of a force X is contained in the empirical equation

$$X = 6\pi\mu a v_1 \left\{ 1 + \frac{l}{a}\left(.874 + .32\epsilon^{-1.5_4\frac{a}{l}} \right) \right\}^{-1} \tag{5}$$

ϵ representing the Naperian base. This means that K of equations 1, 2, and 4, is given by

$$K = 6\pi\mu a \left\{ 1 + \frac{l}{a}\left(.874 + .32\epsilon^{-1.54\frac{a}{l}} \right) \right\}^{-1}. \tag{6}$$

This makes it possible to determine precisely how the Brownian displacements $\bar{D}$ increase as the pressure decreases in the case of a drop of any given radius. A glance at equation (1) shows, since μ is independent of pressure, that

$$\bar{D} \propto \sqrt{1 + \frac{l}{a}\left(.874 + .32\epsilon^{-1.54\frac{a}{l}} \right)}. \tag{7}$$

The accompanying table shows how $\bar{D}$ increases as the pressure falls from 760 mm. to 1 mm. in the case of the smallest drops ($a = .00003$ cm.) upon which accurate experiments have thus far been made in the use of the "oil drop method."

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[1] Harvey Fletcher, Phys. Rev., XXXIII., August, 1911. R. A. Millikan, Popular Science Monthly, 80, p. 438, May, 1912; also Trans. Am. Electrochemical Soc., Vol. XXI., p. 185, April, 1912.

p mm. Hg.	l/a	A'	$\bar{D}/\bar{D}_0$
760	.314	.874	1.13
380	.628	.902	1.25
190	1.255	.968	1.49
100	2.385	1.042	1.88
50	4.770	1.106	2.50
25	9.540	1.142	3.45
10	23.85	1.174	5.39
5	47.70	1.184	7.58
2	119.25	1.190	11.95
1	238.5	1.192	16.89

The column headed A' in the table gives the variable values which must be assigned to the quantity A if the law of motion of a drop through a resisting medium is to be expressed in the form

$$X = 6\pi\mu a v_1 \left(1 + A'\frac{l}{a}\right)^{-1}. \tag{8}$$

A' is constant and has the value .874 so long as $l/a < .4$, after which it rises as shown on the table to the value 1.193, beyond which it never goes (see also equation 3). The column headed $\bar{D}/\bar{D}_0$ in the table gives the ratio between the Brownian displacement at the given pressure, and the displacement at a pressure so high that the term in l/a is negligible in comparison with unity. The values given in this last column are computed by means of (7) but all save the last three of them have been checked experimentally with no larger errors than those mentioned above.

The table brings out clearly the advance which the "oil drop method" introduced into the study of Brownian movements by making it possible to hold a given particle under observation for an indefinite time in a gas at any pressure from 1 mm. up. For example, the ratio between the Brownian displacement $\bar{D}_a$ of a given particle in air at atmospheric pressure where the term in l/a is nearly negligible and the Brownian displacement $\bar{D}_w$ of the same particle in water is

$$\frac{\bar{D}_a}{\bar{D}_w} = \sqrt{\frac{\mu_w}{\mu_a}} = \sqrt{\frac{.0110}{.00178}} = 7.7.$$

The displacement of the particle considered above is multiplied again 7.6 fold by reducing the pressure to 5 mm. of mercury (see table). The change from 760 mm. to 5 mm. corresponds then very closely to the transfer of the particle from water to air.

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[1] l is computed from $\mu = .3502mn\bar{c}l$, in which $\bar{c}$ is the average molecular velocity and $\mu_t = .0001824 + .00017856(t - 23° \text{ C.})\ .00276$.

I have not yet carried the direct quantitative measurements to pressures lower than 5 mm., but I expect to do so in the near future and see no difficulties in the way of working at pressures as low as 1 mm. Under these conditions the Brownian displacements of the particle considered will be more than 100 times greater than they are in water, and the Brownian movement method of determining the elementary electrical charge should begin to be comparable in accuracy with the oil drop method, although its probable error would still be five or six times as great as that inherent in the latter method. The present value of e by this last method is

$$e = 4.774 \pm .009.$$

The value by the Brownian movement method fluctuates between Perrin's value 4.2×10^{-10} and Fletcher's value 5.01×10^{-10} with Svedberg's determination at the intermediate value 4.7×10^{-10}.

De Broglie,[1] who has made qualitative observation on the Brownian movement of smoke particles at low pressures, concluded that $\bar{D}$ was independent of pressure down to about 1 cm. of mercury, where it began to increase rapidly. This would be roughly true for particles ten times the size of those above considered, *i. e.*, for particles of radius $a - .0003$ cm. for the value of l/a at 1 cm. would then be 2.385 (see table) and $\bar{D}$ would be but $1.87/1.13 = 1.66$ times as much as at atmospheric pressure.

Ryerson Laboratory,
The University of Chicago,
December 20, 1912.

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[1] De Broglie, C. R., 154, p. 112, January, 1912.

RECTIFYING PROPERTIES OF A PHOTO-ELECTRIC CELL.

BY S. HERBERT ANDERSON.

IT has long been known that a photo-electric cell in which electrons are freely emitted from a metal under the action of light has distinct rectifying properties. In some investigations by Professor Kunz and Dr. J. G. Kemp it was found that in a photo-electric cell of the type used by the author in the present investigation may be used as a detector of electric waves. When used in connection with a Fleming cymometer it was found much more sensitive than the neon tube furnished with the instrument. Since its use as a detector depends upon its rectifying property, the present investigation was undertaken to determine the adaptability to practical use in wireless telegraphy.

DESCRIPTION OF METHOD AND APPARATUS.

The form of the photo-electric cell used is shown in diagram by Fig. 1, *A*. One electrode is a hemispherical cap of pure potassium deposited in the lower part of bulb *a*, which is 4 cm. in diameter. The other electrode is a platinum point *b*, the distance of which from *c* can be adjusted by the action of a magnet on the iron ring *d*.

The method of preparing the potassium was the same described in a former paper.[1] The potassium collected in *e* was poured into bulb *a* and deposited on the lower half by distillation. It was desired to have the cell as stable as possible and to this end nitrogen was introduced into the tube *A*. For removing all traces of oxygen and water vapor from the nitrogen, tube *B*, Fig. 1, was used. Before the evacuation of the system a piece of potassium was introduced into the tube at *f* which was then sealed off. When the tube was evacuated the potassium was distilled so as to form a brilliant metallic surface all over the inside of *B*. With stopcock *g* closed, dry nitrogen was let into *B* through *h* at atmospheric pressure. The potassium near *h* turned black showing some oxidation. *h* was closed and the tube heated until again there was a bright metallic surface. This insured the removal of all oxygen from the gas. Then *g* was opened and nitrogen let into *A*. *g* was closed and the tube *A* pumped out until the discharge from a small induction coil passed easily between the electrodes *b* and *c*. Thus in a rough way the pressure of minimum sparking potential was obtained. When a potential of 220 volts of a 60-cycle alternating current was applied no discharge occurred. but with 330 volts a current of about 0.1 ampere passed. *A* was then sealed off from the pump at *k*.

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[1] PHYSICAL REVIEW, XXXV., p. 239, 1912.

The method used in examining the rectifying properties of the cell was the same as used by Pierce[1] in examining the crystal and electrolytic detectors and is shown in diagram by Fig. 2. The image of the luminous spot of the fluorescent screen of a Braun tube is brought to a focus on a drum *d* by means of a lens *l*. This drum is connected directly to synchronous motor which operates on a 110-volt, 60-cycle alternating current. The drum is covered with a sensitive photographic film and is enclosed in a light-tight box. The source of the current sent through the photo-electric cell *A* was a 440-volt, 60-cycle alternating current supplied by the same dynamo that furnished the 110-volt circuit that the motor was operated upon. Thus the frequency of the two circuits was the same. By means of a potentiometer scheme *P* any potential difference up to 455 volts (which was the potential across the so-called 440 mains), could be applied to the electrodes of the cell. Connected in series with *A* were a pair of electromagnets, c_1 and c_2. These were placed, one above and the other below, the Braun tube, with their axes in a vertical plane, so as to give a horizontal deflection to the narrow cathode beam which passed through the small hole in the metal screen *s*. The total resistance of the electromagnets was 381.8 ohms. They were fitted with cores of soft Swedish iron, ½ inch in diameter. By means of a double throw switch *S*, the photo-electric cell could be replaced by a non-inductive resistance *R* which was adjusted to allow the same current to go through the electromagnets as passed when the cell was in the circuit. An electrostatic voltmeter *V* gave the fall of potential across the electrodes of the cell.

Method of Taking Oscillograms.

In Plate I., (*a*), (*b*), (*c*), are shown three of the oscillograms taken, These were obtained in the following manner: a photographic film was placed on the drum *d*, Fig. 2, the box closed and a cap put over the lens; the motor was started and adjusted to run in synchronism; the switch *S* was then closed so that the alternating current potential was applied at the electrodes of the cell (the current which passes through the cell also goes through the electromagnets and the alternating magnetic field produced causes an alternating deflection of the cathode beam, so that the spot of light on the fluorescent screen is drawn out into a horizontal line); the cap was then removed from the lens and the image of the spot of light moving on the screen was thrown upon the photographic film. The drum was rotated in synchronism and made a complete rotation for every two cycles of the current. So the spot of light moved over the film with two motions, the horizontal motion being produced by the changing magnetic field and the vertical motion produced by the movement of the drum. The spot of light started in at the same point at the beginning of each rotation. The part of the oscillogram obtained by this much of the exposure is the heavy line showing loops only above the horizontal axis, that is, a *current in one direction only*. The switch was then thrown so that R was in the circuit instead of A. The part of the oscillogram due to this exposure is the sine curve. The switch was then opened so that no current went through the electromagnets and consequently there was no horizontal deflection. This exposure gives the line of the axis of abscissas, the axis of zero current. The time of exposure for oscillogram (*b*) for the curve of the rectified current was 2.5 minutes; for the non-rectified current, 1.5 minutes; for the line of the horizontal axis, 40 seconds. So for the total exposure the spot of light moved over the film 8,400 times. This shows that the motor was running in synchronism.

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[1] Physical Review, XXVIII., p. 153, 1909.

For oscillograms (*a*) and (*b*) the ordinary Eastman kodak films were used. For oscillogram (*c*) an Eastman *extra rapid* film was used. The times of exposures for the three curves of this were 1′ 50″, 50″ and 30″ respectively. Probably 1′, 30″ and 20″ would be plenty of time. The lines are all heavy except in one place, where the film may have been defective or was not evenly developed.

The curve obtained when the current passed through the ohmic resistance R may be called a *potential phase* curve. It is of course a current curve, but since the potential across the ohmic resistance is in phase with the current through that resistance it gives us the phase of the potential about the photo-electric cell, and enables us to determine whether or not there is any lag or advance of the current through the cell. It will be noticed that in every case that the maximum point of the rectified curve is less than that of the potential phase curve. This occurs in spite of the fact that in every case great care was taken to adjust the resistance R before the exposure for an oscillogram so that the maximum deflection of the cathode beam was the same for the current passing through R as it was for the current passing through A. But as the cell is used, that is, as a current passes, the resistance of the cell seems to increase, and consequently the deflection of the cathode beam decreases. This is shown by the broad line of the curve of the rectified current in oscillograms (*a*) and (*c*) especially. During the exposure for each of these curves the current decreased about 0.5 milliampere, or 10 per cent. This increase of the resistance seems to come about by an absorption of the gas (nitrogen) in the cell. The character of the glow discharge changed in such a way as to show a decrease in pressure. The effect seemed to be much the same as that which occurs with a glow discharge through hydrogen with a potassium cathode in which potassium hydride is formed. But with this difference, that in the cell with hydrogen the potassium hydride formed is of a deep violet color, becoming almost black if the discharge is continued for a long time. But with the cell containing nitrogen the color taken on by the potassium under the action of the glow discharge is at first bronze, and with a continuation of the discharge the color becomes a bright blue, with just a suggestion of a greenish tinge. On heating the cell until the potassium is melted the color disappears, the potassium assumes its original bright metallic luster and the pressure is increased. It seems very probable that a potassium nitride is formed by the glow discharge. Since this work was done a reference[1] has been found giving an account of the formation of potassium nitride by a glow discharge between a potassium cathode and silver anode immersed in a mixture of 90 per cent. liquid nitrogen and 10 per cent. liquid argon; and also the account states that the nitride is formed when the liquid nitrogen is replaced by gaseous nitrogen.

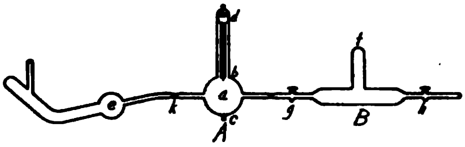

Fig. 1.

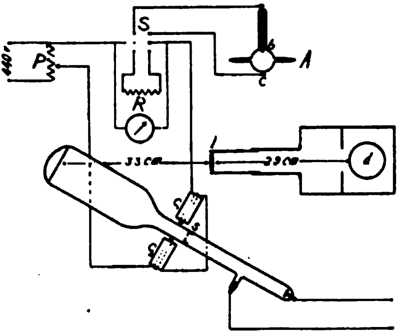

Fig. 2.

In taking all these oscillograms the amount of light falling on the photoelectric cell was rather faint. There was an eight-candle-power red-globed, incandescent lamp about three meters distant from the cell; and the cell was exposed to the faint light from the Braun tube. It was noticed just after oscillogram (*b*) was taken that light falling on the cell had a marked effect on the action. If the cell was in total darkness and the potential applied there was no discharge in the tube and no measurable current passed. But if a 16-candle-power lamp at a distance of a meter was turned on the discharge occurred and continued after the light was turned out. But by the deflection of the cathode beam it was noticed that the current was about 6 per cent. less when the cell was in darkness than when it was illuminated by the 16-candle-power lamp at the distance of 1 meter. If however the lamp was brought up to a distance of 40 cm. from the cell, the discharge stopped, but started again when the light was moved away. This shows that the light intensity has a very marked effect, as well as the gas pressure and the electrode distance. It is very probable that the discharge continued after the light was turned out because of the action of the light produced by the discharge in the cell upon the potassium.

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[1] Ber. der deutschen Chemischen Gesellschaft, Vol. 43. p. 1465, 1910.

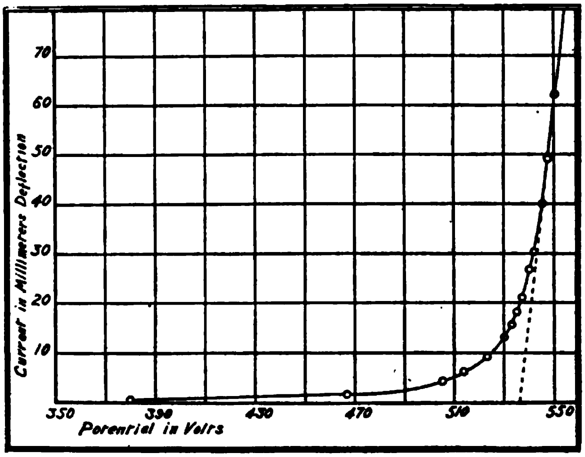


Fig. 3.

TABLE I.

Oscillogram.	I in Amperes.	V R.M.S. Value in Volts.	Electrode Distance (in Centimeters).
(*a*)	0.004	390	4.1
(*b*)	0.005	455	4.7
(*c*)	0.003	370	4.7

DISCUSSION OF OSCILLOGRAMS.

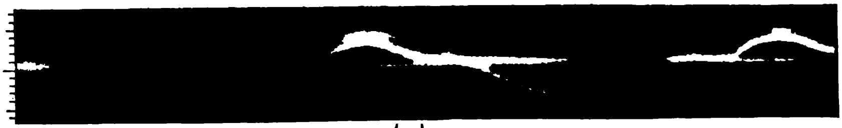
In Table I. are given the maximum values of the rectified current, I; the R.M.S. voltage across the electrodes, V; and the electrode distance for each oscillogram. The divisions of the scales along the oscillograms represent milliamperes. Time is represented in a horizontal direction from left to right.

PHYSICAL REVIEW, VOL. I., SECOND SERIES.
March, 1913

PLATE I.
To face page 226

(a)

(b)



(c)

S. HERBERT ANDERSON.

The author takes pleasure in acknowledging his indebtedness to Dr. J. G. Kemp for the type of cell used and for the curve of Fig. 3, to Professor Kunz for many valuable suggestions and assistance in preparing the cells used, and to Professor Carman and the physics department of the University of Illinois, where the investigation was carried on, for the facilities and means for the work.

PHYSICS DEPARTMENT,
UNIVERSITY OF WASHINGTON,
October, 1912.

IONIZATION OF POTASSIUM VAPOR BY ULTRA-VIOLET LIGHT.

By S. Herbert Anderson.

THE general method of determining the ionization of potassium vapor by ultraviolet light consisted in measuring the current produced between two electrodes which were contained in a highly exhausted tube containing potassium vapor, when a given potential difference was applied across the electrodes and a beam of ultraviolet light passed through the vapor between the electrodes.

The form of the tube used is shown in Fig. 1, *A*. The tube was 4 cm. in diameter and 12 cm. long. It was closed at one end with a quartz plate sealed on with Bank of England sealing wax. The opposite end of the tube was drawn down to about 8 mm. diameter and terminated in a bulb *b* in which the potassium *k* was lodged. The smaller tube was bent around in a semicircle to prevent particles shot off from the potassium from penetrating directly the space between the electrodes. The potassium introduced into *b* was obtained by distillation and was perfectly pure and clean. The tube was exhausted by a Gaede pump and then by a charcoal bulb and liquid air to the best possible vacuum. The electrodes, e_1 and e_2, were platinum plates 6 × 2.5 cm., and were placed parallel to each other 3 cm. apart.

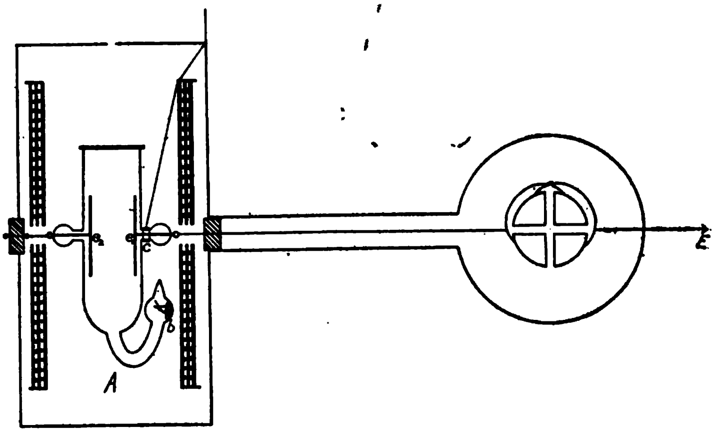


Fig. 1.

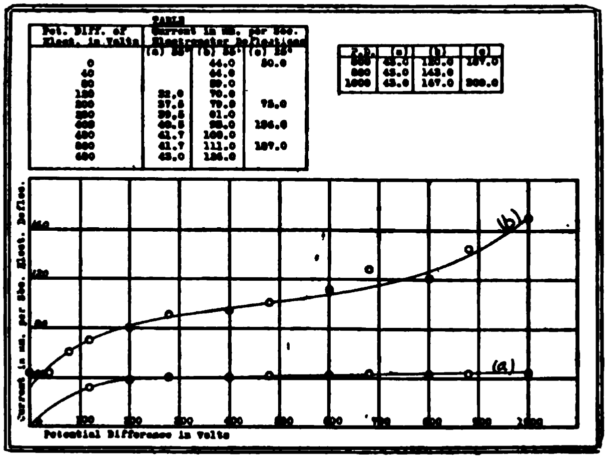
In order to test the ionization at temperatures higher than room temperature the tube was placed in an electric furnace consisting of a copper cylinder on which were wound two layers of wire so connected that the magnetic field within was negligible. The electric furnace containing the tube was placed in a sheet iron box, tightly closed except for a slit 5 × 30 mm. at one end, which could be opened to admit ultraviolet light. This metal box was connected by a metal tube to a tight metal cylinder in which a Dolezalek electrometer was placed. Electrode e_1 was connected to one pair of quadrants of the electrometer. Electrode e_2 could be grounded or connected to one terminal of a battery. c is a metal collar about the tube where electrode e_1 is sealed in. This was grounded so as to prevent conduction over the outside of the tube from effecting the electrometer. The copper cylinder of the heating coil and the metal containers of the heating coil and electrometer was grounded, E.

When the tube was kept in darkness at room temperature, 25° C., no current could be detected with the electrometer when a potential difference of 1,000 volts was maintained between the electrodes. When a beam of ultraviolet light from a spark between zinc electrodes was passed into the tube, but not striking the electrodes, and the same potential difference, 1,000 volts, was maintained, e_2 being connected to the positive terminal of the battery, there was still no current. (The electrometer was capable of indicating a current of 10^{-13} amperes.) If however the beam of light was incident upon the electrode connected to the electrometer, there was a deflection and the direction of the current was from e_2 to e_1. This of course was due to the photo-electric action of the platinum electrode. Hence there was no indication of ionization at a temperature of 25°.

The tube was then heated to 55°, which is the highest temperature to which the sealing wax may be heated without softening. Two hours was required for this heating, in order to get a constant temperature. At this temperature when a beam of ultraviolet light was passed between the electrodes there was a comparatively large current, the values of which for different potential differences are given in column (*a*) of the table. It was found that these readings could not be duplicated but that the current increased as the tube was maintained at this temperature. After four hours another set of readings was taken which is given in column (*b*). The tube was then allowed to cool down to room temperature and the conductivity tested when a beam of ultraviolet light passed between the electrodes. The current was found to be about the same as that found by the last set of readings at 55°, as is seen by comparing column (*c*) with column (*b*) (Fig. 2).

The tube was in perfect condition after the heating. The vacuum was tested by a spark discharge and found to be at the Röntgen ray stage. The discharge showed some of the characteristic color due to potassium vapor.

In the table the current is given in terms of the rate of deflection of the electrometer needle in mm. per second. A deflection of 20 mm. per second corresponds to a current of 10^{-12} amperes. The current observed is surprisingly large if one takes into consideration the fact that at 55° the vapor pressure of potassium is not large. The minimum current observed was 1.5×10^{-12} ampere, that is, an order of magnitude quite different from the currents due to ionization of any other vapor or gas by ultra-violet light.



TABLE

Pot. Diff. of Elect. in Volts	Current in mm. per Sec. Electrometer Deflections (a) [illegible]	(b) 55°	(c) [illegible]
0		44.0	80.0
40		44.0	
80		59.0	
120	32.0	70.0	
200	37.5	79.0	75.0
280	39.5	91.0	
400	40.5	98.0	136.0
480	41.7	100.0	
600	41.7	111.0	187.0
680	43.0	126.0	

P.D.	(a)	(b)	(c)
800	43.0	[illegible]	[illegible]
880	43.0	143.0	
1000	43.0	167.0	200.0

Fig. 2.

The increase of conductivity as the tube was maintained at a temperature of 55° is not easy to explain; nor the fact, that when the tube had cooled to room temperature the conductivity was about the same as that last observed at 55°, while in the first place there was no current at room temperature. However, it is likely that this is closely connected with the phenomenon observed by Wood[1] in the resonance spectra of mercury vapor. He found that there was a true absorption of light only when the pressure of the gas was above 0.01 mm. When a small amount of air was introduced so that the pressure was raised above 0.01 mm. the absorption increased with the pressure. It is not unreasonable to expect that ionization accompanies the transformation of light energy into heat energy that occurs in true absorption. If this is the case an introduction of a small amount of air or other gas into the tube of this experiment would result in an increase of conductivity. In this experiment it is very probable that gas was given off from the electrodes and walls of the tube as the temperature was maintained at 55°. On the basis of Wood's discovery this then would account for the increase in conductivity. No deposit of potassium was observed in the tube, after the temperature had been raised to 55°. Moreover, if there had been a deposit of metal, it would have occurred during the distillation, as the temperature was then above 200°, and yet no conductivity or ionization was noticeable until the tube was heated to 55°. It is the intention of the author to extend this investigation and determine this point.

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[1] Phil. Mag., V., 23, p. 689, 1912.

SUMMARY.

1. Potassium vapor at 55° is readily ionized by ultraviolet light.

2. An increase of ionization is produced by the addition of a small amount of foreign gas.

The writer is indebted to the Laboratory of Physics of the University of Illinois for the facilities and to Dr. Kunz for suggestions for the above work.

UNIVERSITY OF WASHINGTON,
November, 1912.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

A METHOD OF PRODUCTION OF LIGHT-NEGATIVE SELENIUM.[1]

BY F. C. BROWN.

IN the previous work[2] having to do with that variety of selenium that decreases its conductivity with illumination, no definite instructions have been given as to a reliable method of production. In fact no reliable method was known. Recently I have found two methods for producing this light-negative variety which are successful and apparently easy to follow. Both depend on the action of mercury vapor on selenium. The first method requires only that the vitreous or amorphous selenium be crystallized at temperatures anywhere between 170° C. and 200° C., over a sand bath in which there is a trace of mercury. Thirty minutes is usually sufficient time for this crystallization.

The second method has been to place a sample of light-positive selenium such as found in the Giltay cell in a vacuum created by a mercury pump. After several hours in such a vacuum the mercury vapor causes the resistance to fall from upwards of a million ohms to a value as low as 20 ohms without changing from room termperatures. When this lower resistance is reached the selenium becomes light-negative.

It is believed that the mercury vapor is directly responsible for calling forth the light-negative property. This may occur by one of three processes. The mercury may form conducting films through and around the selenium. Accordingly these films are broken by the changing volume and form of the selenium crystals under the action of light. This would require a mercury film of average thickness less than a millionth of a centimeter. Secondly the mercury may act as a catalyzer, thereby merely altering the rates of change between components of the selenium. And another explanation might be on the basis of the formation of a selenide by the mercury. The plausibility of the explanations is ranked in the order in which they are above stated.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

[2] Paper by F. C. Brown, PHYS. REV., 33, p. 1, and Phys. Zeits., 11, p. 482, 1911.

[3] Paper by Lilah B. Crum, PHYS. REV., 33, p. 538.

SUMMARY OF A STUDY OF THE ELECTRIC FURNACE SPECTRUM OF IRON.[1]

BY ARTHUR S. KING.

THE iron spectrum for the range from λ 2500 to λ 7200 as given by the tube resistance furnace in the Pasadena laboratory has been studied at temperatures ranging from 1800° to 2600° C. These temperatures were read by a Wanner pyrometer directed at the interior wall of the heated tube. The photographs of the visible spectrum have been made on large scale plates and at temperatures designated as "low," "medium," and "high," for which the measurements were about 1800°, 2100°, and 2500° C. respectively. Estimates of the intensities of lines in the arc were made in addition to those for the three furnace temperatures. From these intensities, it was possible to form a classification of the lines according to the temperature at which they first appear, and their rate of growth with increase of temperature, one of the main points brought out in the investigation being that some lines strengthen much faster than others as the temperature rises.

Six classes of lines are formed on this basis. Of these, the first two, designated as Classes I *A* and I *B*, appear at the lowest temperature and are strong in the furnace at all temperatures. The peculiarity of Class I *A* is that these lines are very weak in the arc spectrum. The lines of Class II appear also at low temperature but strengthen much more rapidly than the preceding classes as the temperature rises. Class III lines first appear distinctly at about 2100° and show a rapid increase of intensity at higher temperature and in the arc. Lines of Class IV appear only at the highest furnace temperature, while those of Class V are faint or absent in the furnace, though frequently strong in the arc.

The furnace at 2600° gives a spectrum in which a large percentage of the arc lines appear, but the relative intensities are very different from those of the arc spectrum. However, except for the lines of Class I *A*, for which the arc conditions are decidedly unfavorable, the intensities shown in the arc are of the order to be expected from the rate of strengthening with rising temperature in the furnace, if the arc discharge is considered as giving an excitation equivalent to a much higher temperature than is attainable in the furnace, where an upper limit is set by the strong vaporization of the carbon tube. If the melting and boiling points of iron are taken as 1500° C. and 2450° C. respectively, it is evident that the vapor can be made to radiate much below its boiling point. Since the highest furnace temperature is above the boiling point, the arc should give a spectrum similar to that of the furnace if the iron were simply boiling in the arc. It would thus seem that the differences between arc and furnace line intensities are to be attributed mainly to the high electronic speeds given by the arc discharge.

The changes in intensities of lines from the core to the outer vapors of the iron arc were examined by projecting an image of the horizontal cross-section of the arc on the slit of the plane-grating spectrograph. The low temperature furnace lines were found to extend farthest into the outer vapors of the arc, the high temperature lines being giving chiefly by the core. The increase in strength from the flame into the core was also found to be similar to the rate at which the lines strengthened with increasing furnace temperature. The results thus indicate an equivalence between the excitation in the different parts of the arc and that which would result from temperature differences. Chemical processes in the flame of the arc do not appear to be an active agent in the phenomena, since the furnace *in vacuo* at low temperature, which should give minimum chemical action, gives a spectrum most nearly resembling that of the outer vapors of the arc.

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, Nov. 30, 1912.

The furnace material offers many instances where two lines near together show quite different response to temperature change and their relative intensities in a given source of light may be used to indicate its temperature when referred to the furnace spectrum for a known temperature. This method is applied to the flame spectra of iron obtained by Hemsalech and de Watteville for various gas mixtures and the several flames are arranged in order of their effective temperatures in producing radiation from iron vapor.

The ultraviolet furnace spectrum was found to be very rich in lines. As would be expected, the spectrum extended farther toward shorter wave-length as the temperature increased. It was found, however, that the line spectrum did not extend as far as the continuous spectrum given by a plug in the middle of the tube for the same temperature. Aside from this, the distribution of lines through the spectrum bears little resemblance to the intensity gradation shown by the spectrum of an incandescent solid. The low temperature lines are chiefly in the blue and green, higher temperatures being required to give lines in the orange and red.

The furnace experiments do not offer definite evidence as to the possibility of producing a line spectrum by temperature alone, since there is strong ionization from the carbon tube and a limited amount of chemical action. However, the position of temperature as the agent which gives the initial excitation and controls the observed variations in the spectrum seems to be clear. A potential difference is not necessary to the existence of the line spectrum. The spectrum fades gradually when the current is broken, some iron lines having been observed in the heated tube 5 minutes and the *D* lines of sodium 16 minutes afterward. A plug in the tube serves to suppress or throw into absorption all the lines of a vapor also present, indicating that the black body radiation is stronger than that of a metallic vapor for any given wave-length.

Mount Wilson Solar Observatory,
November 7, 1912.

THE EARTH INDUCTOR AS AN INCLINOMETER.[1]

BY N. E. DORSEY.

THE instrument chiefly used for the determination of the magnetic inclination is the dip needle. Although very simple in theory, this instrument in practice is subject to many well known imperfections, the most serious of which are those known as pivot errors. These errors vary with the inclination, and can not be eliminated by any combination of observations. For this reason another universal instrument for the determination of the dip is imperatively needed.

The most promising of such instruments is the continuously rotating flat coil earth inductor, when used with a suitable galvanometer and a two-part commutator. In the past the theory and the use of this instrument has been limited to the simplest possible case, viz., that in which the axis of rotation lies in the magnetic meridian and at such an inclination that the deflection of the galvanometer is zero. Then, except for the effect of thermal electromotive forces, the axis of rotation lies in the direction of the earth's field. The thermal current is eliminated by a suitable combination of observations taken with the coil rotating in opposite directions.

This procedure is very good when the meridian can be readily determined and when the instrument has a firm support, but it breaks down when observations are desired near the poles or at sea. In the first case the meridian can not be readily determined with the requisite accuracy, and in the second the instrument is never at rest. In neither case, however, would there be serious difficulty in the determination of the dip had we but a perfect dip needle. When oscillating, as on ship board, the mean of a sufficient number of turning points would be its correct position; and when at rest the sum of the squares of the cotangents of the inclinations of the needle at two azimuths differing by 90° equals the square of the cotangent of the true dip.

A consideration of the theory of the earth inductor shows that when the brushes are so adjusted that commutation occurs when the mean plane of the coil is parallel to the inclination axis of the instrument, then that inclination of the axis of rotation of the coil for which the deflection of the galvanometer, in both magnitude and sign, is independent of the direction of rotation of the coil (the speed being the same for both directions) is exactly that which would be taken by a perfect dip needle when its pivots are parallel to the inclination axis of the inductor. Furthermore, when the instrument is oscillating, that inclination for which the mean of the turning points of the galvanometer is independent of the direction of rotation of the coil, speed constant, is equal to the mean of the turning points of a perfect dip needle having its pivots parallel to the inclination axis of the inductor.

Consequently, under all conditions the earth inductor can be used in such a manner as to give the same indications as would be given by a perfect dip needle.

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[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

The above assumes that the thermal electromotive force is independent of the direction of rotation, the speed being constant; or that it has been eliminated by suitable reversals of the coil terminals with reference to the commutator segments.

Since there is comparatively little wear upon the brushes it is probable that these will require resetting only at relatively long intervals. They can be set wherever the magnetic meridian is known. To do so, place the inclination axis in the magnetic meridian, and the axis of rotation of the coil vertical; then the brushes will be properly set when the deflection of the galvanometer is independent of the direction of rotation of the coil.

According to advices received by cable from Chile, an earth inductor now in use upon the *Carnegie* is giving perfect satisfaction. The instrument has not yet been tried out in the neighborhood of the magnetic poles.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington.

(To be published in the Journal of Terrestrial Magnetism for March, 1913.)

The Use of Sounding Boards in an Auditorium.[1]

By F. R. Watson.

SOUNDING boards have long been used as an aid to speaking in auditoriums, especially in churches. The author has tried the effect of sounding boards of different kinds in the auditorium at the University of Illinois, where the acoustical properties are unsatisfactory. A plane reflector placed at an angle over the speaker's position proved to be of little benefit. The same was true for a sheet of heavy canvas. A pronounced effect was found when a parabolic reflector was used. The main volume of the sound was directed as desired and a number of troublesome echoes eliminated. The results indicate that the use of a reflector designed for the dimensions and shape of an auditorium will be an aid to good hearing. Experiments are now being conducted in which reflectors of special design are being made to direct the sound more effectively to the audience.

The Effect of Magnetization on Thermal Conductivity.[1]

By N. F. Smith.

CONTRADICTORY results have been obtained by other observers. The present investigation has been confined, thus far, to a determination of the change produced in the thermal conductivity of iron by a longitudinal field. Observations have been made on metal tubes three eighths of an inch in diameter and about three feet in total length. The iron tube and a comparison tube of brass were mounted side by side. One end of each of these tubes was maintained at a constant temperature of from 200° to 225°. The two tubes were surrounded by similar solenoids, connected in series. The coil surrounding the comparison tube was wound non-inductively, while the winding of the other could be changed from non-inductive to inductive by a commutator. By a suitable arrangement of thermo-couples, two points of equal temperature were determined in the two tubes while a non-inductive current flowed in each coil. The magnetic field was then applied to the iron tube by shifting the commutator, and after time had been given for temperature conditions again to become steady, the point of equal temperature in this tube was again determined. Fields up to 163 C.G.S. units were employed. The sensitiveness of the apparatus was such that a change of conductivity of one tenth of one per cent. could have been detected with certainty, yet in no case was an effect as great as this observed.

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[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

CENTRE COLLEGE,
DANVILLE, KY.

LINE STRUCTURE AND SHIFT WITH AN ECHELON SPECTROSCOPE.[1]

BY NORTON A. KENT.

IN an article,[2] dealing with line structure and shift as revealed by an echelon spectroscope, the writer showed that the inequality of wave-length between certain arc and spark lines in the spectrum of zinc could be explained by the appearance of satellites on the red side of the main lines. The results obtained confirmed those of previous investigations with large Rowland gratings and showed how the satellite systems varied with the capacity and inductance in the circuit and were modified by using an alloy for the spark terminals and by throwing upon the slit of the spectroscope various regions of the spark.

The writer's attention has since been called to a paper by Stansfield,[3] and the suggestion made that possibly the satellites obtained were those due to a secondary action of the echelon. The writer after further experimentation concludes that this cannot be the case, for the following reasons:

1. "The secondary bands are superposed on the echelon lines and resemble them in appearance. Their width is about the same as that of the finer spectrum lines and so in the ordinary position of the echelon they are not easily recognized."[4] The satellites observed are by no means "fine," but are as diffuse as the main line; moreover, they are "easily recognized."

2. "When the echelon is rotated the secondary bands move faster than the spectrum lines in the same direction and so move across them. No such phenomenon has been observed in the case of the satellites."

3. The secondary bands are "curved in the same direction" as the spectrum lines "but more strongly." The satellites observed are of the same curvature as the main lines.

4. The secondary bands should be about one seventh the distance between the adjacent orders, as given by the echelon. The satellites are usually sepa-

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[1] Abstract of a paper presented at the Clevcland meeting of the Physical Society, December 30, 1912.

[2] Proceedings of the American Academy of Arts and Sciences, Vol. XLVIII., No. 5, August, 1912.

[3] Philosophical Magazine (6), 18, 383, 1909.

[4] The quotations are from Stansfield's paper.

rated from each other and the main line about one third or one fourth the distance between the orders.

5. The structure is shown by two echelons made by different manufacturers.

Moreover, it may be added that the satellites are not due to false structure owing to the errors of the instrument. Their reality has been tested by the Koch method.[1]

Physics Laboratory,
Boston University.

Vacuum Tube Discharge in a Magnetic Field.[2]

By Norton A. Kent and Royal M. Frye.

ALTHOUGH many have studied vacuum tube discharge in a magnetic field, no investigator has fully identified the resulting spectrum and classified the effects obtained upon the creation of the field.

The writers have found that the decrease of the effective cross-section of the tube, and the sand-blast action of the ions deflected against the glass—both caused by the magnetic field—result in the production of one or more of the following classes of spectra:

1. Those of impurities in the tube; examples, hydrogen appearing in an argon tube, and argon in a nitrogen tube.
2. Those of dissociation products of the original gas; example, the oxygen spectrum appearing in a carbon monoxide tube.
3. Those of dissociation products of the glass; example, sodium and oxygen lines appearing in the spectra given by many tubes when in the field.

The case of nitrogen and argon above mentioned is of special interest, as some of the lines of the blue spectrum of argon persist for at least ten minutes after the magnetic field is removed, and the reappearance of the band spectrum of nitrogen is but gradual. This is a unique phenomenon.

Physics Laboratory,
Boston University.

Resistance of Oxides at High Temperatures.[2]

By A. A. Somerville.

THE resistance of ten oxides is determined for a temperature range between room temperature and higher to about 1000 degrees Centigrade. In some cases the oxides are almost perfect insulators at room temperature. In practically all cases the resistance decreases with rise of temperature.

Cadmium oxide is one exception to the above rule. It is a fairly good conductor at ordinary temperatures. The resistance decreases slowly for a time when heated, and upon continued rise in temperature the resistance rises also, only to decrease again and at 1100 degrees is practically the same as at 20 degrees.

Silicon monoxide is a new product and the lightest of any oxide, its density being .02 grams per cubic centimeter.

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[1] P. P. Koch, Ann. der Phys., 34, p. 386, 1911.

[2] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

A Micropyrometer.[1]

By George K. Burgess.

A SMALL incandescent lamp is mounted within the Huyghens eyepiece of an ordinary microscope which is then used as a Morse or Holborn-Kurlbaum pyrometer. Combined with a suitable furnace, such as a platinum or other metallic strip heated electrically in vacuo or suitable gas as hydrogen, melting points and emissivities of minute specimens (0.01 to 0.001 milligram or less) are readily observed. The apparatus is most conveniently calibrated empirically in terms of known melting points. It is being used for measurements on the rare refractory elements and the author will be glad to receive pure samples of a few hundredths milligram, particularly of the rarer elements, for the determination of their melting points and emissivities.

December 12, 1912.

Optimum Wave-Length in Radiotelegraphy.[1]

By A. H. Taylor.

GIVEN a certain available power, furnished by a self-controlling transformer, and a fixed aerial system, the determination of the optimum wave-length for the station resolves itself into the determination experimentally of the maximum attainable value for the expression $I^2/\lambda^2 P$ which represents the radiant efficiency of the station, and of the maximum value of I/λ, which is proportional to the maximum sending distance in an ideal non-absorbing medium. The first of these quantities is a function of λ but not of P; the second is proportional to $\sqrt{P}$, where I = sending current, λ = wave-length, and P = power input.

The first of these expressions, $I^2/\lambda^2 P$, is proportional to the audibility at close range. The second, I/λ, has its effect modified by the absorption term, $\epsilon^{-\alpha d/\sqrt{\lambda}}$.

Using data obtained by Austin[2] the effect of absorption for over sea transmission can be calculated. This effect depends on the available power and on the height of the aerials, but it seems clear that for low power installations the shortest wave-length consistent with good coupling will give the best results.

At distances of 300 to 600 kilometers, the calculations indicate that a very considerable variation in wave-length will affect the range but little. The short wave-lengths should be much more effective at close range. These conclusions have been partly verified by experiment.

The following variations from the deductions may be predicted:

First. Increase of power will push the optimum wave-length toward longer waves.

Second. Increase of height of sender or receiver will make the optimum wave-length longer.

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[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

[2] L. W. Austin, Bull. Bur. St., Vol. 7, No. 3, 1911.

Third. Inefficient aerials (K smaller) will make the optimum wave-length shorter.

Fourth. Increased absorption will make the optimum wave-length longer.

Fifth. Increased radiation efficiency $i^2/\lambda^2 P$, as by synchronous rotary spark gap, impact excitation, etc., will tend toward a longer optimum wave-length.

THE SIMILARITY OF ELECTRICAL PROPERTIES IN LIGHT-POSITIVE SELENIUM TO THOSE IN CERTAIN CRYSTAL CONTACTS.

BY T. C. BROWN.

SINCE so many of the electrical properties of selenium have been supposed to be unique, it is thought to be significant that most of these properties also appear in connection with crystal contacts. The like phenomena that are compared are:

1. The apparent invalidity of Ohm's law.
2. The variation of the conductivity with pressure.
3. The change of the resistance with time of current action.
4. The effect of alternating currents.
5. The decrease of resistance by slight amalgamation.
6. The effect of abrasion.

Attention is further called to the fact that the light effect could be explained by merely assuming a change in crystal volume or crystal form as the result of light-action. This light effect is the only important electrical phenomenon that is known not to have its counterpart in crystal contacts.

COMPARISON OF SMALL ELECTROSTATIC CAPACITIES.[1]

BY J. C. HUBBARD AND H. F. STIMSON.

THE electrostatic capacities of small bodies such as lead wires, binding posts, electrometers, small spheres, or the change in capacity when two such bodies are joined, as when a lead wire is connected to a sphere, are compared by the method of divided charge with the capacity of a straight wire used as a standard. The instrument making these experiments possible is the gold-leaf electroscope of constant capacity described by one of the authors.[2]

It is first necessary to measure the capacity of the electroscope and its lead wire. The latter was chosen about 1 meter long. About 20 cm. of its free end is made to move through a small angle. In its lowest position this wire touches an earth connection and the comparison capacity (a length of bare wire stretched between two pieces of red sealing wax) thus putting the gold leaf and standard at zero potential. Raising the lead wire breaks, first the earth connection, then connection with the standard. Further raising puts the lead wire in contact with a terminal of a suitable battery (ten volts) which charges the gold leaf. Lowering the lead wire brings it in contact with the standard, with which the charge is thereby divided, and still further lowering earths both the electroscope and wire standard capacity again. In the meantime potential readings for these various positions are taken with the electroscope. The measurements are complicated by the fact that the movement of the electroscope lead wire slightly varies the capacity both of the electroscope system and of the standard. For example, the combined capacity of the two is less in some cases by more than 1 cm. when they are in contact than when they are separated. But it is only when the electroscope lead is within a fraction of a cm. in distance from the standard wire that its influence must be taken into account. In fact one of the most interesting results of these measurements it the fact that wire standards by reason of the considerable extension made possible, are not seriously affected by the movement in their neighborhood of comparatively small bodies charged to low potentials. It was not found necessary to make any correction for change in capacity of the electroscope system when the lead was withdrawn from the charging battery terminal, as the change was well within the errors of observation.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 12, 1912.

[2] PHYS. REV., 33, p. 558, December, 1911.

As an example of the accuracy obtainable there is here given the result of comparing the capacity of the electroscope with the capacities of three wires of different lengths. Let V_1 be the potential of the charged gold leaf before the charge is divided; V_2, the potential of the divided charge; E, the capacity of the electroscope and lead; and C, the capacity of the comparison wire. Then

$$EV_1 = (E + C - d)V_2,$$

where d is the amount by which the capacity of the combined system is diminished on account of the contact of the electroscope lead with the standard. Let l be the length of the standard wire and a, the radius. Instead of using V in volts it is more convenient to express it in terms of electroscope scale readings, here proportional to the potential. All of the wires had a radius of .0253 cm. From the values given in the table, d is found to be 1.51 cm. of capacity, the values of E being respectively 9.53, 9.44, and 9.48 cm., giving a mean of 9.48 cm. It is of interest to note that of this capacity of 9.48 cm., about 6 cm. is due to the lead to the electroscope, leaving about 3.5 cm. as the capacity of the gold leaf and its connection.

l cm.	C cm.	V_1	V_2	E cm.
370.6	19.32	459	160	9.53
269.8	14.55	464	195	9.44
135.5	7.89	460	275	9.48

The capacity of a circular plate was measured in the following manner. Two equal straight wires were attached perpendicularly to the center of the disc, one on each side. The capacity of the whole system was then measured, using one of the wires as the lead. The other wire was then removed and the capacity measured again. The first value minus the second gives the amount by which the capacity is changed by the presence of one lead. Subtracting this amount from the second value should give the capacity of the disc alone.

Thus, with two leads, each two meters long and of a diameter of .1025 cm., and a plate having a diameter of 30.5 cm., the combined capacity was found to be 27.5 cm. Removal of one lead gave 18.5 cm., being less by 9.0 cm. Subtracting this amount from 18.5 cm. leaves 9.5 cm. as the measured capacity of the plate. The capacity as calculated from $C = 2r/\pi$ is 9.7 cm.

One condition very essential to the success of the measurements is that there shall be no alternating current potentials within several meters of any part of the apparatus. When the alternating current was not cut out of the room in which the experiments were carried on the electroscope gave erratic deflections of large magnitude every time connection between it and any other body was broken.

The method here outlined has been used for measuring the capacities of the lead wires, levers, and connections in the oscillation experiments described in the next abstract. It may equally well be used for the measurement of the capacity of bodies used in ionization experiments.

The Determination of the Period of Electrical Oscillations.[1]

By J. C. Hubbard.

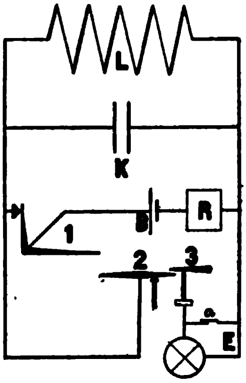

Fig. 1.

The method consists in starting a train of damped oscillations and in determining the times between successive nodes. Let the inductance L (see Fig. 1) and the capacity K, be connected in parallel, and let a battery, B, be connected in series with the inductance and a resistance R. A lever, 1, is in the circuit and, when struck by a falling weight, breaks the circuit and starts a series of damped oscillations. The drop chronograph described by Professor Webster[2] has been used as the circuit breaker. The conditions for breaking the circuit so as to have no spark at the break have been worked out by the author.[3] A carriage moved vertically by a micrometer screw carries levers 2 and 3 and their stops shown in the figure. The falling weight, after striking 1, descends through a measured distance and strikes 2. Lever 2 turns on its axis in the center and lifts lever 3 from the screw which is connected to the quadrant electrometer. The switch at *a* was opened in advance, leaving the quadrants of the electrometer free and at zero potential. It will be seen that connection has been established between the electrometer and the oscillating system *only* during the instant that lever 2 was lifting lever 3 from the supporting screw, for no sooner has lever 3 moved than connection with the electrometer has been broken. Thus, the latter has been connected across the condenser terminals for a very brief instant, and at a time after the starting of the train of oscillations which is controlled by the vertical height of lever 1 above lever 2. If the electrometer reading is zero, connection has been made at a node of potential. The method of procedure consists in finding the first five nodes, then in calculating approximately and ascertaining exactly by trial the positions of the 10th, 15th, 20th, 25th, 50th, and 100th. Or, if the period is very large, sufficient accuracy may be reached with the determination of as few as five consecutive nodes. The height of fall of the projectile may vary from a few cm. to 100 cm. depending upon the frequency to be measured. The greater the height the greater the distance between successive nodes and the greater the accuracy with which they are determined.

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[1] Abstract of a paper presented at the New York meeting of the Physical Society, October 12, 1912.

[2] A. G. Webster, Phys. Rev., VI., pp. 297–314, 1898.

[3] J. C. Hubbard, Phys. Rev., XXII., pp. 129–158, 1906.

Experiments have been carried out during the past summer with the assistance of Mr. H. F. Stimson. Two coils have been experimented upon. One is a coil of 60 layers of wire with 60 turns in each layer. This coil has been quite fully described by J. E. Ives.[1] The other is the single layer standard of inductance wound on a marble cylinder and described by J. G. Coffin.[2] The capacity used is the standard condenser described in the paper of Professor Webster already referred to. The capacity of leads, levers, and connections was measured by the method described in the preceding abstract. Values of capacity used varied from 200 to 1,500 cm., and of inductance, from .018 to 3 henries. The results obtained with the Ives coil show the large distributed capacity in coils of more than one layer. When only ten layers of the coil were used the distributed capacity was several hundred cm., and increased rapidly with the frequency. The effect of the increase is to make the actual period in some cases twice as large as that calculated from the inductance and the known external capacity. Much more simple results were obtained with the marble standard of inductance. A single set of observations is here given for the purpose of illustrating some of the magnitudes involved.

Let $K(obs)$ be the total external capacity including the capacity of the standard and connections, *i. e.*, all but the distributed capacity of the coil. Let $K(calc)$ be the capacity calculated from the formula $T = 2\pi\sqrt{LK}$, where $L = .01822$ henry. The variation of L with the frequency is neglected for the present. The inductance consisted of the double strand marked B on the coil, the two wires being in parallel. The height of fall, h, of the projectile to lever 1 was 53 cm. Let s be the distance passed through by the projectile after striking lever 1 while 100 cycles take place. Then

$$T = \frac{1}{100}\sqrt{\frac{2h}{g}} \cdot \{\sqrt{1 + s/h} - 1\}.$$

The difference between observed and calculated values of the capacity is doubtless mostly due to the distributed capacity of the coil, though the variation of the inductance with the frequency is not without effect. The coil is not

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[1] J. E. Ives, PHYS. REV., XIV., p. 298, 1902.

[2] J. G. Coffin, Bull. Bureau of Standards, II., pp. 87–143. 1906.

TABLE.

s, cm.	$T \times 10^5$.	K (calc.) cm.	K (obs.) cm.	Diff.
.8815	2.723	927.5	845.3	82.2
.7460	2.305	664.8	584.4	80.4
.6650	2.055	528.5	446.9	81.6
.6015	1.861	433.4	351.1	82.3
.5155	1.595	318.2	230.3	87.9

satisfactory for use in these experiments on account of the double winding which makes it impossible to apply known formulæ for either inductance or distributed capacity under the conditions necessary in these experiments. Suitable single layer coils are being designed for use in a systematic study of the effect of varying frequency upon the inductance and the distributed capacity. It may then be possible to get some quantitative data of value with coils of more than one layer of turns.

THE CONSTANTS OF SPECTRAL RADIATION OF A UNIFORMLY HEATED ENCLOSURE OR SO-CALLED BLACK BODY.[1]

By W. W. Coblentz.

IN previous communications to this society accounts were given of the progress made in the investigation of the constants involved in the formulas, proposed by Wien and by Planck, for expressing the partition of energy in the spectrum of a so-called black body.

In view of the fact that the great mass of theoretical speculations seemed entirely out of proportion to the slender experimental data upon which they were based, it seemed desirable to obtain an extensive series of observations under all sorts of conditions, leaving the exact computation of the results until the very last, and then computing all the data on a uniform basis. From the beginning of the work, four years ago, attention was called to the fact that the Wien equation does not fit the observed spectral energy curves, and in later communications, the computations were made on the basis of the Planck equation, introducing all the known correction factors which can effect the observed spectral energy curves. These factors are (1) corrections for the selective reflection of the silver mirrors, the fluorite prism, and the fluorite window which covers the vacuum bolometer; and (2) the corrections for the variation in reflecting power with angle of incidence upon the silver mirrors and upon the fluorite prism.

In the isothermal spectral energy curves the position, λ_{max}, of the maximum emission, E_{max}, is computed by taking the wave-lengths, λ_1 and λ_2 corresponding to equal emissivities, $E_1 = E_2$, on the assumption that the observed energy curve fits the Planck equation:

$$E_\lambda = c_1\lambda^{-\alpha}(e^{c_2/\lambda T} - 1)^{-1},$$

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[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

from which follows

$$(2)\qquad \lambda_{max} = \frac{\alpha(\log\lambda_2 - \log\lambda_1)\lambda_1\lambda_2}{\alpha'(\lambda_2 - \lambda_1)\log e} - \frac{\lambda_1\lambda_2[\log(1 - e^{-c_2/\lambda_1 T}) - \log(1 - e^{-c_2/\lambda_2 T})]}{\alpha'(\lambda_2 - \lambda_1)\log e}.$$

The second term in this equation can usually be abbreviated, since terms involving λ_1 are usually negligible. For values of λ_2 which are less than about 4μ the term $\log(1 - e^{-c_2/\lambda_2 T})$ may be expanded into a series and (by dropping all terms but the first) may be used in the form $-e^{-c_2/\lambda_2 T}\cdot\log e$.

In this equation $\alpha' = 4.9651$ and $c_2 = \alpha'\lambda_m T$. For computing the second term correction factors to λ_m, the value of $c_2 = 14{,}500$ was used. However a variation of 100 units in c_2 (*e. g.*, $c_2 = 14{,}600$) would change the mean value by only 0.0005μ, which is negligible.

For computing the constant, c_2, from an isochromatic energy curve, at any wave-length, λ, Planck's equation is used in the following form:

$$(3)\qquad C_2 = \frac{(\log E_2 - \log E_1)\lambda T_1 T_2}{\log \epsilon(T_2 - T_1)} - \frac{(e^{-c_2/\lambda T_2} - e^{-c_2/\lambda T_1})\lambda T_1 T_2}{T_2 - T_1}$$

where E_1 and E_2 refer to the emissivities corresponding to the temperatures T_1 and T_2 respectively. In this equation the terms $\log(1 - e^{-c_2/\lambda T_1})$, etc., are expanded into a series and only the first term ($-e^{-c_2/\lambda T_1}\log\epsilon$, etc.) is used. As in eq. (2) an approximate value of $c_2 = 14{,}500$ is used in applying the second term correction. For wave-lengths up to 1μ this correction term is small, being only 2.1 and 4.3 for temperatures intervals ($T_2 - T$) of 363° and 623°, respectively, when using $T_1 = 1450°$. However these corrections increase very rapidly with wave-length beyond 1μ so that at 2μ, with the same temperature intervals just mentioned, the second term corrections to the values of c_2 amount to 168 and 227 respectively.

As already stated, it was deemed of greater importance to obtain experimental data than to spend the time discussing the bearing of the data at hand upon existing theories. From the data now at hand, this procedure seems amply justified. More than 180 isothermal energy curves have been obtained and by actual count 75 to 80 per cent. of these curves are found to fit the Planck equation, within the experimental errors of observations. The numerical values of the constants are smaller than the older determinations of Paschen, and of Lummer and Pringsheim (and, for that matter, the earlier values of the present data, obtained by a different system of computation, and not including all the correction factors for reflection). However, as will be shown in the complete paper, the data of previous observers are in agreement with present values, when computed on the same basis.[1]

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[1] For example, Paschen s data, if computed by the present methods would give a value of $\lambda_m T = 2{,}912$ and $c_2 = 14{,}460$. The data of Lummer and Pringsheim are wrong owing to an error in their calibration curve which amounts to $0.02\ \mu$ for the region of the spectrum up to $2.5\ \mu$. This would reduce many of thei rvalues of λ_{max} by almost 1 per cent., and reduces their mean value to $\lambda_m T = 2{,}930$. Their energy curves did not fit the Wien equation and since in the present research, radiators of their design were used, under conditions which were similar to theirs, it is possible to reduce their data by the present methods of computation.

The data now available were obtained with different fluorite prisms, water cooled shutters, air and vacuum bolometers, and thermocouples. The radiators were platinum-wound porcelain tubes of the design used by Lummer and Pringsheim. Energy curves were obtained from these tubes before and after they were blackened with chromium and cobalt oxides. It was found that the percentage of the energy curves fitting the Planck equation was greatly increased when the radiators were blackened. There are still a few outstanding discrepancies which need further investigation. For example, the average value of c_2 for a given set of observations may be 0.2 to 0.5 per cent. higher than a similar series made, under slightly different conditions, but using the same prism, which was flawless. Usually these fluctuations coincide with variations in humidity, but this is not sufficient to fully explain the matter. Again when using a fluorite prism containing numerous flaws, which caused a conspicuous scattering of light, the energy curves appear to be distorted so that only 25 to 30 per cent. of them fit the Planck equation, and the value of c_2 is extraordinarily high, being of the order of $c_2 = 14{,}700$. The data obtained in 1910 and in 1911 give an average value of $c_2 = 14{,}500$. Using the same prism, in 1912, the value of the constant is slightly lower, being of the order of $c_2 = 14{,}480$. Although this difference of 0.3 to 0.4 per cent. in the values of c_2 is systematic it is within the experimental errors of observation. It therefore appears that the weighted value of this constant, as determined from isothermal curves, will be found to be close to $c_2 = 14{,}500$, and $\lambda_m T = 2{,}920$. The values of c_2, obtained from isochromatics, have not yet been computed.

The constant, c_2, is of great importance in optical pyrometry, and heretofore owing to the absence of concordant data experimenters have used $c_2 = 14{,}500$.

The value of the melting point of platinum, on the basis of the optical temperature scale using $c_2 = 14{,}500$, was observed by Waidner and Burgess to be 1,753° C. The latest work on the gas temperature scale by Day and Sosman gives a value of 1750° to 1755°, with a mean value of 1752°, for the melting point of platinum. A melting point of 1755° is equivalent to a value of $c_2 = 14{,}475$, which is close to the predominating values of this constant as observed in the 9 sets of observations, aggregating 90 spectral energy curves which were obtained in 1912. While it is very gratifying to find that the new value of $c_2 = 14{,}470$ to 14,500 places the melting point of platinum within 1 or 2° from the observed values, this coincidence is to be considered somewhat accidental.

Planck's theory of the mechanism involved in the production of "black" radiation admits of the computation of the value of the elementary electrical charge, e^4 by means of the constant of spectral radiation, $b(\lambda_m T = 2{,}920)$, and the constant, σ, of total radiation. The recent determinations of the constant of total radiation, by Gerlach and by Puccianti give a value of $\sigma = 5.9 \times 10^{-12}$ watts per cm^2 per deg.[4] Combining this with the value of $b = 0.2920$ cm. deg.

This gives a mean value of $\lambda_m T = 2{,}911$ and $c_2 = 14{,}450$. If we exclude their last value of $\lambda_m T = 2{,}814$, which is evidently not comparable with the rest, the mean value of $\lambda_m T = 2{,}924$ and $c_2 = 14{,}500$. Considered as a whole, a fair estimate of the older data is $c_2 = 14{,}460$ to 14,500.

gives a value of $e = 5.1 \times 10^{-10}$ E.S.U. This is considerably higher than the experimental value by Millikan, which is 4.77×10^{-10} E.S.U. While the time is still premature to attempt to harmonize the experimental value and the theoretical value as deduced from the radiation laws, it is of importance to notice that there are wide variations in the various determinations of the constant, σ, of total radiation. In future communications it is hoped to contribute values of σ, which are determined with some of the apparatus used in determining c_2.

Heretofore, the experimental work on the spectral energy curves was done in the winter time when the humidity was low. The season of low humidity being close at hand, it is purposed to observe several more series of energy curves, using other fluorite prisms (also quartz prisms for the higher temperatures attainable with a vacuum furnace which has been constructed for the purpose), before publishing a more complete report. As matters now stand it seems highly desirable to put the complete optical path of the spectroradiometric apparatus and of the radiator in a vacuum in order to obtain the complete spectral energy curves free from the atmospheric absorption bands of water vapor, carbon dioxide, and oxygen. Having established the form of the energy curve with some degree of accuracy it is purposed to determine with greater exactness the numerical value of c_2 from isochromatics using a fine wire bismuth-silver thermopile, the radiation sensitivity of which is easier to control than is the vacuum bolometer.

WASHINGTON, D. C., December 12, 1912.

AN APPLICATION OF HIGH TENSION OSCILLATORY SYSTEMS TO IGNITION.[1]

BY M. E. GRABER.

THE principal problem in electromagnetic wave propagation is the storing up of a maximum of energy in the aerial with a minimum heat loss in the spark gap.

The inverse problem presents itself in the application of high tension oscillatory systems to gas engine ignition. Here it is necessary to produce a spark of high calorific quality by using smaller capacities and a short spark gap.

In the present investigation of ignition systems, the primary source of current was a Splitdorf magneto with an armature wound to generate from about 7 to 100 volts at from 80 to 700 R.P.M.

After testing out a number of high tension oscillatory systems, a modified Tesla transformer was found to give the best spark for ignition. The diagram of connections for this system is shown in the accompanying figure.

The armature, A, of the magneto supplies the current to the main circuit, which is made and broken by the interrupter, K, once in every rotation of the armature. The oscillatory discharge across the spark gap, PG, serves as an interrupter for the secondary of the Tesla transformer, S.

When C_1 and C_2 are properly adjusted, the secondary, S, responds strongly to the oscillations set up by each primary spark gap discharge and the potential difference of its terminals reaches very high values. For the best spark at slow speeds the primary spark gap was .0015″ to .0025″. At higher speeds the primary spark gap was .0025″ to .0035″. The high tension oscillatory discharge seems impracticable for automobile ignition because (1) the hot spark requires adjustments of capacity and spark gaps for the transition from high speed to low speed in order that the two systems may have the same oscillation constant; (2) the secondary condenser insulation breaks down under the high potential stress to which it is subjected; (3) undoubtedly the high calorific quality of the spark at the high speeds was due to arcing, as the hot spark differed materially from the ordinary Tesla spark. The system was tested out on a Cadillac car and an E.M.F. car with satisfactory results for high speeds but indifferent success for slow speeds.

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[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

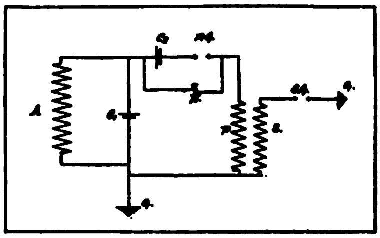

Fig. 1.

Heidelberg University,
Tiffin, Ohio.

A Method for Charging the Needle of a Quadrant Electrometer.[1]

By Henry A. Erikson.

To obtain a conducting quartz fiber of high sensitiveness is difficult. The method here described obviates this difficulty, and has been found very satisfactory.

An insulated ionization chamber *A* is placed above the quadrants and is separated from them by a metal plate *B*, which is connected to the case of the instrument, and prevents any direct induction effect from *A* on the quadrants. The supporting stem of the needle passes through the center of the chamber *A*, and forms a central electrode. The air inside the chamber *A* is ionized by means of the rays from a small polonium plate *P*. The chamber *A* is connected to a battery, giving the desired potential. The ionization current set up charges the needle to the potential of *A*.

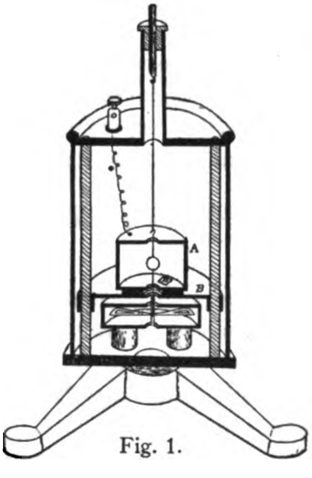

Fig. 1.

Experience thus far has shown that the action of an electrometer with the needle charged by this method is the same as when the needle is charged by means of a conducting fiber.

Physical Laboratory,
University of Minnesota,
December 19, 1912.

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[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

THE DISTRIBUTION OF VELOCITIES AMONG THE PHOTO-ELECTRONS EMITTED FROM THIN CATHODE FILMS WHEN ILLUMINATED BY A MERCURY VAPOR LAMP.[1]

BY PAUL H. DIKE AND F. C. BROWN.

APPARATUS similar to that used by Dike,[2] except the larger dimensions and the more perfect insulation, was constructed with the idea of further identifying the apparent high velocity electrons.

2. The general results, at least as far as the production of high potentials, were verified. Potentials as high as 52 volts were obtained.

3. With a given film the effect decayed by illumination in such a manner as to remove the apparent high velocity electrons first.

4. The apparent high velocity electrons disappeared by resting without illumination, but much slower than by illumination.

5. These apparent high velocity electrons were not obtained by Robinson[3] using apparatus differing apparently only in size and the thickness of the conducting boundary, as well perhaps also in the location and the method of placing the windows.

6. The discrepancy can be explained in one of two ways. Either the reflection of electrons was entirely different in the two cases, or there was in our case something of the nature of an insulating film which gave a condenser effect. In the latter case the quantity of electricity carried over in the deposition of the film would be regarded as bound in a neutral doublet until freed by the ultraviolet light.[4]

7. We tried to detect an insulating film by bringing up a mercury globule, as was done in the previous work referred to, and could detect no such film. This only proves that there was no insulating film formed identical to that previously discovered.

8. Further experiments must be undertaken to settle if the high potentials represent high velocity electrons.

ON THE ORIGIN OF THE EARTH'S MAGNETIC FIELD.[1]

BY L. A. BAUER.

IF attention is confined to the portion of the earth's magnetic field symmetrical about both the axis of rotation and the equator, this portion representing about 90 per cent. of the magnetic energy of the earth's total magnetic field, the following main conclusions have resulted from the author's recent investigations:

1. The portion considered is similar to the field of a uniformly magnetized sphere which has superposed on it another field whose intensity in both hemispheres increases systematically from parallel to parallel, with approach

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[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, November 30, 1912.

[2] PHYS. REV., 34, p. 459, 1912.

[3] Phil. Mag., 6, 23, p. 542, 1912.

[4] See paper by Brown, Phil. Mag., 6, 18, p. 668, 1909.

towards the equator. Thus the equivalent intensity of magnetization, in C.G.S. units, is 0.070 for parallels 60° North and South; it then increases systematically toward the equator until it reaches the value of 0.082, which is about 17 per cent. greater than for 60° North and South.

2. Since electricity of both kinds is now known to be an essential and possibly the only constituent of matter, assume that the earth contains two opposite and practically equal charges distributed throughout its interior in such a manner that if the one body is contained within a sphere (or spheroid) bounded by the earth's surface, the opposite body charge is contained within a sphere of slightly smaller radius; in brief, assume that owing to some cause, the volume density of the one charge is slightly different from that of the opposite charge, then the rotation with the earth of the opposite charges produces a magnetic field. Applying this hypothesis, it is found that to harmonize with the most obvious fact of the earth's magnetism—that the north-seeking end of a magnetic needle points below the horizon in the Northern Hemisphere, the volume density of the negative charge must be smaller than that of the positive, or, in other words, the earth's total negative charge must be distributed through the larger sphere and, if that be the whole earth itself, then, for the chief term involved in the magnetic potential, the surface of the sphere containing the positive charge need be, on the average, only 0.4×10^{-8} cm., below that of the earth's surface, *i. e.*, four-tenths of the radius of an ordinary molecule, to give a magnetic field of the required strength. Taking the average atomic weight of the earth's substance in round numbers as 50, the mean volume density of either charge would be about 3.3×10^{13} electrostatic units.

3. Applying the same hypothesis to the portion of the earth's magnetic field which must be referred to systems in the atmosphere, it is again found that the magnetic facts require the negative charge to be distributed through the larger volume. Hence the same kind of electric distribution suffices to produce both the earth's internal and its external magnetic systems. In short, to account for the earth's magnetic field as observed on the surface, it is necessary to assume that the positive and negative electrons in the earth, and in the atmosphere, are distributed in such a way that, on the average, the negative ones (the more mobile ones) are farther away from the earth's center than the positive ones. (The effect resulting from the fact that the observer who measures the magnetic field moves with the electric charges, is very nearly eliminated, owing to the *double and opposite* distribution assumed; the effect, namely, enters with reversed sign for the two opposite charges.)

4. Endowing next the electrons with inertia, then, because of the earth's centrifugal force, there may result possibly a distribution of the electronic systems departing slightly from the simple spherical distribution at first supposed, so that the effective equivalent distribution approaches that of the

[1] Presented before the American Physical Society and Section B of the A. A. A. S., Cleveland, December 30, 1912. The abstract also contains the chief results of the author's paper on "Cosmical Magnetic Fields," presented before the joint session of A and B, December 31, 1912. To be published in Journal Terrestrial Magnetism and Atmospheric Electricity, Vol. 18, 1913.

type represented by the varying surface density of electricity on a conducting oblate ellipsoid of revolution whose minor axis coincides with that of the earth's rotation. Applying this hypothesis, an expression for the earth's magnetic potential is obtained which with three terms, the first, third and fifth zonal harmonics, reproduces very closely the increase towards the equator in the equivalent intensity of magnetization noted in 1 and gives values for the magnetic components, which agree with the observed ones, on the average, to within 0.4 per cent.

5. Some evidences have been found that the theory, or working hypothesis above outlined, will also render valuable aid in accounting for the non-coincidence of the earth's so-called magnetic axis with the axis of rotation, and in the study of the variations to which the earth's magnetic state is continually subject. There is apparently also some indication that we may not have to refer chiefly to the heterogeneity of structure of the earth the fact that its equivalent magnetic poles are not diametrically opposite to each other.

6. The quantity entering finally into the potential of the magnetic field, caused as we have supposed above, is a volume density into a distance which represents the distance between the surfaces of the two spheres (or spheroids), containing the opposite body charges—in other words, an equivalent surface density of about 1.3×10^4 electrostatic units, on the average. Supposing that the latter, as a first approximation, varies directly as the radius, R, of the revolving body, then, so far as the chief term of the potential is involved, the magnetic intensity at the poles is equal to $\kappa\pi\omega R^2$, where ω is the angular velocity of rotation. For the earth $\kappa = 7.0 \times 10^{-15}$ in C.G.S. magnetic units. Applying this constant to the atmosphere and to sun-spots, magnetic fields of the right order of magnitude are found. If the same constant may be applied to the entire sun, the vertical magnetic intensity at the solar poles would be about 300 gausses.

7. There seems little hope at present that a magnetic field caused as supposed above can be detected in the laboratory. Thus for a sphere of 15 cm. radius, rotating 100 times a second, the magnetic intensity at the poles would be but one hundred-millionth part (10^{-8}) of that of the earth and the possible centrifugal effect would only be about one four-billionth part (2.5×10^{-10}).

8. The electrical system which suffices for the production of the magnetic field of the earth and of the atmosphere[1] also appears to be a gravitation system if we suppose that every mass contains a similar system. The force of gravitation, on this hypothesis, is a residual quantity resulting not from a slight difference in the laws of electrical force for the opposite kinds of electricity, as has been assumed, but rather because of non-coincidence of the "centers of action" of the supposed opposite charges, owing to the difference in the volume distribution and the alteration in the distributions by the mutual influence of the attracting bodies upon one another. The distance between the "centers

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[1] Professor Hale (Terr. Mag., Vol. 17, p. 178, 1912) has found that if he assumes a similar system for the sun he may account for the polarity of the possible solar magnetic field indicated by his preliminary observations over various zones of the Zeeman effect.

of action" of the attracting bodies is accordingly slightly different for the one kind of electric charge from what it is for the other. The apparent alteration in electrical force for the opposite charges is to be referred, therefore, on this hypothesis to the slight change in the denominator (the square of the distance). The final resultant of all the operating forces may possibly be the force of gravitation; however, this will require further study.

A Comparative Study of the Joule and Wiedemann Magnetostrictive Effects in Nickel Rods.[1]

By S. R. Williams.

A PHOTOGRAPHIC method employed in a former work[2] on these effects in iron was applied to a study of the same effects in two nickel rods. Conditions were found under which nickel behaved as iron for the Joule effect, viz., an initial lengthening for small field strengths and a contraction for strong. Other investigators have found only a decrease in length for all field strengths. Maxwell described the Wiedemann effect as a special case of the Joule effect. One difference seems to have been overlooked by him and others, viz., that in the Wiedemann effect the direction of the resultant magnetization is changing as the longitudinal field is increased, whereas, in the Joule effect it remains unchanged. This change in direction of the resultant magnetic field in the Wiedemann effect is apparently one of the causes why the maximum twist for iron occurs at lower field strengths than does the maximum elongation for iron in the Joule effect. The behavior of these two nickel rods points to a crystalline aggregation of the elementary magnets which mechanical stresses may alter and so affect the two phenomena studied.

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Oberlin, Ohio.

Change in Electrical Conductivity Due to the Orientation of Oblate Spheroids within the Conductor.[1]

By S. R. Williams.

CHANGE in electrical resistance due to a magnetic field was first discovered by Lord Kelvin in 1856. He found that qualitatively a magnetic field increased the electrical resistance when the direction of flow was parallel to the field and decreased it when normal. This effect was found in nickel and iron. Later investigators have confirmed these results for the ferromagnetic substances with which they have worked, but extension of the investigations to diamagnetic substances has shown only an increase of resistance for the various angles between the direction of flow and the magnetic field. Various and somewhat contradictory reasons have been given for this phenomenon and many of the suggested causes must be present as factors to modify the effect.

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[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

[2] Phys. Rev., Vol. 34, p. 258, April, 1912.

In many magnetic phenomena I have found[1] that the different effects may be readily explained by the orientation of ellipsoidal particles within the medium. To make an experiment analogous to that of Lord Kelvin I measured the resistance of an electrolyte containing a group of oblate spheroids. When the equatorial planes were normal to the lines of flow the resistance was increased about 15 per cent. over what it was when the equatorial planes were parallel to the lines of flow. This is in qualitative agreement with the effect discovered by Lord Kelvin who worked with strong magnetic fields, in which the ellipsoidal elementary magnets are supposed to be turned with equatorial planes normal to the magnetic lines of force. Hence the electrical resistance is increased in a direction parallel to the magnetic field and decreased when normal. It may happen that other causes enter to cover up completely the change in resistance due to the orientation of the ellipsoids.

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NOTES ON PROBLEMS INVOLVED IN PRODUCING THREE-COLOR TRANSPARENCIES.[2]

BY CHARLES D. HODGMAN.

THE production of colored transparencies by the three-color process has been the subject of experimental study.

To determine the selection of color-sensitive plates various makes of commercial panchromatic plates have been tested spectroscopically. The construction of filters and their adjustment to the plates used involved similar tests of a large number of dyes and the making and testing of many experimental filters. A simple spectrograph was used, fitted with a plane grating. Photographs of the daylight spectrum were made with a slit wide enough to give a nearly continuous spectrum.

The positives were made by the well-known method of Du Hauron, used later by Mr. Ives. Various dyes were used for staining and the effects observed of variation in the concentration of the dye solutions and the time of staining. The best results were obtained with solutions of greater concentration and shorter time of immersion than are usually employed.

CASE SCHOOL OF APPLIED SCIENCE,
CLEVELAND, OHIO.

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[1] PHYS. REV., Vol. 34, p. 40, January, 1912.

[2] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

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THE

PHYSICAL REVIEW.

THE ANNEALING OF STEEL IN AN ALTERNATING MAGNETIC FIELD.

BY H. PENDER AND R. L. JONES.

THE results obtained may be briefly summarized as follows:

1. It has been shown that the treatment of steel by a cyclicly varying magnetic field during its annealing results in a pronounced alteration of its hysteresis loop. The result is to increase largely the permeability at low and moderate inductions with a corresponding increase of the remanent magnetism. The coercive force and the losses are slightly decreased. The maximum value of the permeability was increased as much as 50 per cent. in some cases.

2. The improvement of magnetic quality depends upon the maximum intensity of the force used in the magnetic treatment and shows an approach to a maximum or saturation value when the force is large.

3. The best maximum temperature at which to apply the cyclic treatment has been identified with the critical point Ar_1, about 690° C., on the iron and steel diagram, and through this fact and the evidence of the micrographic studies, it seems very probable that the good results obtained may be ascribed to a preservation of the fineness of the metallographic structure which steel possesses just after it has passed from the non-magnetic to the magnetic condition.

The above results are incomplete but exceedingly suggestive. They point the way to a new line of research on the treatment of steels for electrical purposes and possibly of those for other uses as well. None but magnetic qualities have been observed, but doubtless mechanical characteristics were also affected.

The possible commercial applications of magnetically annealed steel which suggest themselves are numerous. Many forms of direct current apparatus might be decreased in cost by its employment provided the requisite commercial conditions for the treatment could be obtained.

Thus if the fields of direct-current generators and synchronous motors could be made of the material, the weight of iron could be decreased, though not in so large a proportion as the increase of permeability, together with a corresponding decrease in the mean length per turn of

TABLE OF RESULTS.

Sample No.	Maximum Permeability.	Flux Density (C.G.S. Lines per Sq. Cm.) at which Maximum Permeability Occurs.	Permeability at Given Flux Densities (C.G.S Lines per Sq. Cm.).			Iron Loss at Given Flux Densities (Watts per Pound at 60 Cycles).			Explanatory Notes.
			5,000	10,000	12,000	5,000	10,000	12,000	
A	2,620	6,000	2,580	1,900		.490	1.30		Material A as received.
A_1	12,800	8,400	10,200	11,600		.180	.560	.763	Max. temp. 760° C. $H_{\text{max.}}=18.5$ from 760° C. to 340° C.
A_2	8,400	7,200	7,730	6,600		.188	.605	.822	Max. temp. 760° C. Annealed plain.
A_3	13,100	8,500	10,500	11,900	6,350	.222	.630	.837	Max. temp. 870° C. $H_{\text{max.}}=18.5$ from 870° C. to 160° C.
A_4	9,000	6,850	8,400	7,500	4,050	.236	.662	.876	Max. temp. 870° C. Annealed plain.
A_5	13,500	7,200	11,500	11,300	6,200	.216	.650	.850	Max. temp. 790° C. $H_{\text{max.}}=18.5$ from 655° C. to 300° C.
A_6	7,020	6,100	6,780	4,260	1,830				Test on effect of max. temp. of magnetization. Max. temp. 805° C. $H_{\text{max.}}=18.5$ on A_7 from 500° C., on A_8 from 625°, on A_9 from 750° C. A_6 had no winding. Mag. current off at 275° C.
A_7	8,370	6,250	7,920	4,870	1,200				
A_8	10,200	7,000	9,100	6,820	3,000				
A_9	9,620	6,700	8,530	5,570	1,760				
A_{10}	3,200	6,600	3,000	2,200					Max. temp. 625° C. $H_{\text{max.}}=18.5$ from 625° to 180°.
B	3,330	6,100	3,250	2,400		.558	1.55		Material B as received.
B_1	11,500	7,500	10,400	8,650		.194	.600	.790	Max. temp. 795° C. $H_{\text{max.}}=29.6$ from 795° to 400°.
B_2	8,700	6,600	8,040	5,900		.212	.660	.860	Max. temp. 795° C. Annealed plain.
B_3	7,200	5,600	6,980			.240	.680	.870	Max. temp. 765° C. $H_{\text{max.}}=69.2$ from 765° to 485°.
B_4	6,530	5,600	6,360			.240	.680	.870	Max. temp. 765° C. Annealed plain.

Sample No.	Maximum Permeability.	Flux Density (C.G.S. Lines per Sq. Cm.) at which Maximum Permeability Occurs.	Permeability at Given Flux Densities (C.G.S. Lines per Sq. Cm.).			Iron Loss at Given Flux Densities (Watts per Pound at 60 Cycles).			Explanatory Notes.
			5,000	10,000	12,000	5,000	10,000	12,000	
B_5	5,980	5,900	5,860	4,280	2,430				Test of effect of intensity of magnetization. Max. temp. 800° C. At 700°, $H_{max.} = .46$ on B_5, 1.85 on B_6, 5.53 on B_7, and 18.5 on B_8. Mag. current off at about 400° C.
B_6	6,870	6,000	6,720	4,430	2,330				
B_7	7,760	6,700	7,320	5,500	2,880				
B_8	9,300	7,200	8,350	7,300	4,000				
C_1	12,100	9,100	9,200	12,000	9,980	.420	1.28	1.73	Max. temp. 835° C. $H_{max.} = 18.5$ from 835° C. to 200° C.
C_2	9,000	8,100	7,600	8,700	7,500	.440	1.32	1.78	Max. temp. 835° C. Annealed plain.
C_1'	10,900	9,700	7,500	10,750	9,500	.400	1.21	1.60	C_1 after aging at 100° C. for 865 hours.
C_2'	8,400	9,200	6,400	8,300	7,350	.438	1.29	1.68	C_2 after aging at 100° C. for 865 hours.

the winding and a consequent increase in copper efficiency. The power factor of transformers, induction motors, and alternating-current series motors could be improved, and the constancy of ratio of instrument transformers could be increased by employing magnetically annealed metal.

DETAILS.

There is very little published information on the annealing of steel for electrical uses, a subject of great commercial importance. The mechanical properties of metals, however, as dependent upon their microscopic structure have been the subject of extensive research. In the case of steel especially, metallographists have much information showing relations between the microscopic structure and grain size, and the tensile strength, elastic limit, and percentage elongation at fracture.[1] The structure of a metal, although it depends partly upon its chemical composition, may be largely modified by heat treatment. The arrangement of the chemical or metallographic constituents and the size of the *grains* in a steel depends partly on the elevation of the maximum temperature above the principal critical point and partly on the rate of cooling. There is practically no information available on the relations between the electrical and magnetic properties and the *grain* size, and such information as is available connecting them with the metallographic structure is scattered and meager. Hence rules for annealing are qualitative and unsatisfactory.

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[1] Howe, H. M., Iron, Steel, and Other Alloys, page 217 et seq.

The use of a magnetic field as a formative force during the structural arrangement of iron particles has not been entirely unsuggested. In the old experiment of Beetz, recalled by Ewing, a small constant magnetic field acted along the axis of the cathode of an electrolytic cell containing a solution of an iron salt. The iron deposited was strongly magnetic and practically saturated. More recently the work of Maurain[1] and of Gans[2] has thrown further light on the subject. The former found that a field of ten to fifteen gausses acting on the cell was sufficient to saturate the electrolytically deposited iron and that the resulting product had a hysteresis loop which was almost a rectangular parallelogram. Gans has also studied the peculiarities in the hysteresis loop of iron electrolytically deposited in a magnetic field.

The fact that a magnetic field might have a similar effect on the structural formation which takes place with the heat treatment of steel appears to have attracted no serious consideration. A single reference to such a possibility occurs in a discussion before the American Institute of Electrical Engineers,[3] where Mr. P. A. N. Winand states that he once tried to magnetize bar magnets by quenching them while under the influence of a steady magnetizing force. He thought that the magnets were slightly better than those obtained by the common method, but decided that the difference was not sufficiently great to warrant fuller investigation.

This paper reports a method of annealing steel by which its permeability may be permanently increased by an amount considerably greater than that possible by heat treatment alone. The method consists in subjecting the metal to a cyclicly varying magnetic force during the period of slow cooling from a temperature above the recalescence point. The result manifests itself by a change in the shape of the hysteresis loop, as well as by an increase in its ordinates. The remanent field is considerably increased and the coercive force slightly decreased, *i. e.*, the loop becomes more erect and slightly narrower.

The work at first undertaken was along the line suggested in the opening paragraph. Its plan included a study of the effect of the maximum annealing temperature and of the rate of cooling upon the electric properties and microscopic structure of such silicon steels as are used in electrical manufacture. Early in the work it was desired to have a simple method of detecting the point at which the principal critical temperature or point of magnetic transformation is reached in the heating and cooling.

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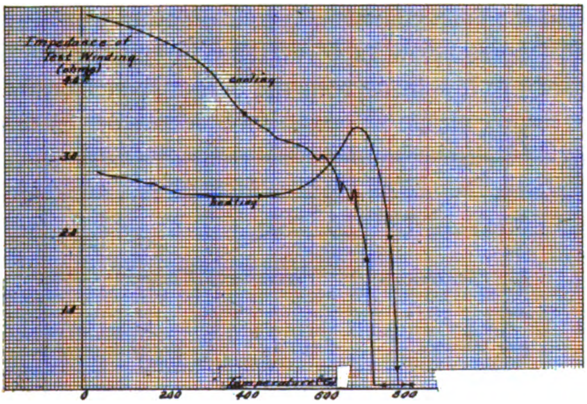
[1] Maurain, C., "Magnetic Deposits Formed in a Magnetic Field," Comptes Rendus, CXXXI., 410 (1900).

[2] Gans, R., "Magnetic Properties of Iron Electrically Deposited in a Magnetic Field," Phys. Zeit., XII., 911 (1911).

[3] Guthe, K. E., "The Influence of Heat Treatment on the Magnetic Properties of Steel and Iron," Trans. A. I. E. E., XIV., 57 (1897).

This temperature, Ar_2 in the equilibrium diagram of iron-carbon alloys, is usually determined by means of the evolution of heat which takes place at that point due to chemical transformations. The fact that a marked change of magnetic permeability also occurs here suggests the use of this phenomenon as a means of the determination. A specimen carrying an asbestos insulated magnetizing winding was placed in the furnace and through the winding a 60-cycle alternating current of 4 amperes constant effective value was maintained during the heating and cooling. The data showing the temperatures and corresponding impedances are shown in Fig. 1. The impedance is approximately proportional to the permeability, since the resistance of the magnetizing winding was low. The plot indicates with a precision of a few degrees the point at which the magnetic property was lost and that at which it was regained. It also has a peculiarity worthy of note.



An attempt to determine the transformation point A_2 by a magnetic method.

Fig. 1.

In the experiments of Hopkinson[1] and Morris[2] on permeability at high temperatures, all the curves of permeability and temperature approach zero temperature in a general horizontal direction. On the other hand the curve obtained here shows a marked improvement in permeability as the sample cooled to room temperature. Apparently this could be explained only on the assumption that the alternating magnetic field exercised a pronounced influence on the beneficial structural changes which accompanied the annealing, and the circumstance was deemed worthy of further examination. The effect was confirmed by further experiment and this led on to an investigation which precluded the original plan of work. Certain interesting and suggestive results were obtained which this paper aims to present.

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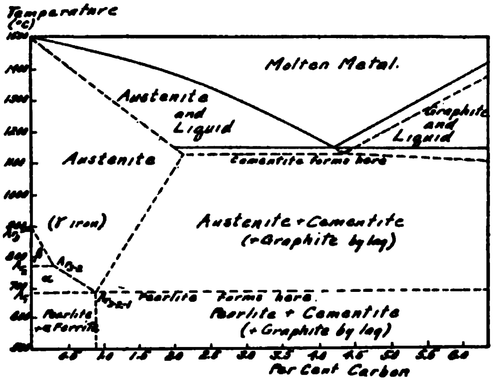
[1] Hopkinson, J., "Magnetic Properties of Iron at High Temperatures," Phil. Trans. (1889), 443.

[2] Morris, D. K., "On the Magnetic Properties and Electrical Resistance of Iron as Dependent upon Temperature," Phil. Mag. (5), XLIV., 213 (1897).

METHODS OF STUDY.

Any study of heat treatment may best be based on the equilibrium diagrams of iron alloys. In all commercial steels carbon is an important constituent, and the transformation diagram of the iron-carbon system is the one best known. The work of Guertler and Tamman[1] on the iron-silicon system shows that for all alloys containing less than about 20 per cent. silicon the mass consists of a solid solution of iron silicide in iron with no critical point below 1200°. Hence in spite of the fact that the samples studied are low in carbon, the iron-carbon transformations are probably of considerable importance in the structural changes accompanying annealing.



The carbon-iron equilibrium diagram. (Howe.)

Fig. 2.

The essential portions of the iron-carbon diagram are reproduced in Fig. 2, which shows transformation temperatures for different percentages of carbon during cooling from the molten condition. Here we are interested simply in the left-hand edge of the diagram, since the samples used are all of low carbon content. Above 900° C. the mass consists of austenite, a solid solution of carbon in iron in the gamma form. At about 900° C. the gamma iron changes to the beta form and the austenite passes into a transition stage to which the name martensite has been given. This transformation point is known as Ar_3. That at which the beta allotropic modification changes to the alpha form, about 770° C., is called Ar_2. Alpha iron is magnetic, while beta and gamma iron are non-magnetic. Hence Ar_2 is the magnetic transformation point. Finally, the temperature where the alpha ferrite and the cementite (Fe_3C) begin their segregation to form the composite lamellar structure known as pearlite, is called Ar_1. This temperature is about 690° C. Two transition stages have been distinguished between martensite and pearlite, and to these the names troostite and sorbite have been given. The former is probably the structure of a tempered steel. The sorbitic stage represents the condition where most of the metal has been transformed into ferrite and cementite but where the two have not yet formed the pearlitic structure. It is probably an ultramicroscopic mixture of ferrite and cementite particles. As the cooling continues slowly to room temperatures, the tendency is for the ferrite and cementite to segregate and form interstratified layers of the two materials. This composite pearlite is the characteristic structure of a slowly cooled steel as ordinarily annealed. The microscope is commonly used as an aid to metallographic research and studies have been made of polished sections of several of the samples in investigating the structural transformations and the resulting changes in properties.

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[1] Guertler, W., and Tamman, G., "On the Compounds of Iron with Silicon," Zeit. für Anorg. Chemie, XLVII., 163 (1905).

The steel used in this research consists of two kinds of silicon alloy sheets such as are used in transformer manufacture. In addition, two samples, C_1 and C_2, of low-carbon dynamo steel were used. Analyses of the former materials as given by the manufacturers are as follows:

	Nos. *A*.	Nos. *B*.
Carbon	.06	.09
Manganese	.13	.17
Phosphorus	.04	.05
Sulphur	.02	.03
Silicon	3.46	3.95

The samples were in the form of hollow squares 3 in. in internal diameter and 5 in. in external diameter. Each sample contained one hundred sheets and weighed about 6½ lbs.

The heat treatment was carried out by means of an electrical resistance furnace in which several samples could be treated at once. The inside of the furnace consisted of a nine-inch iron pipe closed at the bottom and about 15 in. tall. The walls were ¼ in. thick and the furnace chamber had an iron cover. The furnace pot was first covered with a layer of thin mica sheets. The resistance winding was double, each element consisting of 20 turns of nichrome resistance ribbon (3/16 in. × .005 in.) and was wound on the mica under considerable tension. The use of simply a thin layer of mica to insulate the winding from the furnace walls reduced the temperature gradient, and the form of the resistor material gave a large area of contact on the surface of the furnace. Over the winding about ¼ in. of a paste of magnesium oxide and water was put on and securely bound in place with asbestos tape. The whole was then surrounded with a pipe cover and placed in a sheet iron cylinder 24 in. in diameter, and of about the same height. The space of about 6 in. beneath the furnace and between it and the walls of the container were filled in with ground pipe cover, of the ordinary 85 per cent. magnesia variety. When a run was made the top of the furnace was covered to about 6 in. with the same material.

The furnace took about 2 kw. maximum power. Four specimens could receive heat treatment simultaneously and the heating period necessary was about 8 hours.

The temperatures were measured by means of a platinum-platinum-iridium thermocouple and a Hartmann & Braun recording pyrometer. The thermocouple was enclosed in a quartz tube which was bound to the iron of one of the samples under treatment.

The application of an electromagnetic field to magnetize a specimen during its annealing necessitated that each specimen carry an insulated winding. An asbestos-covered stranded copper wire was used for this purpose. As a further precaution each specimen had its edges covered with folded mica strips and was wound with asbestos tape before receiving its winding. Each winding could be used but once owing to the deterioration of its insulation.

The magnetic testing of the specimens was carried out with the aid of a special testing transformer of which the specimen formed the core. This device had hinged coils, thus expediting tests by making it unnecessary to wind each specimen. Two primary coils of ten turns each enclosed two opposite sides of the hollow square sample and two secondary coils of forty turns each, the other two sides. Permeability measurements were made with direct current, a form of ballistic galvanometer being used to measure the induction.

The hysteresis and eddy current losses were measured at 60 cycles by supplying an alternating current to the magnetizing winding and measuring the power used in the specimen by means of a wattmeter. A diagram of connections is shown in Fig. 3. The induction was calculated from the induced voltage as obtained from the corrected reading of the voltmeter. The loss measurement was obtained by subtracting from the wattmeter reading the total losses of the secondary circuit. No separation of the hysteresis and eddy current losses has been made.

The generator used as a source of supply was a Mordey inductor alternator, giving a wave form which closely approximates a sinusoid. The generator was sufficiently large as to be only lightly loaded by the testing current, and the resistance of the primary circuit was small, the voltage of the alternator being controlled by a large variable resistance in its field circuit. In order to keep the magnetizing current low, specimens of large cross-section, and hence of low reluctance, were used. These conditions all tend to secure a sine wave of induced flux in the specimen under test.

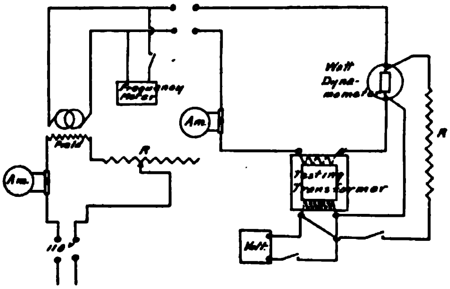


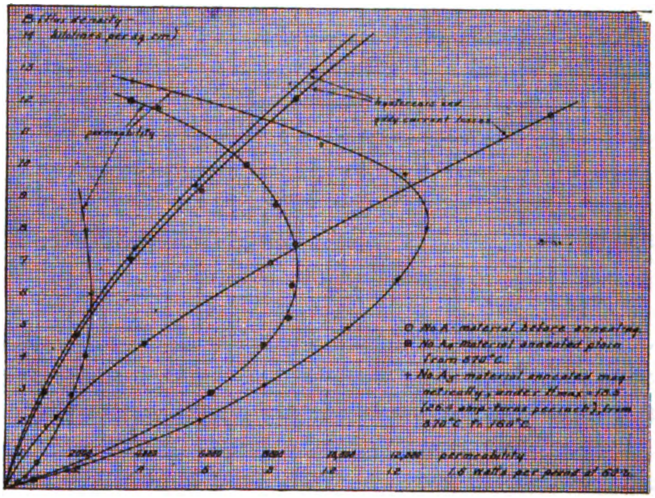
Diagram of connections for magnetic testing with alternating current.

Fig. 3.

Experimental Investigation.

In order to confirm the effect of a cyclicly varying magnetic field during annealing, Nos. B_1 and B_2, after being annealed together from 950° C., were prepared for test. The former was wound with asbestos-insulated wire and placed in the furnace with the latter. Both were then heated to 795° C. and slowly cooled. During the cooling B_1 had an alternating magnetic field of maximum value $H = 29.6$ (42.2 ampere-turns per inch) impressed upon it down to 400° C. Upon testing, it showed a maximum value of permeability about 33 per cent. higher than that of B_2 and slightly lower losses. The result was confirmed by a similar run on Nos. A_1 and A_2. A consideration of the method of experiment shows that the only difference in the treatment of the two samples other than that of the magnetic field is the possibility of more even cooling due to the

I^2R loss in the magnetizing winding. As a final test two runs were made in which the unmagnetized sample had a non-inductive winding of the same number of turns as that of the magnetized specimen. The windings of the two specimens were connected in series, and both occupied the same relative positions in the furnace. Under these conditions, the only possible difference in treatment is that due to the magnetizing field. The results of these two tests are those given for Nos. B_3 and B_4 and for Nos. A_3 and A_4. A_3, which was magnetized during its cooling, has a maximum permeability of over 13,000, about 45 per cent. greater than that of No. 8 and slightly lower losses. The permeability and loss curves of Fig. 4 show the results obtained with A_3 and A_4 and may be taken as typical of these preliminary tests.



Permeability and loss curves after plain and magnetic annealing.

Fig. 4.

In order to determine whether or not as great an improvement in magnetic quality might be looked for in the magnetic annealing of other sorts of steel, two samples of a low carbon dynamo steel were given a treatment similar to that of the other samples. These are Nos. C_1 and C_2. C_1, which was annealed under $H = 18.5$ (26.4 ampere-turns per inch) from 835° C. down to 200° C., showed an increase of permeability over that of C_2 of about the same per cent. as the other samples.

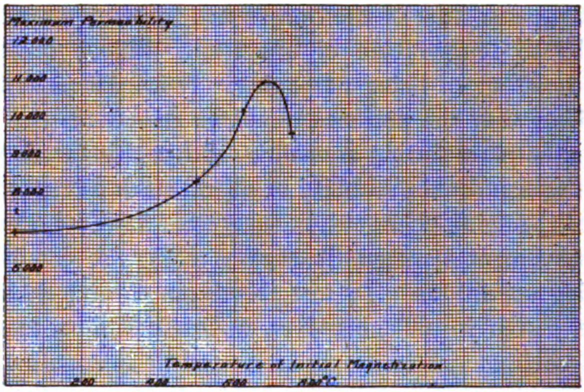
The phenomenon of aging is now of course well recognized in the behavior of iron, and in carbon steels operating under working temperatures an increase of 50 per cent. or more in losses with a corresponding decrease of permeability, is not uncommon. This is due to a gradual alteration of the structure of the iron caused by the prolonged exposure to the moderate temperature of operation. Silicon steels have the important advantage of being non-aging. It is well recognized among metallographists that the presence of silicon stimulates the transformations which take place during annealing, and in this fact is probably to be found the reason for the stability of silicon steels as regards aging.

The fact of the aging phenomenon suggested, however, that the newly effected improvement of magnetic quality might prove only temporary under working conditions. To test this Nos. A_1 and A_2 and Nos. C_1 and C_2 were placed in an aging oven maintained at a temperature of 100° C. and their magnetic properties tested at intervals over a period of about six weeks. A_1 and A_2 show scarcely any decrease in permeability or increase in losses. In the case of C_1 and C_2 about the usual amount of aging was observed. Each showed a decrease in permeability but the percentage improvement of the magnetically annealed sample was about the same after aging as before. This test indicates that the improvement in properties is just as permanent as that due to ordinary annealing.

Experiments were next undertaken to study the best temperature limits of the alternating magnetic treatment. The method of studying the effect of a maximum temperature at which magnetic treatment is applied, was to place several samples wound with an equal number of turns of asbestos-insulated wire in the furnace and turn the current on the samples at different points on the cooling curve. Thus, after heating a number of samples up to a point a little above the principal critical point, the current would be turned on the first sample at 700° C., on the second at 650° C., on the third at 600° C., and on the fourth at 550° C. When all the samples were in circuit they were connected in series so that the same current flowed through the winding of each. The current was maintained constant by means of an external resistance.

The results shown by Fig. 5 were obtained. They indicate that the best effect of the magnetizing field is to be obtained if the field is impressed at a point on the cooling curve of about 690° C. Referring to the transformation diagram, we see that this temperature seems to correspond to the transformation point Ar_1 where pearlite begins to be formed.

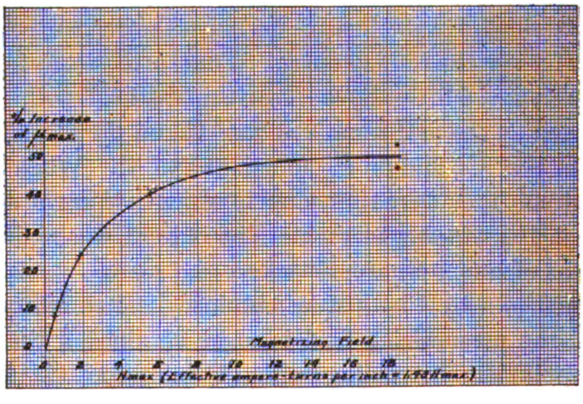
One sample, No. A_{10}, was heated to a temperature of about 600° C. and cooled from that temperature under a magnetizing force of 18.5 (26.4 ampere-turns per inch). Only a very slight improvement in magnetic quality, as compared with that of the other tests, was obtained. This tends to indicate that the critical temperature must be exceeded in order to benefit from the magnetic treatment.



Effect of initial temperature of magnetization on the maximum permeability. Max. temperature of heat = 800° C. Magnetizing force (H max. = 18.5) ceased at 275° C.

Fig. 5.

A study of the effect of various intensities of maximum magnetization yielded the results shown by Fig. 6. The result is what might have been expected from our theories of magnetism and shows that there is a saturation value for the improvement in permeability due to increasingly strong field. This test was made in the following way: A number of samples wound with different numbers of turns and having their windings connected in series, were placed in the furnace and after heating to about 800° C. were slowly cooled under a magnetizing force which in the case of each sample was proportional to its turns. The upper two points of the curve were obtained from two samples, treated under as nearly

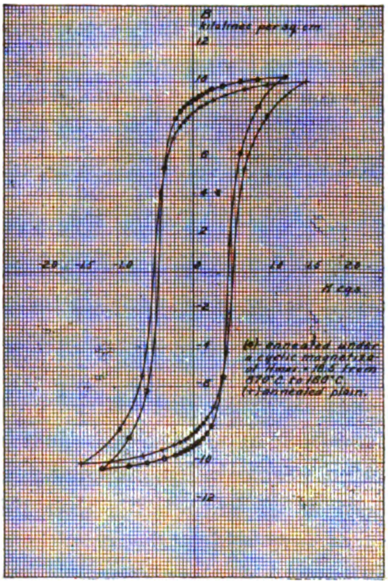


Effect of the magnitude of the annealing magnetizing force on the increase in max. permeability. Max. temperature of heat = 830° C. Magnetizing force on from 750° C. to 200° C.

Fig. 6.

identical conditions as to heat and cooling as possible (Nos. A_1 and B_8) although their treatment was not simultaneous with that of the others. The curve indicates that under the given conditions of maximum temperature and rate of cooling, the maximum percentage increase of permeability, to be obtained by magnetic treatment over that by plain annealing, is about 50 per cent. This is no indication that a larger improvement is not possible under more favorable conditions of temperature treatment and point of application of the magnetizing force on the cooling curve. However, the same shape of curve for the effect of the strength of the magnetizing field in increasing the permeability would doubtless be obtained.

Such a large increase in permeability as is evidently obtained by appropriate magnetic treatment without a corresponding decrease in eddy-current losses and hysteresis, is somewhat surprising. Hysteresis loops taken with a fluxmeter for samples (1) after plain annealing, (2) after magnetic annealing, are shown by Fig. 7. The effect of the treatment has evidently been to increase the permeability of the silicon steel at moderate and low flux densities without a corresponding increase at the higher densities. It is noticeable that the value of the remanent flux is increased with a slight decrease in the coercive force. Thus although the permeability in moderate fields is much greater after the treatment, the area of the hysteresis loop is only slightly lessened. The effect seems to be rather an increase in the power factor than an improvement in losses.



Effect of magnetic annealing on the shape of the hysteresis loop.

Fig. 7.

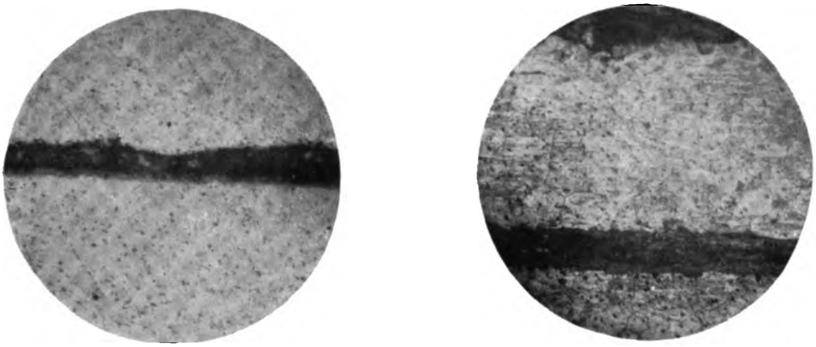
The data for Nos. C_1 and C_2 indicate that in the case of a carbon steel the treatment described would result in a larger improvement at higher flux densities than in the case of the silicon steel. A study of the shape of the permeability curves of all the different samples shows that the magnetic treatment shifts the point of maximum permeability toward higher flux densities, thus giving the largest increases in permeability in the range of flux densities used in the design of transformers. In the case of carbon steels the improvement is sufficiently marked, even up to densities such as are used in generating machinery, to be worthy of consideration. The accompanying table gives a summary of the magnetic data obtained.

A micrographic study of several of the specimens has been made in order to see if any connection might be traced between the effect of the magnetic treatment and the metallographic structure. The results of this study are shown by the micrographs of Figs. 8 and 9, which are but two of a number of specimens. Those specimens which were annealed under a magnetizing force seem to show a more homogeneous structure than those which were annealed plain. The micrographs of the etched sections show the grain size of corresponding samples to be about the same and give no further light as to structural difference due to magnetic annealing. The grain of the etched sections has been found, by the investigations of various metallographists, to be a function of the maximum temperature of the heat treatment and the grain structure to undergo a marked change at the transformation point Ar_1.

STRUCTURAL HYPOTHESIS.

A slight consideration of the molecular theory of magnetism in its relation to the structural forms of steel offers an apparent explanation for the results that have been obtained. Considering the case of a low carbon steel and again referring to the iron and steel diagram, we see that the point Ar_2, where iron regains its magnetic properties on cooling, comes at about 770° C. Just below this point steel shows a remarkably high permeability, especially for low magnetizing forces, and it seems not at all improbable that this characteristic is due to the very fine structure which the sorbitic steel possesses at this point of its heat history. As the cooling progresses further and Ar_1 is reached, the sorbite is gradually transformed into a conglomerate structure containing pearlite with some excess magnetic ferrite. Pearlite is a lamellar structure consisting of curving layers of alpha ferrite and cementite, and under the higher power microscope appears as in Fig. 10. The dark layers of highly magnetic ferrite are partitioned off by interstratified bands of lower permeability cementite. Pearlite always occurs in the constant proportions of about six parts by weight of ferrite to one of cementite.

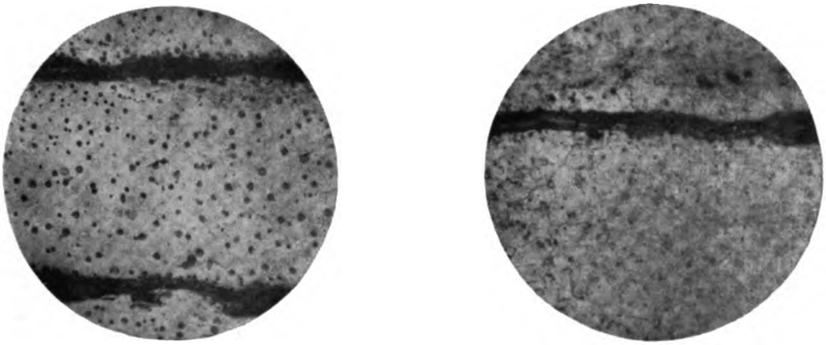
This structure would seem to divide the material somewhat into molecular groups, each of which has its own arrangement of magnetic molecules. Upon applying a magnetizing force each group has a certain stability of its own and hence the particular field intensity at which the arrangement succumbs to the applied force varies in the different minute groups of ferrite molecules. This characteristic of the pearlite structure shears over and rounds off the hysteresis loop for such a steel, making it



a. Unetched. × 80. *b.* Etched 25 sec. in HNO_3 solution.

No. A_5 heated to 790° C. and magnetically annealed from 655°.

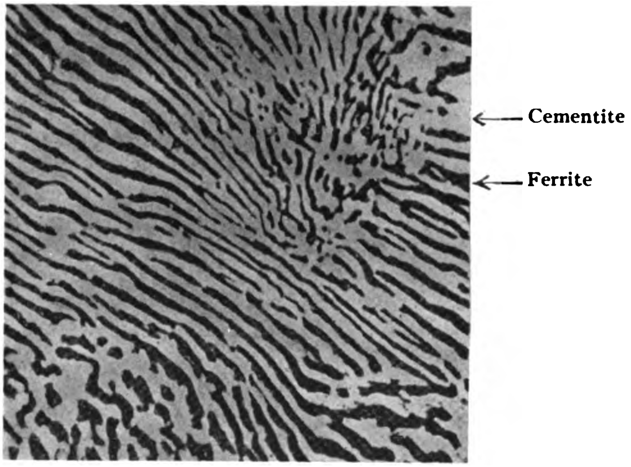
Fig. 8.



a. Unetched. × 80. *b.* Etched 25 sec. in HNO_3 solution.

No. A_3 heated to 870° C. and annealed without magnetization.

Fig. 9.



Pearlite (Osmond).

Fig. 10.

PENDER AND JONES.

less erect than that of the ferrite would probably be, if simply diluted by the cementite present. Hence it may very well be that if we could preserve the more homogeneous sorbitic structure in some way, a very satisfactory electric steel would be the result. The greatest difficulty in the achievement of this result is the fact that practically all the transformations of the iron-carbon system take a considerable length of time and show a large time lag in completing their reactions and physical arrangement. Thus if we were to attempt to quench a steel as sorbite we should probably get some austenite and beta iron remaining, together with some early developed pearlite, and the sample would be likely to prove quite unsatisfactory. Hence slow cooling is quite necessary to the production of a satisfactory electrical steel, as ordinarily this allows the formation of pearlite.

The treatment of the steel by a cyclicly varying field during its cooling probably has the effect of making the structure more homogeneous than would otherwise be the case, preventing the pearlite from forming and keeping the steel largely in the form of sorbite. This it probably does by immediately polarizing the alpha molecules as soon as they are formed by transformation, and thereafter keeping them under control as units of a single magnetic system. Thus they are prevented from forming small secondary groups of stable arrangement.

This suggests the possibility that the peculiar structure of pearlite, always occurring as it does in constant proportions, may very well be due to the magnetic qualities of the molecules of its two constituents and their consequent grouping tendency as soon as the magnetic alpha iron cools out from the beta form.

The results of this investigation seem to lend support to the hypothesis proposed. The most favorable temperature for applying the magnetic treatment was found to be about 690° C., which is about the temperature where a sorbitic steel starts to acquire the structure of pearlite. Then the change in form of the hysteresis loop by the magnetic treatment is precisely what would be expected by the hypothesis. The loop becomes more erect, showing that the steel changes its magnetic condition more as a unit, and thus indicating the greater homogeneity of its structure and the greater uniformity of the type of magnetic groups that prevail throughout. This results in a greater stability of the arrangement acquired under the influence of an externally applied magnetizing force and hence a greater value for the remanent magnetism. This too is confirmed by the hysteresis loops of Fig. 7. The micrographs also lend slight support in that those taken show a finer and more homogeneous structure for the magnetically annealed samples than for those annealed without the alternating field.

CONDITIONS OF SENSIBILITY OF PHOTO-ELECTRIC CELLS WITH ALKALI METALS AND HYDROGEN.

BY J. G. KEMP.

THE object of this investigation is a systematic quantitative study of the conditions of sensibility of photo-electric cells of alkali metals with hydrogen and a determination of the work required to draw an electron out of an atom.

The photo-electric phenomena are very important in connection with the theory of radiation, and for photometric purposes. Planck's theory of radiation requires that the potential due to incident light should increase proportional to the frequency of the light. Moreover, if the theory of the units of energy strikes reality, we should expect that the photo-electric current for very weak intensities of light becomes intermittent, as the electrons are given out only from time to time, so that the phenomenon should resemble the radioactive scintillator. If the intensity of the incident light falls below a critical value, an electron can only escape from the metal, if its kinetic energy is at least equal to the unit of energy received from the beam of light. Photometric measurements have already been carried out by Elster and Geitel, by Richtmeier,[1] by E. L. Nichols and E. Merritt.[2] As the sensitiveness of the photo-electric cells is very great, comparable indeed with the sensitiveness of the selenium cell, we hope to use these photo-electric cells for the measurement of the light from the fixed stars.

The sensitiveness of the photo-electric cells has been increased considerably by the formation of an alkalihydride and by replacing hydrogen by helium. This has been done by Elster and Geitel.[3]

The work necessary to produce an ion can be determined by two independent methods. One method is based on the ionization by α particles, the other on the potential difference which in a discharge tube is necessary to produce ionization by collision. The results of the two methods do not satisfactorily agree with each other. A larger number of determinations of this quantity is necessary before we can explain the difference of the results.

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[1] PHYSICAL REVIEW, 29, p. 71, 1909.

[2] PHYSICAL REVIEW, 34, p. 475, 1912.

[3] Physik. Zeitschrift, 11, April, 1910, and August, 1911.

Description of Apparatus.

It was required to find the sensitiveness of the photo-electric cells as function of the gas pressure, the electrode distance, the area of metal illuminated, the voltage applied to the electrodes and the intensity of illumination. Fig. 1 shows the glass tube used in making the cell. A spherical bulb, 2.5 cm. in diameter, has two tubes 1.0 cm. in diameter, sealed horizontally and diametrically opposite each other. A vertical tube, 1.8 cm. in diameter, about 15 cm. long is sealed in the top of the bulb.

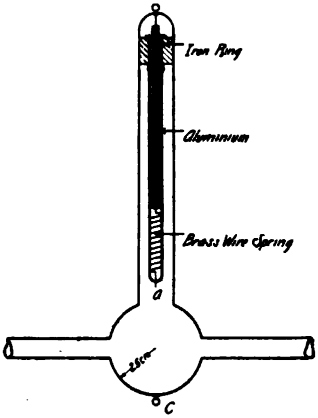


Fig. 1.

At the top of this vertical tube a platinum wire is sealed and fused to an aluminum rod 0.4 cm. in diameter and 10 cm. in length. To the lower end of the aluminum rod is attached a brass spiral spring to which is connected the platinum wire anode. The anode, *a*, being sealed through the lower end of the glass tube which telescopes the aluminum rod. At the upper end of this glass tube is attached an iron ring which fits neatly inside the larger tube. By means of an electromagnet, using a current of 2 amperes, the inner tube carrying the anode, *a*, can be held in any desired position relative to the cathode, *c*, at the bottom of the bulb.

At the bottom of the bulb and diametrically opposite the anode, *a*, is sealed the cathode, *c*, the upper point of which does not extend beyond the surface of the inside of the bulb. In some of the cells the inside of the lower surface of the bulb was silvered, the metal distilled into it and deposited upon the mirror surface. In this way a good contact was insured between the platinum and the metal. In some of the tubes the metal was not distilled into the bulb but poured into it while in the molten state and allowed to solidify over the platinum electrode. The metal in all cases was used as the cathode of the cell. In order to form the sensitive hydride the alkali metal, that is, the cathode, was connected to the negative terminal of a battery of 300 or 400 volts while a resistance of 3,000 ohms and a galvanometer was placed in series with the positive terminal of battery and the anode of the cell. The pressure of the hydrogen gas in the cell was then reduced until the current flowing between anode and cathode caused a faint glow to fill the whole tube. The metal surface, which was very bright before the illumination appeared in the tube, afterward became colored, being brownish for sodium, bluish violet for potassium, and light greenish for rubidium and cæsium. These colors are due to the formation of a compound of the hydrogen and the metal, which is called a hydride. When the hydrogen is replaced by argon or helium, the cell maintains its high sensitiveness constant for a long time. And even if the metal remains in contact with hydrogen, the sensitiveness did not seem to change during the few days in which the readings were taken.

The distance between the electrodes could be changed by means of an electromagnet, arranged inside the light-tight box, and a cathetometer was used for the accurate determination of the electrode distance.

Fig. 2 shows the entire arrangement of the apparatus for the investigation. The photo-electric cell is enclosed in a light-tight box. Wires connected to the anode and cathode, and to the electromagnet, *M*, pass through insulators in the walls of the box. The cell is sealed to the system containing a Macleod gauge for measuring pressures up to 0.24 cm., a closed manometer for measuring the higher pressures, a regulator for obtaining small variations in pressure, a tube containing palladium metal strips for supplying pure hydrogen gas, and an air pump The palladium metal was charged with hydrogen gas by the electrolytic method. With an electrolyte of one part of H_2SO_4 and three parts of H_2O, the anode being platinum, and the palladium metal the cathode, hydrogen gas was absorbed by the cathode when 2.5 volts was connected across the electrodes. After charging the palladium, the tube containing it was sealed to the system. When the tube is heated with a small bunsen flame, the metal gives off pure hydrogen gas. The whole glass system, when the pump was cut off, could be filled to a pressure of about 25 cm. when the palladium was heated.

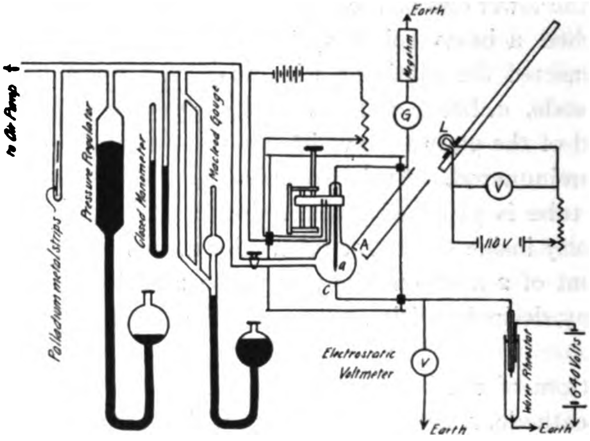


Fig. 2.

The galvanometer used is a Leeds and Northrup type HS, the sensibility being 3.78×10^{-10} amp. per mm. Deflection for 2 meters scale distance. The anode of the cell was connected to the earth through the galvanometer and a megohm resistance. The cathode of the cell was connected to a variable point in a water rheostat which is in series with about 640 volts from a storage battery. The voltage applied to cathode could be varied by means of the water rheostat and it was measured by a Kelvin electrostatic voltmeter reading 0–600 volts.

The variation of the area of the metal illuminated was obtained by varying the opening of the iris diaphragm A, which is placed at the lower end of a brass tube. This tube was blackened on the inside to prevent reflection of light.

By moving the lamp on the guide the intensity of illumination on the metal in the photo-electric cell could be varied, and this variation calculated directly by means of the inverse square law.

METHOD.

Since the most sensitive conditions for the photo-electric effect are being sought, it is necessary to study the effect due to varying all the possible conditions in order to find the most effective set of conditions. The variables in this work are the following: P the pressure of the gas, D the distance between the electrodes, V the potential difference applied to electrodes, A the area of metal illuminated, L the intensity of illumination, t the temperature of the cell, and d the galvanometer deflection which is proportional to the current.

Two sets of readings are possible for each cell, namely, before forming and after forming the hydride surface. In this paper this process will be called forming.

Four cells were studied: one with cæsium and three with potassium metal. The readings were taken in the order as follows: With t, L, A, P and D constant the values of the deflections, d, of the galvanometer were read for increasing values of V. Thus values of current and voltage were obtained for an ionization curve. This was repeated for three and in some cases four distances of D.

From the above data four ionization curves are obtained which show the effect of varying the distance between the electrodes for constant values of t, L, A and P. If the above three or four ionization curves be called a set, then it is possible to get as many sets as there are values of P, the gas pressure. From three to five different values of P were selected for each cell and in this way the effect due to change of pressure was studied.

Ionization curves were also obtained in which A, L, P and D are constant for three or four different temperatures. After the forming process similar sets of ionization curves were taken except those for temperature changes. In addition to the ionization curves taken after forming the cell No. 4, sets of data were taken in which L, V, P, t and D are constant while A and the current varied. Also readings were taken for A, V, P, t and D constant while L and the current varied.

A total of thirty-six plates, each containing four or five curves were obtained for the four cells. On account of the similarity of the large number of curves and data taken only representative curves will be given for potassium cell number four.

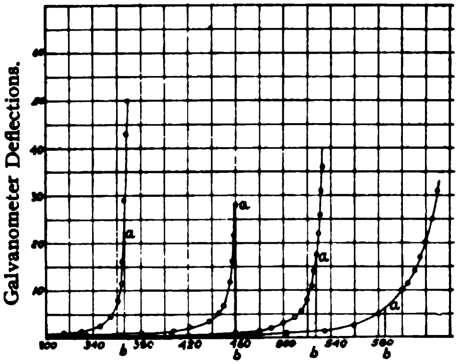
CURVES.

A in cm^2. represents the area of metal illuminated.
L in candle feet represents the intensity of illumination.
t in ° C. the temperature of the cell.
P in mm. mercury represents the pressure of the hydrogen gas.
D in cm. represents the distance between the electrodes.
V in volts represents the potential difference between electrodes.
d in mm. represents the galvanometer deflections.

CRITICAL VOLTAGE AND CURRENT.

It was found that when the voltage was applied to the cells, it could be increased to a certain definite maximum value before a deflection of the galvanometer was noticeable when the light was not acting. If this voltage was exceeded by an amount hardly readable on the voltmeter a deflection of the galvanometer resulted. Furthermore, if the light was permitted to act and the voltage applied, equal to the maximum value determined as stated above, a definite deflection of the galvanometer was produced; and, when the light was suddenly turned off the galvanometer deflection always became zero. However, if this maximum value of the voltage were exceeded and the light turned off, the deflection of the galvanometer was decreased but never became zero.

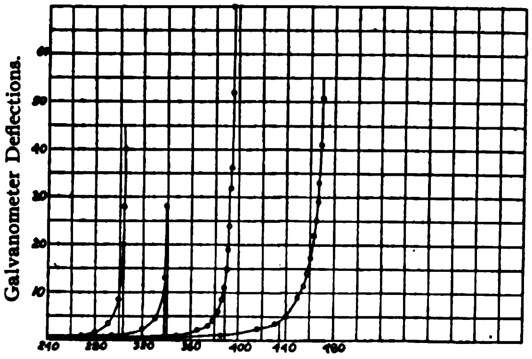
This voltage, therefore, represents the maximum which may be applied to the cell, in this particular case, and at the same time have the ionization current produced only by the action of light. This value of the voltage I shall call the "critical voltage," and the corresponding current the "critical current" for this particular condition of the photo-electric cell. The critical voltage and the critical current taken together give a definite criterion for determining the best conditions for sensitiveness. When the critical current is a maximum and the critical voltage is a minimum, then the most sensitive conditions obtain. Therefore, the critical voltage and the critical current are given in the data and shown by the vertical lines as for example *ab* in Fig. 3. If the voltage is increased beyond the critical value, when the cell is in the dark, the current will increase suddenly, so that it cannot be measured with the galvanometer. This indicates that the sparking voltage is not much greater than the critical voltage. Some physical results, chosen from a large number of observations are given in Figs. 3 to 17. Four groups of curves will be given. In the first group, Figs. 3 to 10, relating to a potassium cell before formation of the hydride, the two most important independent variables, the electrode distance and the gas pressure are studied. The same holds for the second group of curves, Figs. 11–15, which however have been taken after the formation of the hydride. The variation of the area illuminated is represented in Fig. 16, and the variation of the intensity of illumination in Fig. 17.



Voltage on Metal.

Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Gas pressure 5 mm. Before forming.

Fig. 3.



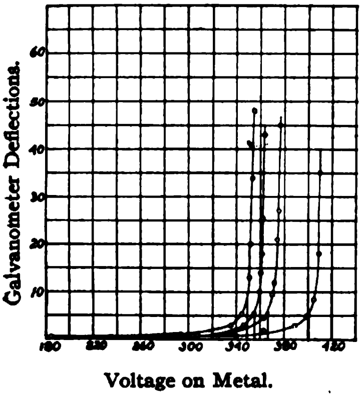
Voltage on Metal.

Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Gas pressure 2 mm. Before forming. $A = 4.36 \text{ cm}^2$. $L = 0.22$ C.f. $t = 26°$ C.

Fig. 4.

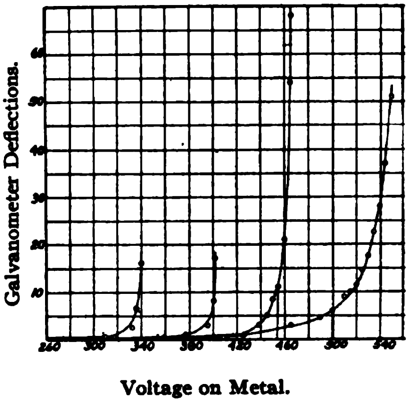
The following remarks may be made on curves Figs. 3–10. The illuminating voltages are in the order of the values of D.

Fig. 3. The critical voltage for $D = 0.5$ cm. is the smallest, while the critical current for $D = 1.0$ cm. is the largest; therefore, the best condition for this pressure is for some value of D between 1 and 2 cm.



Variation of electrode distance reading from left to right 1–0.5–2–3 cm. Gas pressure 1 mm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 26°$ C.

Fig. 5.

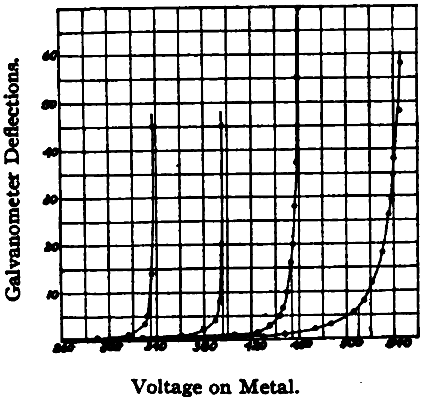


Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Temp. salt and ice bath. Gas pressure 3 mm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 20°$ C.

Fig. 6.

Fig. 4. The order of the illuminating voltages is the same as that for values of D. The critical voltage for $D = 0.5$ cm. is the smallest and the critical current is the largest; therefore, the best conditions for sensitiveness are shown by this curve.

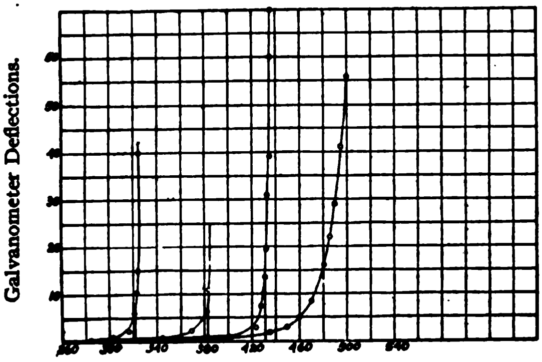
Fig. 5. The order of the illuminating voltages is not the same as that



Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Temp. 0° C. Gas pressure 3 mm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 0°$ C.

Fig. 7.

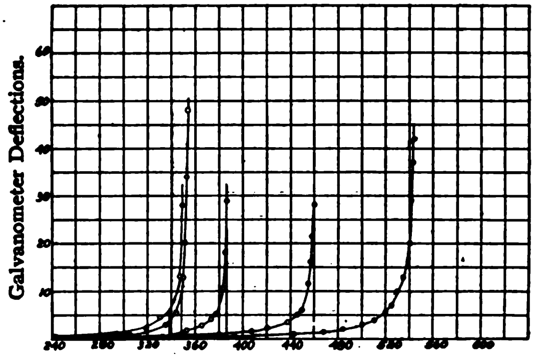
for values of D. The curve for $D = 0.5$ cm. lies between curves for $D = 1.0$ cm. and $D = 2.0$ cm. The curve $D = 1.0$ cm. shows best conditions for sensitiveness.



Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Temp. 38° C. Gas pressure 3 mm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 38°$ C.

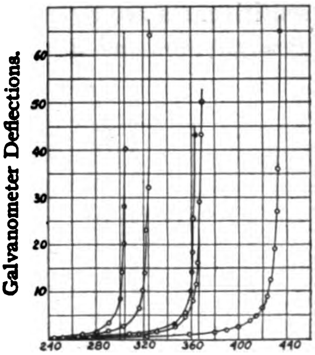
Fig. 8.

Fig. 6. The order of the illuminating voltages is regular. The conditions of sensitiveness are much better as represented in curve for $D = 2.0$ cm.



Variation of gas pressure reading from left to right 2–1–3–5–8 mm. Electrode distance 1 cm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f.

Fig. 9.

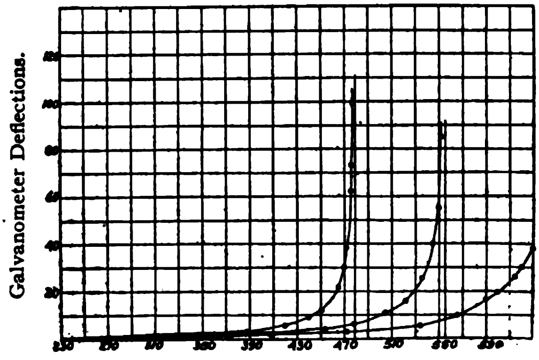


Variation of gas pressure reading from left to right 2–3–1–5–8 mm. Electrode distance 0.5 cm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f.

Fig. 10.

Fig. 7. The order of the illuminating voltages is regular. The best conditions of sensitiveness are represented by a curve that would lie between curves for $D = 0.5$ cm. and $D = 1.0$ cm.

Fig. 8. The order of the illuminating voltages is regular. The best conditions for sensitiveness are represented by a curve which will lie between curves for D = 1.0 cm. and D = 2.0 cm.

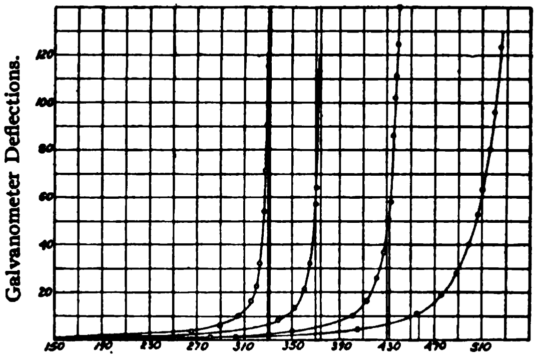


Voltage on Metal.

Variation of electrode distance reading from left to right 0.5–1–2 cm. Gas pressure 10 mm. After forming. A = 4.36 cm². L = 0.22 C.f. t = 24° C.

Fig. 11.

Fig. 9. The curve for P = 1.0 mm. lies between curves P = 2.0 mm. and P = 3.0 mm., showing that the critical pressure for minimum illuminating voltage is in the region of 2 mm. The best conditions of sensitiveness are represented by a curve which lies between curves for P = 3.0 mm. and P = 5.0 mm.



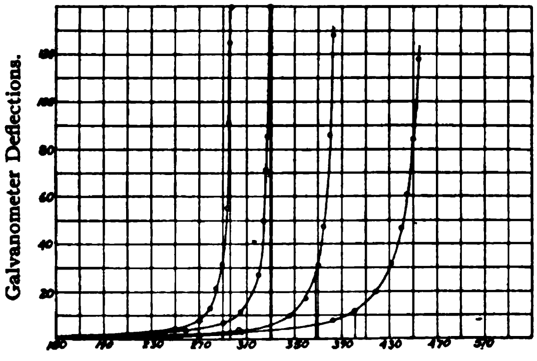
Voltage on Metal.

Variation of electrode distance reading from left to right. 0.5–1–2–3 cm. Gas pressure 3 mm. After forming. A = 4.36 cm². L = 0.22 C.f. t = 25° C.

Fig. 12.

Fig. 10. The curve for P = 1.0 mm. lies between curves for P = 3.0 mm. and P = 5.0 mm. This indicates that the critical pressure at

which the illuminating voltage is a minimum lies in the region of $P = 2.0$ mm. The best conditions for sensitiveness are represented by a curve which lies between curves for pressure between 2 and 3 mm.



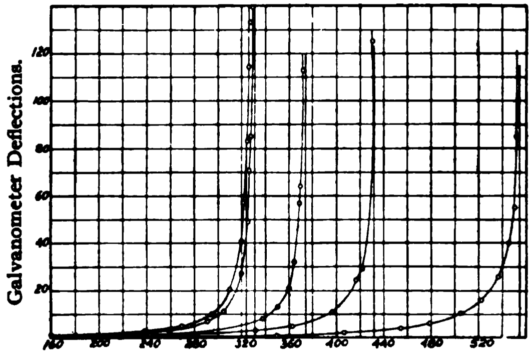
Voltage on Metal.

Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Gas pressure 2 mm. After forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 26°$ C.

Fig. 13.

The next set of figures, 11 to 15, represent values observed after the formation of the hydride. The following notes may be made.

Fig. 11. The order of illuminating voltages is regular. The best conditions for sensitiveness are shown by curve for $D = 0.5$ cm.



Voltage on Metal.

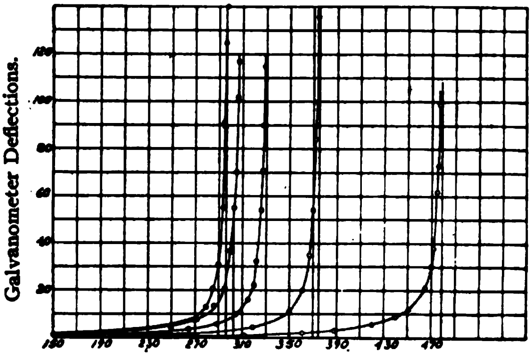
Variation of gas pressure reading from left to right 1–2–3–5–10 mm. Electrode distance 1 cm. After forming. $A = 4.36$ cm². $L = 0.22$ C.f.

Fig. 14.

Fig. 12. The order of illuminating voltages is regular. The best conditions for sensitiveness are shown by curve for $D = 0.5$ cm.

Fig. 13. The order of illuminating voltages is regular. The best conditions for sensitiveness are shown by a curve which lies between curves for $D = 0.5$ cm. and $D = 1.0$ cm.

Fig. 14. These curves show conditions of sensitiveness for different pressures and $D = 1.0$ cm. The curve representing best conditions for sensitiveness lies near the curve for $P = 3.0$ mm.

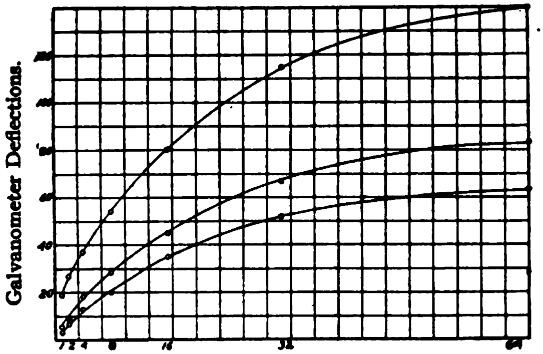


Voltage on Metal.

Variation of pressure reading from left to right 2–1–3–5–10 mm. Electrode distance 0.5 cm. After forming. $A = 4.36$ cm^2. $L = 0.22$ C.f.

Fig. 15.

Fig. 15. The curves show the conditions for sensitiveness for different pressures and $D = 0.5$ cm. The curve representing best conditions for sensitiveness lies near the curve for $P = 3.0$ mm.



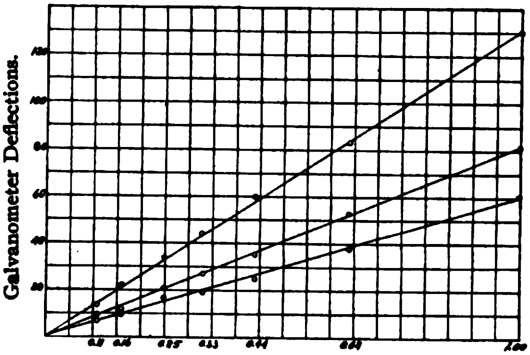
Area of Metal Illuminated.

Variation of area of metal illuminated. Intensity of illumination 0.22 C.f. Gas pressure 2 mm. Electrode distance 0.5 cm. Voltage on metal reading from top downward 295, 294, 293 volts.

Fig. 16.

The curves of Fig. 16 show the variation of area of metal illuminated. The form of these curves indicates that variations in small areas illuminated produce large changes in the current, and variations in large areas illuminated produce small changes in the current. These facts show that there is a maximum area of illumination for this particular type of photo-electric cell which, if exceeded, will not increase the sensitiveness.

The form of the curve should be a straight line if the electric field were uniform, the surface conditions uniform, with no reflection of the light, and no shadow caused by the electrode. In this cell the above conditions were not fulfilled, therefore the form of the curve is not a straight line. It follows finally a system of curves showing the variation of the intensity of illumination for three different voltages, the observations are taken after the forming of the cell. In Fig. 17 showing variation of current with intensity of illumination for $P = 5$ mm. the points for the smaller voltages lie more nearly on a straight line. The largest error in the intensity for the point farthest from the line is 7 per cent. of .15 C.f. or .01 C.f. The points for smaller intensities show much smaller deviations than those for larger intensities. The higher voltages applied caused unsteadiness of the current, hence, the galvanometer deflections are liable to larger errors.



Intensity of Illumination = K/P^2.

Variation of intensity of illumination. Electrode distance 0.5 cm. Voltage on metal reading from top downward 366, 365, 364 volts. After forming.

Fig. 17.

Best Conditions for Sensitiveness.

By comparing the values for the critical voltages and currents obtained from the curves the best sensitive conditions may be selected.

A table of these values is given below for both before and after forming the hydride on the surface of the metal.

Table to Show Best Conditions for Sensitiveness.

V = critical voltage.
I = critical current in galvanometer deflections.
D = best distance between electrodes in cms.
P = best pressure in mm.

Before Forming Metal.

Figure.	V	I	D	P
3	460 to 527	16 to 28	1 to 2	5.0
4	304	26	0.5	2.0
5	349 to 366	6 to 9	1 to 2	1.0
6	454	8	2	3[1]
7	335 to 396	10 to 45	0.5 to 1.0	3[2]
8	383 to 431	7 to 11	1 to 2	3[3]
9	387 to 460	28 to 29	1	3 to 5
10	304 to 324	26 to 32	0.5	2 to 3
		After Forming Metal.		
11	479	280	0.5	10
12	331	off scale	0.5	3
13	296 to 331	123 to 190	0.5 to 1.0	2
14	374	203	1	3
15	331	off scale	0.5	3

By inspection of the table above, it is seen that the best conditions for sensitiveness before forming are about

$$V = 300 \text{ volts.}$$
$$D = 0.5 \text{ cm.}$$
$$P = 2 \text{ to } 3 \text{ mm.}$$
$$t = 25^\circ \text{ C.}$$

And the best conditions for sensitiveness after forming are about

$$V = 330 \text{ volts.}$$
$$D = 0.5 \text{ cm.}$$
$$P = 3 \text{ mm.}$$
$$t = 25^\circ \text{ C.}$$

The cell is about 100 times more sensitive after forming than before forming.

Theoretical Deduction of Measurement of Intensity of Illumination.

If the intensity of illumination varies directly with the current for very small intensities, then it is possible to calculate the intensity measurable with an instrument of given sensibility.

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[1] About – 20° C. salt and ice.

[2] 0° C.

[3] 38° C.

From Fig. 18 below it is seen that for the curve of 366 volts,

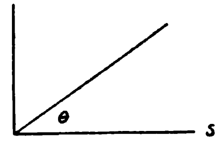

Fig. 18.

$$\tan\theta = \frac{d}{s} = \frac{130}{200} = 0.65,$$

or

$$S = \frac{d}{\tan\theta} = \frac{d}{0.65},$$

$$cd = I \text{ current flowing,}$$

$$c = 3.78 \times 10^{-10} \text{ amp. per mm. deflection,}$$

$$d = \frac{I}{0.65 \times 3.78 \times 10^{-10}} \text{ mm.,}$$

but

$$S = \frac{K}{l^2},$$

and for particular values of $S = 200$, and $l^2 = (100 \text{ cm.})^2$,

$$K = Sl^2 = 200 \times (100)^2 \text{ cm.} = 2 \times 10^6 \text{ cm.}$$

$$S = \frac{2 \times 10^6}{l^2}, \quad \text{or} \quad l = \sqrt{\frac{2 \times 10^6}{S}}.$$

Substitute value of

$$S = \frac{I}{0.65 \times 3.78 \times 10^{-10}}$$

in eq. above, then

$$l = \sqrt{\frac{2 \times 10^6 \times 0.65 \times 3.78 \times 10^{-10}}{I}} = \sqrt{\frac{4.9 \times 10^{-4}}{I}}.$$

By means of an electrometer a current $I = 10^{-12}$ can be measured. Substituting this value of I in the equation above, then,

$$l = \sqrt{\frac{4.9 \times 10^{-4}}{10^{-12}}} = 2.21 \times 10^4 \text{ cm. or 221 meters,}$$

the distance the 2.47 c.p. lamp could be removed from the cell and still be detected. By means of a tilted electroscope a current $I = 10^{-15}$ amperes can be measured.

Substitute this value in the equation,

$$l = \sqrt{\frac{4.9 \times 10^{-4}}{10^{-15}}} = \sqrt{4.9 \times 10^{11}} = 7 \times 10^5 \text{ cm. or 7 kilometers or 4.3 miles.}$$

The 2.47 c.p. lamp could be detected by means of an electroscope at a distance of 4.3 miles. To detect a candle instead of the 2.47 c.p. lamp at 4.3 miles distance by means of the tilted electroscope of 10^{-15} sensibility the distance could be as follows: Since the intensities of illumination of cell must be the same, then,

$$\frac{2.47}{(4.3)^2} = \frac{1}{l^2}, \quad l = \frac{4.3}{\sqrt{2.47}} = 2.7 \text{ miles.}$$

Professor Joel Stebbins,[1] of the University of Illinois, in his work on measuring the variation of intensity of illumination of the variable stars Algol and others used a selenium cell. It is possible to detect a candle at a distance of 500 meters, or 0.3 mile, with such a selenium cell.

The equation for the distance at which this potassium cell is sensitive when the current is measured with a tilted electroscope is

$$l_1 = \sqrt{\frac{K}{I_1}}$$

and for some other distance it is

$$l_2 = \sqrt{\frac{K}{I_2}},$$

$$\frac{l_1^2}{l_2^2} = \frac{I_2}{I_1}, \quad l_1 = 7 \times 10^5 \text{ cm.}, \quad l_2 = 5 \times 10^4 \text{ cm.},$$

then

$$\frac{l_1^2}{l_2^2} = \frac{49 \times 10^{10}}{25 \times 10^8} = 2 \times 10^2 \text{ approximately.}$$

Thus it is seen that the potassium cell is about two hundred times more sensitive than the selenium cell.

Energy Required to Produce an Ion.

To produce an ion a certain minimum amount of energy is required. This energy is that required to draw an electron out of an isolated molecule against the force of attraction of the positive charge of the molecule. Let R_1 be the radius of the molecule, e_1, the positive charge, e_2, the negative charge of electron. If the electron be displaced a distance dR the work done will be

$$dW = \frac{e_1 e_2}{R_1^2}\, dR.$$

The total energy required to draw an electron outside of the influence of the positive charge is

$$W = \int_{R_1}^{\infty} \frac{e_1 e_2}{R^2}\, dR = \frac{e_1 e_2}{R_1} \text{ units of work.}$$

$$e_1 = e_2 = 4.67 \times 10^{-10} \text{ E.S.U.}$$

$$R_1 = 10^{-8} \text{ cm. for hydrogen.}$$

$$(1) \qquad W = \frac{(4.67)^2 \times 10^{-20}}{10^{-8}} = 2.18 \times 10^{-11} \text{ ergs,}$$

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[1] Astrophysical Journal, October, 1911.

the minimum amount of energy required to draw an electron from an isolated molecule of hydrogen or to produce a hydrogen ion.

For an isolated molecule it is necessary to withdraw the electron to an infinite distance from the center. If the molecule, however, is in an electric field the force of attraction between the molecule and the electron is zero at a distance say R_2 from the center. The effect of the electric field tends to decrease the energy required to withdraw the electron on one side of the molecule (with references to the direction of external electric field) while that on the opposite side is increased.

Let Fig. 19 represent the molecule of radius R_1; H is the direction of the external field in the direction of motion, Oa, of the electron. Let R_2 be the distance beyond which the electric attraction between the positive charge of the molecule and the electron is zero. The work required to withdraw the electron beyond the influence of the molecule is

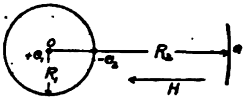

Fig. 19.

$$W = \int_{R_1}^{R_2} \frac{e_1 e_2}{R_2} dR = \frac{e_1 e_2}{R_1} - \frac{e_1 e_2}{R_2},$$

but $e_1 = e_2 = e$; and it is reasonable to assume $R_2 = 4R_1$.

$$W = e^2\left(\frac{1}{R_1} - \frac{1}{4R_1}\right) = \frac{3}{4}\frac{e^2}{R_1}.$$

Substituting the values for R_1 and e,

$$(2) \qquad \frac{W = 3(4.67)^2 \times 10^{-20}}{4 \times 10^{-8}} = 1.63 \times 10^{-11} \text{ ergs}.$$

Thus the value of W in calculation (2) is three fourths that in calculation (1). Hence 2.18×10^{-11} ergs is not the minimum value.

The minimum amount of energy required to produce an ion by collision can be determined roughly from the data taken in this investigation. For the conditions of this work the minimum energy is $W = Eel$, where E is the potential gradient, or electric force, e is the charge of a negative ion or electron, and l is the mean free path of an ion. The force E can be determined roughly as follows: The voltage applied to the electrodes of the cell necessary to produce velocities high enough to cause ionization by collision, is obtained from the ionization curves. This voltage is shown very distinctly at the point on the curve where the ordinate or current shows an increase after the saturation state has been reached. Let V be this voltage taken from the curve, and let D be the distance between the electrodes. Then,

$$E = \frac{300D}{V} \text{ in E.S.U.}$$

(*a*) FIRST ASSUMPTION REGARDING MEAN FREE PATH OF ELECTRONS.

Assuming that the negative ions or electrons and the molecules of the hydrogen gas act as a mixture of two gases, the equation of the mean free path given by Maxwell in the kinetic theory of gases is

$$l_1 = \frac{1}{\pi N_1 \sigma_{11}^2 \sqrt{2} + \pi N_2 \sigma_{12}^2 \sqrt{1 + \frac{m_1}{m_2}}}$$

for the electrons,

$$l_2 = \frac{1}{\pi N_2 \sigma_{22}^2 \sqrt{2} + \pi N_1 \sigma_{12}^2 \sqrt{1 + \frac{m^2}{m_1}}}$$

for hydrogen molecules.

For an electron σ_{11} the diameter of the sphere of action when two electrons collide is practically zero. But σ_{12} the diameter of the sphere of action when an electron collides with a molecule of hydrogen is assumed equal to the radius of the molecule or 10^{-8} cm.

m_1 is the mass of an electron equal to 8.8×10^{-28} grams.

m_2 is the mass of a hydrogen molecule equal to 1.6×10^{-24} grams.

$$\frac{m_1}{m_2} = \frac{8.8 \times 10^{-28}}{1.6 \times 10^{-24}} = 5.5 \times 10^{-4} \text{ gram.}$$

This value is negligible in comparison with unity or $\sqrt{1 + \frac{m_1}{m_2}}$. The equation for the mean free path of the electron is

$$l_1 = \frac{1}{\pi N_2 \sigma_{12}^2} \text{ cm.}$$

In this equation N_2 is the number of molecules per cubic centimeter. From the kinetic theory of gases N_2 may be calculated. $PV = RT$, the gas law, which becomes $PV = \frac{2}{3}L = \frac{2}{3}N' = \frac{1}{2}mC^2$.

Where L is the average total kinetic energy.

N' is the number of molecules in volume V.

C^2 is the square of average velocities of molecules.

$\frac{1}{2}mC^2 = \alpha T$, where α is the universal constant.

$PV = \frac{2}{3}N'\alpha T$, or

$P = \frac{2}{3}N\alpha T$ for unit volume.

$N = \frac{3}{2} \cdot P/\alpha T$ the number of molecules per unit volume

$$= \frac{3}{2} \frac{13.6 \times 980h}{2 \times 10^{-16}T}; \; h \text{ is in cm.}$$

Suppose $T = 27 + 273 = 300$ for this work.

$$N = \frac{3}{2} \frac{13.6 \times 980}{2 \times 10^{-1} \times 300} h = 33 \times 10^{16} h \text{ in cm. mercury.}$$

Substitute this value for N_2 in the expression for the mean free path.

$$l_1 = \frac{1}{\pi \times 33 \times 10^{16} h \times 10^{-16}} = \frac{965 \times 10^{-5}}{h} \text{ cm.}$$

Applying the equation $W = Eel$,

$$e = 4.67 \times 10^{-10} \text{ E.S.U.}$$

$$E = \frac{V}{300D} \text{ E.S.U.}$$

$$l_1 = \frac{965 \times 10^{-5}}{h}.$$

For $h = 0.5$ cm., $D = 1.0$ cm., $V = 330$ volts,

$$W = \frac{4.67 \times 10^{-10} \times 965 \times 10^{-5}}{300 \times 1.0 \times 0.5} = 1.09 \times 10^{-11} \text{ ergs.}$$

An average of ten determinations gave 1.05×10^{-11} ergs for the minimum energy required to produce an ion in hydrogen by collision.

(*b*) Second Assumption Regarding Mean Free Path of Electrons.

If it be assumed that the negative ions or electrons occupy no space in the gas, or, that the hydrogen gas acts as though the electrons were not present, then the value of the mean free path is the same as that of the molecules. That is,

$$l_2 = \frac{1}{\pi N \sigma^2 \sqrt{2} h}$$

$$l_2 = \frac{685 \times 10^{-5}}{h} \text{ cm.}$$

For $h = 0.5$ cm., $D = 1.0$ cm., $V = 330$ volts,

$$W = \frac{4.67 \times 10^{-10} \times 685 \times 10^{-5} \times 330}{300 \times 0.5} = .70 \times 10^{-11} \text{ ergs.}$$

About 35 per cent. decrease from 1.05×10^{-11} ergs.

(*c*) Third Assumption Regarding Mean Free Path of Electrons.

By using the assumption that the mean free path of the electrons is $4\sqrt{2}$ times mean free path of the hydrogen molecules, as Bishop[1] did, the following results are obtained.

$$l_1 = 4\sqrt{2} l_2 = \frac{4\sqrt{2}}{\pi 33 \times 10^{-16} h \times 4 \times 10^{-16}} = \frac{1{,}360 \times 10^{-5}}{h},$$

$$W = \frac{4.67 \times 10^{-10} \times 1{,}360 \times 10^{-5}}{300} \times \frac{V}{Dh} = 21 \times 10^{-15} \frac{V}{Dh}.$$

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[1] Physical Review, 325, November, 1911.

For $P = 0.3$ cm., $D = 1.0$ cm., $V = 260$ volts,

$$W = 21 \times 10^{-15} \frac{260}{1 \times 0.3} = 1.82 \times 10^{-11} \text{ ergs.}$$

An average of 10 calculations gives 1.77×10^{-11} ergs.

To recapitulate, the values determined above and those by other investigators are:

1. For an isolated molecule the theoretical value is 2.18×10^{-11} ergs.
2. For a molecule in an electric field the theoretical value is 1.63×10^{-11} ergs.
3. The average of ten values determined from the data in accordance with the first assumption is 1.05×10^{-11} ergs.
4. The value determined in accordance with the second assumption is 0.70×10^{-11} ergs.
5. The average of ten values determined in accordance with the third assumption is 1.77×10^{-11} ergs.
6. Bishop obtained a value, by a method similar to that used in this work and in accordance with the third assumption used in the fifth determination, 1.58×10^{-11} ergs.
7. Rutherford[1] determined the energy required to produce an ion by the alpha particle. His value is 2.7×10^{-11} ergs.
8. Geiger,[2] and later Taylor,[3] using the same method, obtained about 5×10^{-11} ergs, and other investigators obtained values even as large as 10×10^{-11} ergs.

In the method, based on the ionizing power of alpha particles, it is assumed that their loss of kinetic energy is entirely transformed into energy of ionization, and that the decrease of the kinetic energy over a certain range divided by the total numbers of ions produced gives the energy required to produce one ion. Since a part of the kinetic energy of the alpha particle increases the average kinetic energy of the gas without producing ions, the ionizing energy is taken too large and the energy required to produce an ion is necessarily too large.

Value number (1) represents the minimum energy required to produce an ion when a molecule is isolated. This value is much larger than that for a molecule in an electric field. Rutherford's[4] determination is much nearer the value number (1) than any of the others.

Bishop's determination is very close to the value number (2), while my determination of the ionizing energy is slightly larger. Since the assumption regarding the mean free path in numbers 5 and 6 are the same, and these values differ very slightly from the theoretical value number (2), it indicates that the assumptions made are not far from the truth.

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[1] Radio-Activity, second edition, p. 552.

[2] Proc. Royal Soc., Vol. 82, p. 486, 1909.

[3] Phil. Mag., p. 670, April, 1912.

[4] Radio-Activity, second edition, p. 552.

Owing to lack of time and space an exact determination of the minimum energy required to produce an ion by collision is impossible; in the near future, however, it is hoped that this can be done with the data already in hand.

A design has been made for a sensitive photo-electric cell for photometric work in astronomy. It is expected to get a cell which will be sensitive enough to use instead of the erratic selenium cell now used.

Summary and Conclusions.

The following facts are established by this investigation for this type of photo-electric cell.

1. Owing to the low melting temperature of cæsium the use of this metal in photo-electric cells for photometric use is very impractical.

2. The temperature at which it is best to operate a potassium cell is about 25° C.

3. Cooling the potassium cell much below 25° C. does not increase its sensitiveness.

4. The sensibility of a potassium cell can be increased more than 100 times by the process of forming the hydride surface.

5. The distance between the electrodes for best sensitiveness is about 0.5 cm.

6. The hydrogen gas pressure at which the cell is most sensitive lies between 2 and 3 mm. of mercury.

7. The potential difference applied to the electrodes for most sensitive conditions is about 330 volts.

8. The minimum energy required to produce an ion by collision was calculated from the data and found to be of the order 1.77×10^{-11} ergs, while the theoretical value determined is 1.63×10^{-11} ergs.

9. Assuming that the straight lines obtained which show the relation between current and intensity of illumination hold for exceedingly small intensities, then by using a tilted electroscope of sensibility 10^{-15} amperes, a candle could be detected at a distance of 2.7 miles. This indicates that it is highly probable that a photo-electric cell could be used in astro-photometric work.

The author takes great pleasure in acknowledging his indebtedness to Professor A. P. Carman for the facilities for this investigation, and to Professor Jakob Kunz, both for his general supervision of the work and for many valuable suggestions.

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THE OPTICAL CONSTANTS OF SODIUM AND POTASSIUM.

BY R. W. AND R. C. DUNCAN.

INTRODUCTION.

DURING the early part of the nineteenth century it was found that light was partially polarized by reflection from the polished surface of a metal, and also that plane polarized light became, in general, elliptically polarized upon reflection from such a surface. Later this was shown to be due to a relative change in the phase of the two components. The amount of this change was first measured in 1847 by Jamin,[1] who was thus enabled to calculate, theoretically, the index of refraction of the metal used. The next important work along this line was done by P. Drude,[2] who, about 1890, completed a series of measurements from which he determined the optical constants of most of the common metals.

A large amount of work has since been done, so that at the present time the optical constants of almost all of the metals are known, with the exception of a few of the more highly oxidizable ones. On account of the rapid oxidation of these, the difficulty has been to obtain a bright metallic surface and to preserve it long enough to examine its optical properties. The few attempts that have been made, while not at all satisfactory, have shown that these metals offer an interesting field for further investigation. In 1898 Drude[3] made a single measurement with sodium light reflected from the molten surface of metallic sodium. This measurement gave an index of refraction of 0.0045 which, if correct, means that light travels 220 times as fast in that metal as in air. After making due allowance for some possible errors, which Drude admits might have lowered his result, he says that the index could not be greater than 0.054. But this value is still remarkably low compared to the indices of other metals.

Since Drude's value of the index of sodium was made with sodium light, the question arises is there any connection between the low value of the index and the fact that the light used was of a wave-length peculiar to that emitted by the sodium molecule. Might this be a phenomenon akin to resonance, or is the value of the index equally low for all colors? These questions, together with the extraordinarily low index obtained by Drude, suggested the investigation reported on the following pages.

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[1] Ann. Chim. Phys. (3), Vol. 19, p. 296, 1847.

[2] Ann. d. Phys., Vol. 39, p. 481, 1890.

[3] Ann. d. Phys., Vol. 64, p. 159, 1898.

The Preparation of the Surface.

In his work with sodium, Drude obtained a bright metallic surface by melting the metal in an atmosphere of rarified hydrogen, but found it necessary to renew the surface every few minutes on account of its rapid oxidation. During one of these attempts to remove the thin layer of oxide the vessel containing the sodium was broken and he was unable to verify his first determination. In the present investigation a number of preliminary experiments were made to find, if possible, a method of obtaining a more permanent surface, free from oxide. As the metal can be kept indefinitely under oil, it seemed possible that a bright surface might be obtained and preserved by melting the metal under some light transparent oil. The attempts, however, were not satisfactory for, while a bright surface could be obtained in this way, it would remain bright but a few minutes. Then again on account of the large surface tension and small density of sodium, the molten surface assumed a shape almost spherical, so that a comparatively large surface would be required in order to obtain a small part of the surface approximately plane.

Some efforts were made to obtain the desired surface by melting the sodium in a vacuum, but here, too, it was difficult entirely to prevent oxidation. It was noticed, however, that bright surfaces were often obtained against the inside wall of the glass container. These surfaces remained bright after the sodium had solidified and could be preserved indefinitely by filling the vessel with dry air or oil. This suggested the possibility of obtaining a permanent sodium mirror against a plane glass plate. With this end in view a number of experiments were made. Many of these ended in complete failure, but others gave more promising results. Finally, by combining and improving several plans, a method was devised which has proved quite satisfactory.

The Method in Brief.—A little box, or capsule (Fig. 2), was made by cementing two plane glass plates to the ends of a short iron tube, or ring, from the side of which a small glass tube extended. The latter tube was about 5 cm. long and at its outer end was drawn out so as to leave a very small opening. This capsule was suspended in a cylindrical glass vessel, in the lower end of which had been placed some metallic sodium. The vessel was then closed and evacuated and the lower end immersed in an oil bath, which was heated to a temperature of about

110° C. Meanwhile the suspended capsule was being heated up to approximately the same temperature by an electric current passing through resistance grids held against the glass ends. When the sodium below had melted the capsule was lowered until the small end of the glass tube dipped into the molten metal. Then by admitting dry air into the containing vessel the liquid sodium was forced up through the glass tube into the capsule and pressed firmly against the plane glass ends, thus forming two bright sodium mirrors. After the whole apparatus had been allowed to cool, the sodium capsule was taken out, the small glass tube removed, and the opening closed with paraffin. In this convenient form the sodium mirrors could be kept indefinitely. The method was found to work equally well with potassium, and all sodium or potassium surfaces studied were obtained in this way.

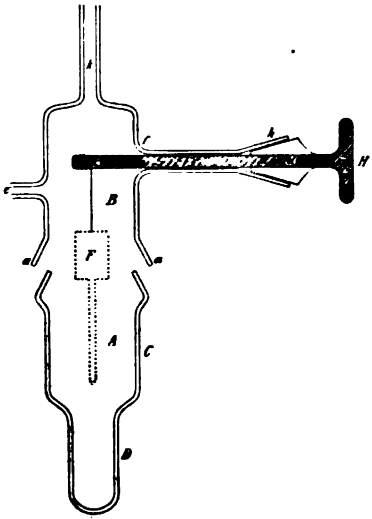

Fig. 1.

Apparatus. — The containing vessel was made of glass and consisted of two parts (*A* and *B*, Fig. 1) fitting together at *a* in a ground junction. The lower part *A* was made by sealing the tube *D*, 5 cm. long and 1.5 cm. in diameter and closed at one end, into the larger tube *C*, 7 cm. long and 4 cm. in diameter. The upper part *B*, 9 cm. long and 4 cm. in diameter, was made from a glass tube closed at one end. At *f* was sealed in a side tube, which ended in the ground cone *h*. A glass rod *E* carried a cone *d* ground to fit *h*, and by means of the T-shaped end *H*, could be rotated about the line *cd* as an axis. It thus served as a windlass to raise and lower the capsule *F* which was suspended by a cord wrapped around the rod at *c*. The wires, carrying the current used to heat the capsule, were led in through the small tube *e*, which was closed with sealing wax. A rubber tube connected the outlet *k* to a two-way stopcock, one branch of which led to the air-pump, while the other, through drying tubes, led to the outside air. The vessel *AB* could thus either be evacuated through *k*, or when evacuated, could be refilled with dry air through the same tube.

The short iron tube (*R*, Fig. 2) used in making the capsule, was 0.8 cm. in length and 1.5 cm. in diameter. A hole *a* 0.45 cm. in diameter was drilled radially through the side of the tube midway between the ends.

A fillet *s* 0.08 cm. wide and of the same depth was cut around the outside edge of each end, thus leaving the shoulder *t* on the inside edge. The end faces of these shoulders were made as plane as possible by grinding on fine emery paper backed by a surface plate. The two pieces of plane glass *gg*, after being carefully cleaned, were clamped tightly against these shoulders. The fillet along the outside edge of the metal ring, together with the adjacent glass plate, now formed a little groove which was filled with cement, while the shoulder pressing against the glass prevented the cement from creeping inside. The capsule was completed by cementing the side tube *T* into the opening *a*. It was then baked at a temperature of about 120° C. for several hours, until the cement, which was at first a light yellow, became dark brown. In some cases a tube of brass (*S*, Fig. 3) was used for the side tube *T*. It was made from a brass rod, 0.45 cm. in diameter and 5 cm. long, which was tapered at one end almost to a point. A hole *o*, 0.3 cm. in diameter, was drilled along the axis of the rod to the point *n*. Then a very small hole 0.03 cm. in diameter was drilled from *m* to *n*, to connect with the larger opening.

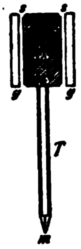

Fig. 2.

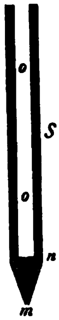

Fig. 3.

Precautions to Prevent Oxidation.—As the most difficult problem in the whole research was the obtaining of a surface free from oxide, the precautions taken to prevent oxidation will be described in some detail.

In preparing to make a mirror, the containing vessel *AB* was first thoroughly cleaned. It was then placed in an upright position, and the capsule (*F*, Fig. 1) suspended from the rod *E* as already described. The vessel was alternately evacuated and filled with dry air[1] a number of times, in order to remove any water vapor or other gases from the interior. Finally it was left closed, full of dry air. The vessel was further protected from moisture by a U-tube of phosphorus pentoxide, placed between it and the pump.[2]

[1] The air was dried by drawing it through U-tubes of sulphuric acid and phosphorus pentoxide.

[2] Gaede's combination of oil and mercury pumps was used.

The quantity of sodium, a part of which was to be used in making the mirror, was immersed in a light paraffin oil. By means of a cork borer of a diameter just smaller than that of the tube (*D*, Fig. 1), a cylindrical stick about 3 cm. long was cut out of the metal. Any heavy coating of oxide still adhering to the ends of the stick was removed with a knife. The cylinder of sodium, thus obtained, was quite free from oxide, since all the cutting had been done under oil. The stick was lifted out and rinsed thoroughly in benzine to remove the oil, the benzine, in turn, being removed by rinsing in toluene. Now, as quickly as possible, the glass vessel *AB* was opened, the sodium cylinder dropped into the receptacle *D*, and the vessel immediately closed and evacuated. The remaining toluene was therefore rapidly removed by evaporation at a reduced pressure. It will be noted that at no time during the whole process was the sodium exposed to the open air. In spite of all these precautions, however, a surface film of oxide would always form before the sodium was entirely melted. Therefore, in order to pierce this surface film, and to permit as little as possible of it from entering the capsule, it was necessary to make the tube (*T*, Fig. 2) pointed, with a very small opening in the end (as already described).

It was possible in this way to obtain bright surfaces, but, upon a close examination, they proved to be covered with an extremely thin filmy veil. This film, however, was not spread uniformly over the surface, but was distributed in curved and ring-like figures, leaving in a number of cases, small areas which were entirely free from it. Further experiment proved that this thin film was due to a spreading out of a small quantity of oxide which had entered the tube *T* when it broke through the surface of the molten metal.

In order to prevent this, two methods were used with about equal success. In the first method a wedge-shaped device was used to cut the surface film and open a way through it for the tube *T*. This device (Fig. 4) was made by fastening two very thin flat steel springs *pp*, each 1.3 cm. long and 0.5 cm. wide, to the metal disk *d* so that the outer ends of the springs were held together, thus making a sort of hollow wedge as shown in the figure. A hole, 0.6 cm. in diameter, was drilled through

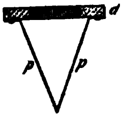

Fig. 4.

Fig. 5.

the center of the disk, which was 1.2 cm. in diameter, just smaller than the inside diameter of *D* (Fig. 1). This was suspended from the capsule so that the tube *T* extended through the hole in the disk, with the pointed end inside the hollow wedge. Now as the capsule was lowered, the wedge pierced the oxide film and entered the sodium, stopping when the disk rested upon the surface of the metal. By a further lowering of the capsule, the sides of the wedge were forced apart by the pointed tube. This pushed aside the surface film so that the point of the tube could enter directly the bright surface of the metal.

In the second method a glass tube (*W*, Fig. 5), whose outside diameter was slightly less than the inside diameter of the receptacle *D*, was drawn down at one end to an opening *h* about 0.8 cm. in diameter. This tube was then suspended, small end downward, from the capsule (*F*, Fig. 1), so that the lower end of the tube *W* was about 1 cm. below the point *m* of the small tube *T*. When, upon lowering this contrivance, the lower end of *W* reached the metal some of the surface film would, of course, enter at *h*. As the tube *W* continued to descend, the liquid metal, entering through *h* from below, burst through and pushed aside this film (which adhered to the sides of the tube *W*), thus permitting the point of the tube *T* to enter a bright globule of melted sodium.

Both of these devices were of considerable help in obtaining bright surfaces, but neither could be said to be consistently successful. By making a number of mirrors, however, a few were obtained which were practically free from this surface film. The results given in this report were obtained from these surfaces.

Other Surface Defects.—In some cases the surface of the mirror was found to be marred by numerous fine lines along which the sodium seemed to have drawn away from the glass. These lines suggested a crystalline formation and probably had been caused by the contraction of the sodium upon solidifying. This configuration on the surface was especially noticeable when the mirror was examined with polarized light, for, since the light totally reflected from the back surface of the glass along these lines differed in phase from that reflected from the sodium surface, both could not be extinguished at the same time. Fortunately, however, only a few of the mirrors showed this defect, and they were either entirely discarded or measurements made only on that part of the surface from which these lines were entirely absent.

A number of failures proved that unless the glass plates, upon which the mirrors were to be formed, were heated to approximately the temperature of the melted sodium, the metal would solidify as soon as it touched the plates, leaving streaks or striations across the surface.

It was also found necessary to heat the tube T, through which the sodium was forced up into the capsule, for, otherwise the sodium would solidify while yet in the tube, and so never reach the glass plates. On this account a brass tube proved much more convenient than the glass one originally used, since the whole of the brass tube could be heated by merely heating the capsule.

Cement.—In making the capsule quite a little difficulty was experienced in finding a satisfactory cement, *i. e.*, one which would fasten together glass and iron in an air-tight junction and would not soften or become porous when heated in a vacuum to the required temperature (about 110° C.). A number of cements were tried, some being commercial products, while others were mixed in the laboratory according to various formulæ. The one finally selected is on the market under the trade name "Rock Cement." When first tried, this was used according to the printed directions, but was not satisfactory. Further experiment, however, showed that it could be made so if thoroughly dried by heating. It was also noticed that better results were obtained by allowing the cement to thicken somewhat by evaporation before using.

MEASUREMENTS.

Method and Apparatus.—Drude's method for obtaining the optical constants of metals was used throughout this investigation. It consists in an examination of polarized light reflected from the polished surface of the metal. If the incident light be polarized in a plane making an angle of 45° with the plane of incidence, the quantities to be measured are ϕ, the angle of incidence; Δ, the phase change introduced by reflection; and ψ, the azimuth of restored polarization. The optical constants (n, the index of refraction and κ, the index of absorption) are obtained by substitution in the following formulæ, given by Drude.[1]

$$n^2(1 - \kappa^2) = S^2 \cos 2Q + \sin^2 \phi, \tag{1}$$

$$2n^2\kappa = S^2 \sin 2Q, \tag{2}$$

where

$$\cos 2P = \sin 2\psi \cos \Delta, \tag{3}$$

$$\tan Q = \tan 2\psi \sin \Delta, \tag{4}$$

$$S = \sin \phi \tan \phi \tan P. \tag{5}$$

Writing

$$S^2 \cos 2Q + \sin \phi = A, \tag{6}$$

$$S^2 \sin 2Q = B, \tag{7}$$

then

$$2n^2 = \sqrt{A^2 + B^2} + A, \tag{8}$$

$$\kappa = \frac{\sqrt{A^2 + B^2} - A.}{B} \tag{9}$$

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[1] Ann. d. Phys., Vol. 64, pp. 161–2, 1898.

The principle angle of incidence $\bar{\phi}$, the principal azimuth $\bar{\psi}$, and the reflecting power R, are given by:

$$\sin^4\bar{\phi}\tan^4\bar{\phi} = n^4(1+\kappa^2)^2 - 2n^2(1-\kappa^2)\sin^2\bar{\phi} + \sin^4\bar{\phi}, \tag{10}$$

$$\bar{\psi} = \tfrac{1}{2}\tan^{-1}\kappa, \tag{11}$$

$$R = \frac{n^2(1+\kappa^2)+1-2n}{n^2(1+\kappa^2)+1+2n}. \tag{12}$$

Since for sodium and potassium the constant A (Eq. 6) is negative, the calculations may be much simplified. Making A negative and expanding the radical, equation (8) becomes

$$2n^2 = A\left(\frac{B^2}{2A^2} - \frac{B^4}{8A^4} + \text{etc.}\right).$$

As B is small compared to A, all terms except the first may be neglected. This gives

$$n = \frac{B}{2\sqrt{A}}. \tag{13}$$

In a similar way, equation (9) may be simplified, giving

$$\kappa = \frac{2A}{B}. \tag{14}$$

In all calculations made in this report an error of less than one per cent. results from using equations (13) and (14).

As in Drude's experiments, the instrument used was a spectrometer fitted with a polarizing and an analyzing nicol and a Soleil-Babinet compensator. The desired wave-length was obtained by means of a monochromatic illuminator, the source of light being an electric arc. For each particular wave-length used, the zero position and the constant of the compensator were accurately determined.

Since the formula for n involves the tangent of the angle 2ψ, usually large, a small error in measuring this angle has a much greater effect upon the result than a corresponding variation in Δ. This is especially true of the light reflected from a sodium surface, for which 2ψ is about 89°.

A number of careful measurements, made to determine the most accurate method of setting the nicol, showed that the best results were obtained by removing the eyepiece of the telescope and looking directly at the nicol. Viewed in this way, there appeared, at the extinction point, a broad and quite black horizontal band which never entirely covered the field.[1] Upon sliding the wedge in the compensator this band would move up or down in the field, while upon slightly moving the nicol in either direction from an extinction point, it would slowly fade away, usually moving across the field in the direction of its length. The cause of this unexpected appearance of the field has been traced to a fault in the construction of the compensator, the principal axes of the quartz plates not being exactly perpendicular to each other.

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[1] This is not the phenomenon so often noticed with crossed nicols, for the band appeared only when the compensator was in position. With the new compensator (mentioned elsewhere in this article) the field was uniform, but if the quartz plates were rotated slightly upon each other, the band appeared and became quite distinct.

The extinction point for both the compensator and the nicol was taken to be that position for which the dark band was centered in the field. For the compensator, the value used for the extinction point was the mean of eight or ten settings. For the nicol, a larger number of readings was necessary, since, under the best conditions, the individual settings varied by approximately thirty minutes of arc. Therefore, in order to get readings in every possible position of both the polarizer and the analyzer the following method was adopted. The polarizer was set in four positions, each 45° from the plane of incidence. At each of these positions, seven to ten readings of the analyzer were taken for each of its two extinction points, 180° apart. Therefore the value obtained for 2ψ was the mean of 50 to 80 readings.[1]

Mounting the Mirrors.—The mirrors were mounted so as to be viewed through a prism. A drop or two of cedar oil, which had approximately the same index of refraction as the glass, was placed upon the surface of the mirror and the mirror pressed firmly against the hypotenuse side of a right-angled prism. The light entered one leg of the prism normally and, after reflection at the sodium surface, emerged normally through the other leg. By this plan all the troublesome reflections from the front surface of the glass of the mirror were avoided, and, since the light was normal to both prism faces, it suffered no change of phase or of azimuth, either at the point of incidence or emergence. In order to remove any film of grease or other foreign matter, which might cause a phase change, the mirror and all of the surfaces of the prism were carefully washed before mounting.

Tests for Possible Corrections.—Should there be a slight difference between the value of the refractive index of the cedar oil and that of the glass, a change in the azimuth of polarization would be produced at the boundary between the two media. A test for such an effect was made in two ways, as follows:

First, a piece of plate glass was fastened with the cedar oil to the hypotenuse side of the prism. Polarized light was sent through the prism in the same way as before, but was now totally reflected from the back surface of the glass plate. A careful examination of this light which, it will be noticed, had passed twice through the oil film, each time at an angle of 45°, showed no change in the azimuth of polarization. The phase change was also measured and found to agree closely with that to be expected from total reflection.

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[1] A new compensator combined with a half shadow analyzer (Zehnder, Ann. d. Phys., Vol. 26, p. 985, 1908) has since been received from Germany, and the few measurements taken with it agree closely with those reported in this paper.

Secondly, five square pieces of glass, cut from the plate used in making the sodium mirrors, were placed between 60° prisms as shown in Fig. 6. A drop of cedar oil was placed between each plate, and between the first and last plates and the prism faces, and the whole pressed firmly together. Polarized light was sent through the combination in such a way as to pass normally through the outer prism faces (as indicated in the figure). The light therefore traversed twelve surfaces of contact between glass and oil, the angle at each being 60°. Here again a careful set of measurements failed to show any change in the azimuth of polarization. Hence, any effect, if present, must be negligible.

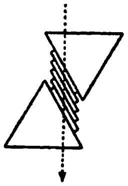
Fig. 6.

Another possible source of error was the presence of the oxide film, of which, as already mentioned, almost every sodium mirror showed at least a trace. In order to determine the optical effect of this film, the phase change and restored azimuth were measured on six different sodium mirrors. A week later a second set of measurements were made on the same surfaces. In the results, which are given below (Table I.), the phase change is expressed in head divisions of the micrometer screw and may be reduced to degrees by multiplying by 0.3937. The numbers given in the first column are used merely to distinguish the individual surfaces.

TABLE I.

Mirror No.	Phase Change.			Azimuth.		
	1st.	2d.	Av.	1st.	2d.	Av.
1	329	327	328	89° 21′	89° 23′	89° 22′
2	322	334	328	89° 20′	89° 24′	89° 22′
3	310	306	308	89° 6′	89° 13′	89° 10′
4	320	305	312	89° 3′	88° 54′	88° 58′
5	330	329	329	88° 54′	89° 3′	88° 58′
6	316	317	317	88° 50′	89° 0′	88° 55′

The results have been arranged in the above table in descending order of the average azimuth, and an examination of the individual surfaces showed that in general the film became more pronounced in the same order. No such variation of phase change could be observed. A comparative study of the first and second columns in both the phase change and azimuth above indicates no consistent change in the surfaces during the week which elapsed between the two sets of readings.[1]

As Nos. 1 and 2 were practically free from film, they were chosen for the determination of the optical constants of the metal as given below. It is believed that the results obtained from these mirrors are very near to those that would be obtained from a perfect surface, since the rather heavy film on Nos. 5 and 6 reduced the azimuth by less than thirty minutes.

Nos. 1, 3 and 4 were made from (Merck) sodium, guaranteed to be free from other metals, while Nos. 2, 5 and 6 were made from ordinary commercial sodium. The results did not show any consistent difference between the two specimens of the metal.

It is interesting to note that Nos. 5 and 6 were mirrors on the opposite sides of the same capsule. No. 5 was made directly on the hypotenuse face of a prism (by substituting a prism for one of the glass plates of the capsule), and so could be examined without the use of the oil film, while No. 6 was formed on the glass plate and examined through the oil. The results indicate that the oil film had no appreciable effect.

Method Applied to Mercury.—Exactly the same method was used throughout to determine the optical constants of mercury. As the constants for this metal are well known the results (Table II.) serve as a check upon the method. It will be noticed that the values for n are slightly higher than those obtained by Drude[2] or by Meier.[3] This may be due to the particular specimen of mercury used.

TABLE II.

Mercury. $\phi = 45°$.

λ	Δ	2ψ	n	κ
6,650	162° 11′	82° 57′	2.34	2.47
5,893	160° 4′	83° 1′	1.92	2.78
4,720	156° 14′	83° 18′	1.35	3.42

Former values obtained are:

Drude, $\lambda = 5{,}893$, $n = 1.73$, $\kappa = 2.87$.
Meier, $\lambda = 5{,}893$, $n = 1.62$, $\kappa = 2.71$.

Specimen Set of Readings in Detail.—In order to give an idea of the accuracy of setting and the method of procedure, a complete series of readings, taken to determine a single value of n and κ, is given in Table III. Every result was determined from a similar series of readings.

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[1] Measurements taken on some of these surfaces three months later showed no change in the character of the surface.

[2] Ann. d. Phys., Vol. 39, p. 530, 1890.

[3] Ann. d. Phys., Vol. 31, p. 1031, 1910.

TABLE III.

A. *Determination of Zero Point and Constant of Compensator.*

Sodium mirror No. 1. $\phi = 44° 59'$.

	For Phase Difference of		
	-2π	Zero.	$+2\pi$
	(0) 80.0	(12) 37.0	(23) 91.0
	81.0	36.0	95.0
	79.2	36.0	98.0
	81.0	38.5	99.0
	78.3	38.0	(24) 1.0
	81.0	38.0	(23) 95.0
	78.2	36.5	96.2
Averages.......	(0) 79.8	(12) 37.0	(23) 96.3

Numbers in ( ) indicate whole number of turns of the micrometer screw. The readings are in head divisions.

2,316.5 divisions = 720° (4π). Therefore 1 div. = 720/2,316.5 deg.

B. *Determination of Phase Change upon Reflection.*

	Readings for Extinction	
	At Beginning.	At End of Series.
	(16) 90.0	(16) 81.5
	89.0	83.0
	88.0	84.2
	86.0	84.0
	86.5	82.5
	87.2	85.0
	88.6	86.0
Av.	(16) 87.9	(16) 83.7

1,685.8 − 1,237 = 448.8 divisions.

Therefore $\Delta = 139° 29'$.

General average, 1,685.8.

C. *Determination of Restored Azimuth.*

Polarizer.	49° 39′		229° 39′	
Analyzer..........	360° 9′	179° 29′	179° 48′	359° 42′
	359° 22′	179° 17′	180° 7′	359° 40′
	359° 30′	180° 2′	180° 31′	359° 42′
	360° 20′	179° 4′	180° 32′	359° 4′
	360° 8′	179° 33′	179° 46′	359° 40′
	359° 59′	179° 23′	179° 44′	359° 32′
	360° 11′	179° 52′	179° 40′	359° 46′

General average (reduced to one position), 359° 46.2′.

Polarizer.	139° 39′		319° 39′	
Analyzer..........	270° 40′	90° 53′	90° 27′	270° 1′
	270° 24′	90° 9′	90° 22′	270° 14′
	270° 22′	90° 18′	90° 28′	270° 53′
	270° 1′	90° 32′	89° 48′	270° 11′
	270° 24′	90° 28′	90° 28′	270° 31′
	270° 11′	90° 9′	89° 53′	270° 10′
	270° 42′	90° 19′	90° 2′	270° 54′

General average, 270° 20.9′. Therefore $2\psi = 89° 25.3'$.
By substitution $A = -3.1553$, $B = 0.1135$. Giving $n = 0.032$, $\kappa = 55.6$.
Index of refraction for glass plate = 1.5 (approximately).[1]
Therefore the index of refraction from air to sodium is:

$$n = 1.5 \times 0.032 = 0.048.$$

RESULTS.

TABLE IV.

Sodium.

Sodium mirror No. 1. $\phi = 44° 59'$.

λ	Δ	2ψ	n	κ
6,650	139° 29′	89° 25.3′	0.048	55.6
5,893	134° 46′	89° 22.9′	0.042	54.3
5,460	130° 34′	89° 14.1′	0.045	44.7
4,720	124° 34′	89° 0′	0.051	34.2
4,350	117° 58′	88° 46.7′	0.053	26.4

Sodium mirror No. 2. $\phi = 44° 59'$.

λ	Δ	2ψ	n	κ
6,650	142° 1′	89° 25.9′	0.053	54.5
5,893	137° 10′	89° 24.7′	0.045	55.8
5,460	136° 6′	89° 9.8′	0.060	40.6
4,720	130° 20′	88° 57.5′	0.062	32.4
4,350	127° 7′	88° 32.0′	0.063	37.0

The averages from the above are given in the second and third columns below. The values of $\bar{\phi}$, $\bar{\psi}$, and R, given in the last three columns, are obtained by using these averages in the formulæ (10), (11), and (12).

λ	n	κ	$\bar{\phi}$	$\bar{\psi}$	R
6,650	0.051	55.0	72° 11′	44° 29′	97.7
5,893	0.044	55.0	68° 51′	44° 29′	97.1
5,460	0.052	42.6	68° 48′	44° 20′	96.5
4,720	0.057	33.3	66° 29′	44° 9′	95.2
4,350	0.058	31.7	66° 0′	44° 6′	94.8

TABLE V.

Potassium.

Potassium mirror No. 1. $\phi = 44° 59'$.

λ	Δ	2ψ	n	κ
6,650	123° 23′	88° 49.7′	0.057	28.2
5,893	116° 26′	88° 36.7′	0.059	22.7
4,720	106° 18′	88° 27.6′	0.060	15.5

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[1] Any error in this approximation is within the limits of experimental error.

Potassium mirror No. 2. $\phi = 44° 59'$.

λ	Δ	2ψ	n	κ
6,650	128° 29′	88° 39.0′	0.075	25.4
5,893	123° 19′	88° 25.5′	0.077	21.6
4,720	114° 19′	88° 5.2′	0.079	13.1

Averages, etc.

λ	n	κ	$\bar{\Phi}$	$\bar{\Psi}$	R
6,650	0.066	26.8	65° 27′	43° 56′	93.8
5,893	0.068	22.1	62° 58′	43° 42′	92.0
4,720	0.070	14.3	57° 9′	43° 0′	86.9

Although the two potassium mirrors studied seemed to be entirely free from the oxide film, the results are not considered conclusive until more surfaces have been examined. No explanation is offered, at present, for the difference in the results obtained from the two specimens.

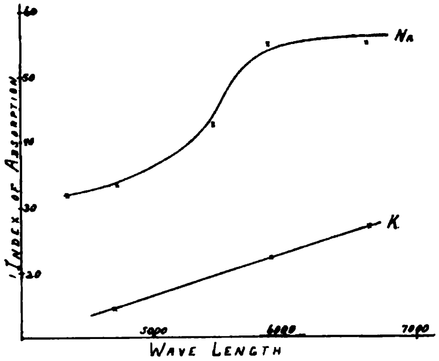

Fig. 7.

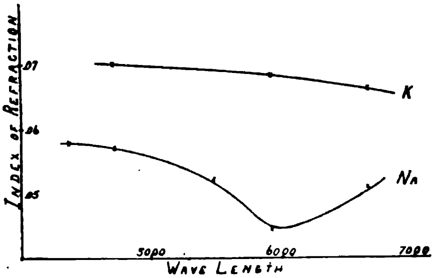

Fig. 8.

The variations of the optical constants with wave-length are shown in Figs. 7 and 8. The curve for the index of refraction of sodium has a distinct minimum near the D-line.

Conclusions.

1. It is possible to obtain and preserve indefinitely bright surfaces of both sodium and potassium.

2. Metallic sodium has the lowest index of refraction and the highest reflecting power of any metal known. This is in agreement with Drude's observations. It is interesting to note that the value for the refractive index for sodium light is very near (slightly less than) the upper limit set by Drude.

3. While the refractive index for sodium is very low for all wavelengths, it apparently has a minimum close to the sodium line.

4. Next to sodium, potassium has the lowest index of any metal. Its reflecting power, however, is slightly less than that of either silver or sodium.

5. The method seems applicable to the sodium-potassium alloy, and possibly may be adapted to other highly oxidizable substances. By using quartz, or uviol glass, plates and prisms the investigation may be carried into the ultra-violet by means of Minor's method.[1]

While the results reported above seem to be fairly consistent among themselves, the authors intend to verify them by further investigation. They hope, also, to be able to determine the optical properties of the sodium-potassium alloys, and probably of some other substances, to which the same method is applicable.

In conclusion we wish to acknowledge our indebtedness to Professor Arthur W. Goodspeed, who placed at our disposal all the facilities of the laboratory and kindly procured for us additional apparatus. We wish also to thank Professor Horace C. Richards for suggesting the subject of this research and for his continued interest and coöperation.

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May, 1912.

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[1] Ann. d. Phys., Vol. 10, p. 581, 1903.

A METHOD OF PRODUCING KNOWN RELATIVE SOUND INTENSITIES AND A TEST OF THE RAYLEIGH DISK.

By G. W. Stewart and Harold Stiles.

THE absolute intensity of sound has been measured principally in four ways, viz., by the use of the Rayleigh disk placed directly in the sound,[1] by the measurement of the increased pressure at a reflecting wall,[2] by measuring pressure changes at nodes of stationary waves by a manometer,[3] and by optical interference methods.[4] In some of the experiments to which reference has just been made, varying measurable sound intensities have been produced, but in such a manner as to be unavailable for the calibration of intensity measuring devices of various kinds. Indeed, we have found no record of a successful effort to produce known varying intensities available for testing purposes. The application of the inverse square law is quite inaccurate, even out of doors. The construction of a sound-proof or a "silence" room will probably not reduce the reflection sufficiently to justify the assumption of the variation of the intensity inversely as the square of the distance.

The theory[5] of the acoustic shadow produced at any distance from a rigid sphere with the source located on the sphere suggested a method of producing known variations of intensity and thus obtaining a calibration device for sound-measuring apparatus.

The theory can be briefly stated. Let the source be confined to a small area on the surface of the sphere within which $P_n(\mu) = 1$. Let the velocity of this source region be simple harmonic and let it have the same magnitude U throughout. The following notation and equations are assumed:

ψ represents the velocity potential,
a represents the velocity of sound,
r represents the distance from center of sphere,
c represents the radius of the sphere,

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[1] Zernov, Annal. d. Phys., 26, 1908, p. 79, Fig. 10.

[2] Altberg, Annal. d. Phys., 11, 1903, p. 405, and Zernov, Annal. d. Phys., 21, 1906, p. 131.

[3] Raps, Annal. d. Phys., 36, 1889, p. 273.

[4] Raps, Annal. d. Phys., 50, 1893, p. 193, and Sharpe, Science, 9, 1910, 1909, p. 808.

[5] Stewart, Phys. Rev., Vol. XXXVIII., No. 6, December, 1911.

dS represents an element of surface,

$$K = \frac{2\pi}{\text{wave length}},$$

$$\gamma = k(at - r + c),$$

$$F = \sum \frac{2n + 1}{2} P_n(\mu) \frac{\alpha\alpha' + \beta\beta'}{\alpha^2 + \beta^2},$$

$$G = \sum \frac{2n + 1}{2} P_n(\mu) \frac{\alpha\beta' - \alpha'\beta}{\alpha^2 + \beta^2},$$

[2] $f_n(ikr) = \alpha' + i\beta'$,

[2] $F_n(ik\epsilon) = \alpha + i\beta$.

Then

$$\psi = \frac{ka}{2\pi r}(F \sin \gamma + G \cos \gamma) \iint U dS. \tag{1}$$

The energy per unit volume is $\frac{1}{2}\rho_0 a^2 s^2$, where ρ_0 is the density and s the condensation. But s equals $-\dot{\psi}/a^2$. Therefore we have for the mean potential energy the following expression,

$$\text{Energy per unit volume} = \tfrac{1}{2}\rho_0 \frac{\dot{\psi}^2}{a^2} = \tfrac{1}{2}\rho_0 (F^2 + G^2)\left(\frac{k}{2\pi r}\iint U dS\right)^2. \tag{2}$$

We determined to construct a sphere with a satisfactory source of sound thereupon, and thus to secure in the region of the sphere varying sound intensities of known relative values.

As shown in Fig. 1 the sphere was mounted on the edge of the roof of the Physics Building. It was placed on the side where the building was 21 meters high. There were neither buildings nor trees within several hundred feet, and this, combined with the high elevation, made the location very satisfactory. Indeed, the only reflecting surface was the roof. The arrangement of the apparatus shows that the error due to the reflection from the roof would be very small, and this error was further reduced by a covering of three fourths of an inch of hair felt.

The sphere was constructed of cement, the wall thickness being 5 cm. The diameter of the opening, the source of sound, was about 5 cm. The circumference of the sphere was 135.9 cm. As shown by the figure, the sphere could be rotated readily, the angle being indicated at the water seal. The sphere was supported by a horizontal 5 cm. pipe 230 cm. above it, and this pipe was in turn supported by two 5 cm. pipes and an iron flagstaff, all three being distant from the sphere at least 250 cm. The reflection from these supports was practically nil.

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[1] $f_n(ikr)$ and $F_n(ikc)$ are defined in Rayleigh's Theory of Sound, Vol. II., and in article by Stewart, loc. cit.

The sound was produced by an electromagnetically operated tuning fork mounted on a resonator, the latter being introduced into a funnel located at a distance of 700 cm. from the sphere and connected to it through an iron pipe as suggested by Fig. 1. The frequency used was 256, and thus *kc* was very approximately unity.

The apparatus to the left of the sphere in Fig. 1 is a Rayleigh disk device. This was used to prove the practicability of this method of producing known relative intensities. This device is a modification of the one suggested by Rayleigh[1] and is drawn to scale in Fig. 2. It was made of brass tubing. The constriction in the tube was introduced to increase the sensitiveness. The dimensions needed were calculated by an approximate formula and then tested experimentally before constructing the apparatus.

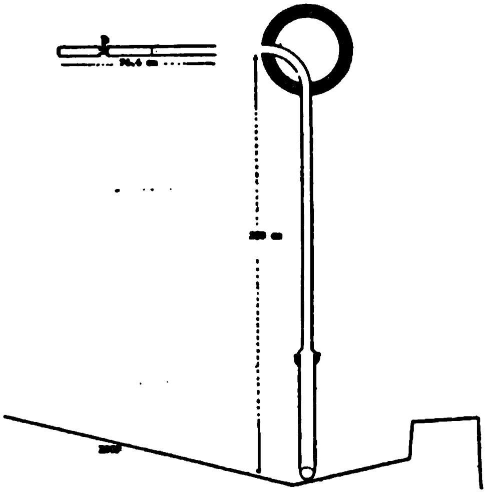

Fig. 1.

The mirror, 0.6 cm. in diameter, was made from a very thin microscope cover glass. It was suspended by a quartz fiber. The complete period was 6 seconds.

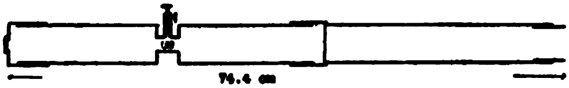

Fig. 2.

The observing telescope was placed along the axis of the tube and the scale parallel to the tube and in front of the mirror window. The apparatus had a high sensibility giving a definite deflection for what would be termed a "faint" sound. The sensitiveness could have been increased greatly by reducing the size of the fiber suspension, but for work out-of-doors a short period was highly desirable.

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[1] Rayleigh, Phil. Mag., Vol. XIV., p. 186, 1882.

The diaphragm was a piece of thin letter paper, but the selection of this material has no significance as it was not the result of investigation. The Rayleigh disk as used undoubtedly gives a deflection which is practically proportional to the kinetic energy on the interior at the disk itself. Indeed this has been demonstrated[1] experimentally. So far as we have been able to ascertain there is no experimental evidence that the kinetic energy at the disk itself is proportional to the potential energy which would exist at the opening of the resonating tube if the presence of the apparatus produced no distortion. Yet we here tentatively assume this to be the case. In this paper "energy" refers to potential energy unless otherwise designated.

We used the disk at three distances, viz., $kr = 2$, $kr = 3$, and $kr = 4$. The computations for these distances were made in accord with the formula (2) and the relative values of $F^2 + G^2$ obtained. The values for $f_n(ikr)$ and $F_n(ikc)$ were computed from equations defining these expressions.[2]

The values of the terms of Legendre's series $P_n(\mu)$ for the angles used from 0° to 180° were ascertained from tables and the relations

$$P_{2n+1}(90^\circ + \theta) = P_{2n+1}(90^\circ - \theta)$$

and

$$P_{2n}(90^\circ + \theta) = P_{2n}(90^\circ - \theta).$$

The terms were retained as far as $P_6(\mu)$. The computations are probably sufficiently accurate for the purposes of this paper. The results are shown in the accompanying tables.

The accompanying curves (Fig. 3 and Fig. 4) show these computations plotted with the value at 0° taken as unity. This is the position of the sphere when the source of sound is directly in front of the Rayleigh disk. The points indicated by small circles are the results of observations with the Rayleigh disk. The relative deflections are plotted. Our maximum deflection from the 45° position of the disk (*i. e.*, 45° between the normal to the mirror and the direction of the undisturbed stream) was 7°. An inspection of the formula[3] derived for the disk shows that the assumption of proportionality of energy to deflection does not introduce an error we need here consider.

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[1] Zernov, Annal. d. Phys., No. 26, p. 79, 1908.

[2] Stewart, loc. cit.

[3] Konig, Wied. Annal., XLIII., 1891, p. 51.

Table I.

$kc = 1$, $kr = 2$.

	0°		30°		60°		90°	
	F	*G*	*F*	*G*	*F*	*G*	*F*	*G*
0	+0.2500	−.2500	+ .2500	−.2500	+.2500	−.2500	+.2500	−.2500
1	+ .7500	.0000	+ .6495	.0000	+.3750	.0000	.0000	.0000
2	+ .3020	+.2669	+ .1887	+.1669	−.0377	−.0334	−.1510	−.1335
3	+ .0949	+.1352	+ .0308	+.0439	−.0415	−.0591	.0000	.0000
4	+ .0398	+.0616	+ .0009	+.0014	−.0115	−.0178	+.0150	+.0231
5	+ .0188	+.0292	− .0042	−.0065	+.0017	+.0026	.0000	.0000
6	+ .0091	+.0142	− .0034	−.0053	+.0029	+.0046	−.0029	−.0044
	+1.4646	+.2571	+1.1123	−.0496	+.5389	−.3531	+.1111	−.3648
$F^2 + G^2$	2.2111		1.2397		0.4151		0.1454	

	120°		150°		180°	
	F	*G*	*F*	*G*	*F*	*G*
0	+.2500	−.2500	+.2500	−.2500	+.2500	−.2500
1	−.3750	.0000	−.6495	.0000	−.7500	.0000
2	−.0377	−.0334	+.1887	+.1669	+.3020	+.2669
3	+.0415	+.0591	−.0308	−.0439	−.0949	−.1352
4	−.0115	−.0178	+.0009	+.0014	+.0398	+.0616
5	−.0017	−.0026	+.0042	+.0065	−.0188	−.0292
6	+.0029	+.0046	−.0034	−.0053	+.0091	+.0412
	−.1315	−.2401	−.2399	−.1244	−.2629	−.0717
$F^2 + G^2$	0.0749		0.0730		0.0742	

The agreement between the theory of the acoustic shadow and the performance of the disk is not good in Fig. 3 for $kr = 2$, but is quite satisfactory in Fig. 4, $kr = 3$, conditions considered. If the disk itself were suspended in the open air it would give correct relative values of the mean kinetic energy per unit volume at the point. But the disk is enclosed in order to utilize the magnifying effect of resonance. This introduces several sources of error. A slight breeze interferes with the resonance of the tube, and doubtless this error has not the same relative value for all values of resonance. We worked under the best conditions obtainable and yet there was always a perceptible motion of the atmosphere. Our observations indicate that the small readings were greatly in error. It should be stated that the observed points are not averages of large numbers of readings, but represent different sets of observations. Another source of error is introduced by the absorption of energy by the resonating disk tube. This must disturb the distribution of sound intensity. It would seem that this distortion would tend to "iron out" the curve, or to produce higher readings on the steeper portions. One would also expect the distortion at $kr = 3$ to be less than at $kr = 2$.

TABLE II.

$kc = 1$, $kr = 3$.

	0°		30°		60°		90°	
	F	*G*	*F*	*G*	*F*	*G*	*F*	*G*
0	+0.2500	−0.2500	+0.2500	−0.2500	+0.2500	−0.2500	+0.2500	−0.2500
1	+0.7000	+0.1000	+0.6062	+0.0866	+0.3500	+0.0500	0.0000	0.0000
2	+0.1311	+0.2904	+0.0819	+0.1815	−0.0164	−0.0363	−0.0656	−0.1452
3	−0.0122	+0.0876	−0.0039	+0.0285	+0.0053	−0.0383	0.0000	0.0000
4	−0.0082	+0.0212	−0.0002	+0.0005	+0.0024	−0.0061	−0.0031	+0.0079
5	−0.0026	+0.0057	+0.0006	−0.0013	−0.0002	+0.0005	0.0000	0.0000
6	−0.0008	+0.0017	+0.0003	−0.0007	−0.0003	+0.0006	+0.0003	−0.0005
	+1.0573	+0.2566	+0.9349	+0.0451	+0.5908	−0.2796	+0.1816	−0.3878
$F^2 + G^2$	1.1841		0.8760		0.4274		0.1834	

	120°		150°		180°	
	F	*G*	*F*	*G*	*F*	*G*
0	+0.2500	−0.2500	+0.2500	−0.2500	+0.2500	−0.2500
1	−0.3500	−0.0500	−0.6062	−0.0866	−0.7000	−0.1000
2	−0.0164	−0.0363	+0.0819	+0.1815	+0.1311	+0.2904
3	−0.0053	+0.0383	+0.0039	−0.0284	+0.0122	−0.0876
4	+0.0024	−0.0061	−0.0002	+0.0005	−0.0082	+0.0212
5	+0.0002	−0.0005	−0.0006	+0.0013	+0.0026	−0.0057
6	−0.0003	+0.0006	+0.0003	−0.0007	−0.0008	+0.0017
	−0.1184	−0.3040	−0.2709	−0.1824	−0.3131	−0.1301
$F^2 + G^2$	0.1065		0.1065		0.1150	

This expectation seems to be realized in the observations as shown in Figs. 3 and 4.

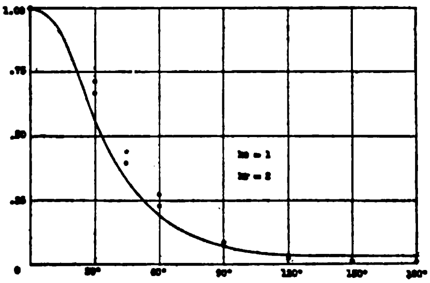

Fig. 3.

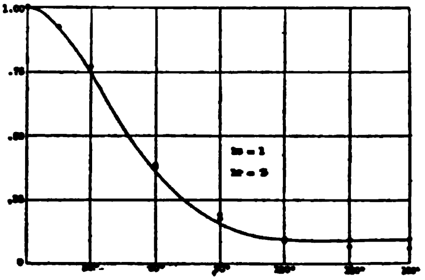

Fig. 4.

The theory, equation (2), shows that the intensities at different distances are proportional to $(F^2 + G^2)/r^2$. The results of a brief test at three different distances, $kr = 2$, $kr = 3$ and $kr = 4$, are presented in the accompanying table.

TABLE III.

kr	$F^2 + G^2$	$A = \frac{F^2 + G^2}{r^2}$.	B = Deflection.	Ratio of A to B.
2	2.206	0.551	11.80	21.4
3	1.145	0.127	2.80	22.0
4	0.915	0.0572	1.28	22.4

The theory is verified in that the ratio between the observed and theoretical relative values is practically constant. It is interesting to note that if the observed values are tested in a similar manner, but assuming the inverse square law, the results for the last column differ as much as 130 per cent.

The results presented in this paper certainly demonstrate that the method of producing known relative sound intensities is a practicable one, although attended with some difficulty of operation and limited both by the absorption of the instrument to be calibrated and the inconstancy of the source of sound. So far as the experiments with the Rayleigh disk are concerned, the results may be regarded in either of two ways. One may consider that they show the theory to be correct, assuming the deflection of the Rayleigh disk to be proportional to the energy. The writers, however, regard the theory of the acoustic shadow as more reliable than the assumption as to the action of the disk. The experiments call attention to certain errors in the operation of the disk which are unavoidable in any measuring instrument which utilizes resonance.

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THE POTENTIAL AND ELECTROSTATIC FORCE IN THE FIELD OF TWO METAL SPHERICAL ELECTRODES.[1]

III. CHARGE AND CAPACITY.

BY GEO. R. DEAN.

THE surface integral of normal force intensity over any closed surface drawn in the field is equal to 4π times the total charge of electricity inside the surface. When the charges are equal, all the lines of force start from one sphere and end on the other; they all cross the plane of zero potential at right angles to that plane. Hence, if G_0 denote the electrostatic force at any point of the plane, Q the charge on either sphere, and z the distance of the point from the common axis, we have

$$4\pi Q = \int_0^\infty 2\pi z \cdot G_0 \cdot dz. \tag{52}$$

The potential at any point (see 46, Part I.) is given by

$$V = \frac{V_1}{2} 2\sqrt{(\cosh\eta - \cos\xi)} \sum_0^\infty \frac{\sinh(m+\frac{1}{2})\eta}{\sinh(m+\frac{1}{2})\alpha} P_m(\cos\xi) e^{-(m+1/2)\alpha}, \tag{53}$$

the potentials of the spheres being $V_1/2$ and $-V_1/2$.
The electrostatic force at any point of plane of zero potential is

$$G_0 = \left(\frac{\partial V}{\partial r}\right)_{n=0} = \left(\frac{\partial V}{\partial \eta}\frac{\partial \eta}{\partial r} + \frac{\partial V}{\partial \xi}\frac{\partial \xi}{\partial r},\right)_{n=0}, \tag{54}$$

and since V is constant over the plane, this reduces to

$$G_0 = \left(\frac{\partial V}{\partial \eta} \cdot \frac{\partial \eta}{\partial r}\right)_{z=0},$$

r and z having same meaning as in § I.
Differentiating (1),

$$\frac{\partial V}{\partial \eta} = \frac{V_1}{2} \frac{\sinh\eta}{\sqrt{2(\cosh\eta - \cos\xi)}} \sum_0^\infty \frac{\sinh(m+\frac{1}{2})\eta}{\sinh(m+\frac{1}{2})\alpha} P_m(\cos\xi) e^{-(m+1/2)\alpha}$$

$$+ \frac{V_1}{2}\sqrt{2(\cosh\eta - \cos\xi)} \sum_0^\infty \frac{(m+\frac{1}{2})\cosh(m+\frac{1}{2})\alpha}{\sinh(m+\frac{1}{2})\alpha} P_m(\cos\xi) e^{-(m+1/2)\alpha} \tag{55}$$

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[1] For Part I. and Part II. see THE PHYSICAL REVIEW, p. 459, December, 1912.

Then

$$\frac{\partial V}{\partial \eta}_{n=0} = \frac{V_1}{2}\sqrt{2(1-\cos\xi)}\sum_0^\infty \frac{(m+\frac{1}{2})P(\cos\xi)\, e^{-(m+1/2)\alpha}}{\sinh(m+\frac{1}{2})\alpha}. \tag{56}$$

From (3), Part I.,

$$\eta = \tfrac{1}{2}\log\left[\frac{z^2+(c+r)^2}{z^2+(c-r)^2}\right];$$

$$\frac{\partial\eta}{\partial r} = \frac{c+r}{z^2+(c+r)^2} + \frac{c-r}{z^2+(c-r)^2},$$

and

$$\left(\frac{\partial\eta}{\partial r}\right)_{r=0} = \frac{2c}{z^2+c^2} = \frac{1-\cos\xi}{c}.$$

Then

$$G_0 = \frac{2V}{c}\sin^2\frac{\xi}{2}\sum_0^\infty\frac{(m+\frac{1}{2})P_m(\cos\xi)e^{-(m+1/2)\alpha}}{\sinh(m+\frac{1}{2})\alpha} \tag{57}$$

For points in the plane of symmetry,

$$\tan\tfrac{1}{2}\xi = \frac{c}{z}.$$

Then

$$z^2 = c^2\cot^2\tfrac{1}{2}\xi,$$

and

$$2z\,dz = -\,c^2\cot\tfrac{1}{2}\xi\,\mathrm{cosec}^2\tfrac{1}{2}\xi\cdot d\xi.$$

When $z = 0$, $\xi = \pi$, and when $z = \infty$, $\xi = 0$. Substituting value of G_0 and $2z\,dz$ in (52),

$$4\pi Q = -2\pi c V_1\int_0^\pi \cos\tfrac{1}{2}\xi dz\cdot\sum_0^\infty\frac{(m+\frac{1}{2})P_m(\cos\xi)}{\sinh(m+\frac{1}{2})\alpha}e^{-(m+1/2)\alpha},$$

$$= -\,2\pi c V_1\sum_0^\infty\frac{(m+\frac{1}{2})e^{-(m+1/2)\alpha}}{\sinh(m+\frac{1}{2})\alpha}\int_0^\pi P_m(\cos\xi)\cos\tfrac{1}{2}\xi d\xi. \tag{58}$$

We have now to evaluate the integral,

$$\int_0^\pi P_m(\cos\xi)\cos\tfrac{1}{2}\xi d\xi.$$

On page 9, Todhunter's Functions of Laplace, etc., we find

$$P_m(\cos\xi) = 1 - \frac{(n+1)n}{1^2}\sin^2\frac{\xi}{2} + \frac{(n+2)(n+1)n(n-1)}{1^2\cdot 2^2}\sin^4\frac{\xi}{2} - \cdots$$

by means of which we find

$$\int_0^\pi P_m(\cos\xi)\cos\tfrac{1}{2}\xi d\xi = \frac{2}{2m+1}.$$

Substituting in (58), we get

$$4\pi Q = -\,2\pi c V_1\sum_0^\infty\frac{e^{-(m+1/2)\alpha}}{\sinh(m+\frac{1}{2})\alpha},$$

or

$$Q = cV_1 \sum_0^\infty \frac{1}{1 - e^{(2m+1)\alpha}}$$

$$= cV_1 \left\{ \frac{1}{1 - e^{\alpha}} + \frac{1}{1 - e^{5\alpha}} + \frac{1}{1 - e^{7\alpha}} + \cdots \right\}.$$

The value of α is given in Part I., x being distance between surfaces, and a the radius:

$$\alpha = \log_e \frac{\sqrt{x^2 + 4ax} + x}{\sqrt{x^2 + 4ax} - x},$$

and

$$2c = \sqrt{x^2 + 4ax}.$$

For the capacity between the spheres, whose difference of potential is V we have

$$C = \frac{Q}{V} = \frac{\sqrt{x^2 + 4ax}}{2} \left\{ \frac{1}{1 - e^{\alpha}} + \frac{1}{1 - e^{5\alpha}} + \frac{1}{1 - e^{7\alpha}} + \cdots \right\}.$$

This is also the capacity of a sphere of potential V against a conducting plane of potential *zero*, *i. e.*, capacity of sphere to ground. Here $x/2$ is the distance from plane to surface of sphere. If D be the distance from plane to center,

$$C = \sqrt{D^2 - 2aD} \left\{ \frac{1}{1 - e^{\alpha}} + \frac{1}{1 - e^{5\alpha}} + \frac{1}{1 - e^{7\alpha}} + \cdots \right\}.$$

The charge on the sphere $\eta = \alpha$ is given by

$$Q_\alpha = \frac{1}{4\pi} \int \left(\frac{\partial V}{\partial v} d\omega \right)_{\eta=\alpha}, \tag{59}$$

where dv is an elemental arc of the line of force, and $d\omega$ is the elemental area on the surface of the sphere. The element of the line of force coincides at the surface with the element of the circle ξ = constant. Hence, we have

$$\frac{\partial V}{\partial v} = \frac{\partial V}{\partial \eta} \cdot \frac{\partial \eta}{\partial v}, \tag{60}$$

$\partial V/\partial \xi$ being zero at that point.

From (20), Part I.,

$$x = \frac{c \sinh \eta}{\cosh \eta - \cos \xi}, \quad y = \frac{c \sin \xi}{\cosh \eta - \cos \xi}. \tag{61}$$

Taking ξ constant,

$$dx = \frac{c(1 - \cosh \eta \cos \xi)}{(\cosh \eta - \cos \xi)^2}, \quad dy = \frac{c \sinh \eta \sin \xi}{(\cosh \eta - \cos \xi)^2}. \tag{62}$$

Then

$$dv = \sqrt{(dx)^2 + (dy)^2} = \frac{c\,d\eta}{\cosh\eta - \cos\xi},$$

and

$$\frac{\partial\eta}{\partial v} = \frac{\cosh\eta - \cos\xi}{c}. \tag{63}$$

For the element area of the surface, we have

$$d\omega = 2\pi y\,d\sigma, \tag{64}$$

$d\sigma$ being an element of a great circle.

Taking η constant, we find, by means of (61)

$$d\sigma = \frac{c\,d\xi}{\cosh\eta - \cos\xi}, \tag{65}$$

$$d\omega = \frac{2\pi c^2 \sin\xi\,d\xi}{(\cosh\eta - \cos\xi)^2}, \tag{66}$$

$$\frac{\partial\eta}{\partial v}\cdot d\omega = \frac{4\pi c \sin\xi\,d\xi}{2(\cosh\eta - \cos\xi)}, \tag{67}$$

$$\left(\frac{\partial\eta}{\partial v}d\omega\right)_{\eta=\alpha} = \frac{4\pi c \sin\xi\,d\xi}{2(\cosh\alpha - \cos\xi)}. \tag{68}$$

When the potential of the sphere $\eta = \alpha$ is V_1, and that of the sphere $\eta = -\alpha$ is zero, the potential at any point is given by

$$V = V_1\left\{\sqrt{2(\cosh\eta - \cos\xi)}\sum_0^\infty \frac{\sinh(m+\frac{1}{2})(\alpha+\eta)}{\sinh(m+\frac{1}{2})(2\alpha)}P_m(\cos\xi)e^{-(m+1/2)\alpha}\right.. \tag{69}$$

$$\begin{aligned}\frac{\partial V}{\partial\eta} = V_1\Bigg[&\frac{\sinh\eta}{\sqrt{2(\cosh\eta - \cos\xi)}}\sum_0^\infty\frac{\sinh(m+\frac{1}{2})(\alpha+\eta)}{\sinh(2m+1)\alpha}P_m(\cos\xi)e^{-(m+1/2)\alpha}\\ &+\sqrt{2(\cosh\eta - \cos\xi)}\sum_0^\infty\frac{\cosh(m+\frac{1}{2})(\alpha+\eta)}{\sinh(2m+1)\alpha}\\ &\times(m+\tfrac{1}{2})P_m(\cos\xi)e^{-(m+1/2)\alpha}\Bigg],\end{aligned} \tag{70}$$

$$\begin{aligned}\left(\frac{\partial V}{\partial\eta}\right)_{\eta=\alpha} = V_1\Bigg[&\frac{\sinh\alpha}{\sqrt{2(\cosh - \cos\xi)}}\sum_0^\infty P_m(\cos\xi)e^{-(m+1/2)\alpha}\\ &+\sqrt{2(\cosh\alpha - \cos\xi)}\\ &\sum_0^\infty(m+\tfrac{1}{2})\coth(2m+1)\alpha\cdot P_m(\cos\xi)e^{-(m+1/2)\alpha}\Bigg],\end{aligned} \tag{71}$$

$$\begin{aligned}\left(\frac{\partial V}{\partial\eta}\right)_{\eta=-\alpha} = V_1\Bigg[&\sqrt{2(\cosh\alpha - \cos\xi)}\\ &\times\sum_0^\infty(m+\tfrac{1}{2})\coth(2m+1)\alpha P_m(\cos\xi)e^{-(m+1/2)\alpha}\Bigg],\end{aligned} \tag{72}$$

Now

$$\sum_0^\infty P_m(\cos\xi)e^{-(m+1/2)\alpha} = \left\{2(\cosh\alpha-\cos\xi)\right\}^{-1/2}.$$

Therefore

$$\left(\frac{\partial V}{\partial\eta}\right)_{\eta=\alpha} = V_1\left[\frac{\sinh\alpha}{2(\cosh\alpha-\cos\xi)}\right.$$

$$+\sum_0^\infty(m+\tfrac{1}{2})\coth(2m+1)\alpha \tag{73}$$

$$\left.\times e^{-(m+1/2)\alpha}P_m(\cos\xi)\sqrt{2(\cosh\alpha-\cos\xi)}\right].$$

To find Q_α, we have then to evaluate the following integrals:

$$\int_0^\pi\frac{\sin\xi d\xi}{[2(\cosh\alpha-\cos\xi)]^2} = -\tfrac{1}{4}\Big[(\cosh\alpha-\cos\xi)^{-1}\Big]_0^\pi = \frac{1}{2\sin^2 h\alpha}; \tag{74}$$

$$\int_0^\pi\frac{P_m(\cos\xi)\sin\xi d\xi}{\sqrt{2(\cosh\alpha-\cos\xi)}} = \int_0^\pi\sum_0^\infty e^{-(\eta+1/2)\alpha}P_\eta(\cos\xi)P_m(\cos\xi)\sin\xi d\xi. \tag{75}$$

$$\int_0^\pi[P_m(\cos\xi)]^2(\sin\xi d\xi) = \int_{+1}^{-1}-[P_m(x)]^2dx = +\frac{2}{2m+1};$$

$$\int_0^\pi P_n(\cos\xi)P_m(\cos\xi)\sin\xi d\xi = -\int_{-1}^{+1}P_n(x)P_m(x)dx = 0.$$

Therefore (75) becomes

$$\int_0^\pi\frac{P_m(\cos\xi)\sin\xi d\xi}{\sqrt{2(\cosh\alpha-\cos\xi)}} = \frac{2e^{-(m+1/2)\alpha}}{2m+1}.$$

Accordingly, substituting in (59), we find

$$Q_\alpha = cV_1\left[\operatorname{cosech}\alpha + 2\sum_0^\infty\coth(2m+1)\alpha\cdot e^{-(m+1/2)\alpha}\right],$$

$$Q_{-\alpha} = 2cV_1\sum_0^\infty\operatorname{cosech}(2m+1)\alpha\cdot e^{-(m+1/2)\alpha}.$$

OPTIMUM WAVE-LENGTH IN RADIOTELEGRAPHY.

By A. H. Taylor.

GIVEN a definite maximum of available power, and a fixed aerial system, the question as to a suitable choice of wave-length presents itself. The fairly well established facts influencing the choice of wave-length are as follows: First, the radiation of the aerial is greater for short waves. Second, the aerial current with a given potential maximum is greater for short waves. Third, the absorption is greater for short waves than for long ones. Fourth, the absorption increases with the distance.

Austin has investigated the formula[1] $I = \frac{kh_1h_2I_1e^{-\alpha D/\sqrt{\lambda}}}{\lambda D}$, where

h_1 = height of sender, h_2 = height of receiver,
λ = wave-length, I_1 = sending aerial current,
I_2 = received current through 25 ohms equivalent resistance,
D = distance between stations,
α = .0015 for transmission over salt water.

The lengths are all in kilometers and the currents in amperes. The constant K has been determined by Austin to be 4.25 for the flat-topped aerials on the cruisers used in his experiments.

As far as I am aware, no publication has been made of similar long distance experiments over land, from which K and α could be determined for any given character of soil and type of aerial. It is known that the absorption over land[2] is in general greater, although the surface absorption of the earth[3] has not been separated from the atmospheric absorption which produces the great difference between day and night messages.

The constant K would depend on local conditions at sending and receiving stations, and for properly located land stations should probably be a little larger than 4.25. The other factors of the formula have been found to be nearly correct for overland transmission and it is natural to assume that the received current will follow the same general law of variation over land distances as over the sea, especially over the flat prairie where the author's experiments are being undertaken.

[1] Austin, Bull. Bur. St., Vol. 7, No. 3, 1911.

[2] Zenneck, Hun. d. Phys., 23, 1907.

[3] Pierce, Wireless Telegraphy, p. 131, 1910 edition.

It has been shown by Austin[1] and others that within practical ranges of usefulness, nearly all detectors respond approximately in such a manner that the audibility A of the signal is proportional to the square of the received current. This makes the audibility inversely proportional to D^2, and directly proportional to i_1^2/λ^2, neglecting the absorption. The installation at the University of North Dakota is at present limited to 1 kw. output, furnished by a Type E self-controlling transformer on a 110-volt 60-cycle circuit. The primary condenser usually had a capacity of .02 Mf. and was coupled inductively to the aerial, loosely enough so

TABLE I.

$\lambda = .400$ km., $C_1 = .01$ Mf.

P Kilowatts.	I Aerial Current.	i^2/λ^2P.
.220	2.1	138
.320	2.6	132
.400	2.85	128
.500	3.1	120
	Average. . 130	

TABLE II.

$\lambda = .457$ km., $C_1 = .02$ Mf.

P Kilowatts.	I Aerial Current.	i^2/λ^2P.
.370	3.3	141
.450	4.0	169
[illegible]	4.3	143
.870	5.7	177
	Average. . 157	

TABLE III.

$\lambda = .560$ km., $C_1 = .03$ Mf.

P Kilowatts.	I Aerial Current.	i^2/λ^2P.
.580	3.8	79
.685	4.2	82
.850	4.8	86
	Average. . . 82	

TABLE IV.

$\lambda = .590$ km., $C_1 = .03$ Mf.

P Kilowatts.	I Aerial Current.	i^2/λ^2P.
.570	3.75	70.5
.700	4.2	72
.800	4.6	76
1.050	5.4	76
	Average. . 74	

TABLE V.

$\lambda = .660$ km., $C_1 = .02$ Mf.

P Kilowatts.	I Aerial Current.	i^2/κ^2P.
.300	3.0	69
.380	3.4	69.5
.460	3.65	66.5
.620	3.9	56
	Average. . 65	

TABLE VI.

$\lambda = .730$ km., $C_1 = .01$ Mf.

P Kilowatts.	I Aerial Current.	i^2/λ^2P.
.252	2.35	41
.400	3.25	49.5
.500	3.45	44.6
.600	3.7	43
	Average. . 44.5	

TABLE VII.

$\lambda = .390$ km. Free aerial operated by induction coil on D.C.

P Kilowatts.	I Aerial Current.	i^2/λ^2P.
.140	2.85	382
.120	2.6	372
.065	1.7	315
	Average. . 356	

TABLE VIII.

$\lambda = .390$ km. Induction coil on A.C. 60-cycle.

P Kilowatts.	I Aerial Current.	i^2/λ^2P.
.037	1.35	323

TABLE IX.

Wave-length.	Av. i^2/λ^2P.	i/λ for P = 1Kw.
.730	44.5	6.65
.660	65.	8.06
.590	74.	8.6
.560	82.	9.05
.457	157.	12.55
.400	130.	11.4

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[1] Austin, Bull. Bur. St., Vol. 6, No. 4, 1910.

that the two waves transmitted did not differ more than 7 per cent. The aerial was of the inverted L type, 27 meters high, 3 meters wide, 40 meters long, of 6 No. 14 aluminum wires. The vertical lead was a single No. 12 stranded copper wire. Its free wave-length was 390 meters. The following tables show the dependence of the factor i_1^2/λ^2 upon the power and upon the wave-length.

There was some irregularity in the action of the zinc spark gap which may account for the variation in the factor $i^2/\lambda^2 \cdot P$, but as a first approximation I propose to call this factor a function of λ but not of P. It represents the intensity or energy radiation factor for a given wave-length and determines the audibility of the signals received at short distances, where the absorption may be neglected. For minimum energy required to produce audible signals at moderate distances, it is evident that the wave-length .457 km. is best. This conclusion has been verified by an operator at Inkster, N. D., thirty-five miles distant, receiving messages sent from the university at 100 watts power.

It is hardly fair to include the data for the free aerial, operated by the secondary of an induction coil directly connected to the aerial, since the maximum amount of energy which can be radiated by the system is reached in the case of direct current and interrupter at 140 watts and with alternating current at 65 watts. This low limit is due in the latter case to arcing at the spark gap and in the former case to limiting insulation. The very high value of $i^2/\lambda^2 \cdot P$ which is obtained by this simple coupling recommends it for very short range work, not requiring much power. It is interesting to note that this factor is not very different for D.A. and A.C. although the spark frequency is much higher for A.C.

In regard to the data on the coupled aerial it is to be noted that at $\lambda = .400$ km. there were so few turns used on the secondary helix that the degree of coupling was insufficient. The radiant efficiency factor increases to a maximum and then falls off rapidly as the wave-length approaches the free wave-length of the aerial where the coupling is zero.

Austin puts $I_2 = 4 \times 10^{-5}$ through a receiving aerial of 25 ohms equivalent resistance as sufficient for good communication. Assuming this value we have

$$D = \frac{4.25 k_1 h_2 \epsilon^{-\frac{4D}{\sqrt{\lambda}}} i_1}{4 \times 10^{-5} \quad \lambda}$$

for the maximum distance for good working. This can be readily solved by successive approximations, and it is at once evident that this distance is not necessarily greater for the short waves which give the strongest signals at closer ranges. The factor i_1/λ, determined for the maximum power of the station, may be regarded as the *distance factor*, since it would determine the maximum distance through non-absorbing space, as approximately, for night transmission at sea. For such transmission the shortest wave possible with proper coupling will again give the best result. Also, if the heights h_1 and h_2 are not sufficient to give a range of over say 200 km. the absorption factor $\epsilon^{-aD/\sqrt{\lambda}}$ will be so small that the distance factor i/λ, greatest for short waves, will control the result. The same argument holds for very low power stations. As a sample calculation, the working distance of this station, sending at 1 kw. to a similar aerial is calculated for different observed values of i/λ at different wave-lengths.

Assuming temporarily $K = 4.25$, $\alpha = .0015$:

λ	i/λ maximum	D (K. M.)
.457	12.53	343
.560	9.05	320
.590	8.66	312
.660	8.06	307
.730	6.65	272

It will be noted that the sending range for 1 kw. is the maximum obtainable with the shortest practicable wave-length, 460 meters. $\lambda = .450$ to .600 probably would give very nearly the same working range for this station to a similar one. It is to be noted that the wave-length $\lambda = .457$, for which $i^2/\lambda^2 \cdot P = 157$ will give at close range twice the audibility of $\lambda = .590$, for which $i^2/\lambda^2 \cdot P = 74$, although their maximum working distance is not much different.

If we assume with Austin that 1×10^{-5} amperes give the limit of extreme audibility, the wave-length for maximum distance is still .457, as Table XI. shows.

λ	D
.457	702
.560	640
.590	648
.660	643
.730	617

Table XII. gives the variation of extreme range of audibility with wave-length for $P = 4$ kw., on the assumption that the quantity $i_1^2/\lambda^2 \cdot P$ is constant for λ constant, and that i_1/λ is proportional to $\sqrt{P}$.

λ	D
.457	905
.560	860
.590	859
.660	865
.730	828

These conditions could probably be realized closely with a transformer of the same type, but of higher power. There is, within the limits of experimental error, but little preference for any wave between $\lambda = .457$ and $\lambda = .660$, although the audibilities at close range have the ratio $157/65 = 2.4$ in favor of $\lambda = .457$.

On increasing the height of the aerials longer waves will obviously be necessary, but the concomitant increase in power and range will also throw the optimum towards longer wave-lengths. As far as this station is concerned, it is evident that the shortest wave-length consistent with good coupling gives the best result under all conditions, *unless* the absorption is very much greater than the assumed value, in which case a somewhat longer wave will be better. Experiments are now under way which will test these points as far as is possible for a station of this low power.

UNIVERSITY OF NORTH DAKOTA.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SIXTY-SIXTH MEETING.

A REGULAR meeting of the Physical Society was held in the Sloane Physical Laboratory of Yale University, New Haven, on Saturday, March 1, 1913, with President Peirce in the chair.

The following papers were presented:

The Actinodielectric Effect of Sulphur. CHESTER A. BUTMAN.

Note on the Ageing Effect as an Influence on the Number and the Velocities of Photo Electrons from Thin Cathode Films. OTTO STUHLMANN, JR.

A Powerful Source of Ultraviolet Light for Photo-electric Work. ALBERT W. HULL AND ANCEL ST. JOHN.

On the Beaded Character of the Deflected Cathode Ray Line as Revealed by Instantaneous Photographs taken at Short Range. CHARLES T. KNIPP.

Dynamic Characteristics of the Wehnelt Rectifier. R. A. PORTER.

The Dimensions of Electric and Magnetic Quantities. DAVID L. WEBSTER.

The Forced Vibration of a Circular Plate. A. G. WEBSTER.

On the Occurrence of Hydrogen Lines in the Spectrum of the "Tube-Arc," and Application to the Nature of Enhanced Lines. ARTHUR S. KING.

The Dispersion of Metals. LYNDE P. WHEELER.

On a Fundamental Law of Mechanics. H. M. DADOURIAN.

A New Vacuum Gauge of Extreme Sensitiveness. I. LANGMUIR.

The Extinguishment of Phosphorescent Light. CHESTER A. BUTMAN.

Adjourned at 1 P. M.

ERNEST MERRITT,
Acting Secretary.

ON THE BEADED CHARACTER OF THE DEFLECTED CATHODE RAY LINE AS REVEALED BY INSTANTANEOUS PHOTOGRAPHS TAKEN AT SHORT RANGE.[1]

BY CHAS. T. KNIPP.

J. J. THOMSON[2] in a recent article on "Positive Rays" calls attention to two kinds of beading. One he styles atomic, and the other is found on the molecular lines. The first is explained by some of the atoms having a double charge. This is shown on the photographic plate as a beak or hook-like projection extending beyond the head of the parabola and towards the vertical axis. The second, the molecular beading, extends away from the vertical axis. His photographs often show as many as three and four beads on a single molecular line. As to the explanation of the latter he is not so sure, but suggests two possible causes. The most likely of the two is that the cathode rays by impact on the walls of the discharge vessel give rise to secondary rays of great ionizing power, producing positive carriers having less energy than those which started from the negative glow.

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[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] Phil. Mag., Vol. 24, p. 235, Aug., 1912.

Recently while photographing retrograde rays the writer was impressed with the beaded character of the cathode ray line in many of the photographs. The plate was mounted in the apparatus so that it could be revolved. Thus instantaneous photographs were obtained, each impression on the plate being due nominally to a single discharge of the induction coil. The cathode rays after passing through the tube (3 cm. long by about .2 mm. internal diameter), were deflected by a strong electrostatic field. The distance of the photographic plate from the mouth of the tube was 1.62 centimeters.

Under these conditions the instantaneous photographs generally reveal a central undeflected spot, and *one*, *two*, and in several instances, *three* and *four* spots all in a straight line along the direction of the electrostatic field, which in this instance was horizontal. Time exposures show the central spot clearly, and a continuous though in general beaded line. The positions of the spots on the instantaneous photographs are more or less rhythmical and if superposed, as in a time exposure, may readily produce the beaded line.

The explanation given by Thomson for the atomic carriers does not apply in this case, for we are dealing with the elementary charge. While the secondary ray explanation may apply, it seems to the writer that the true explanation for the case of the beaded cathode ray line is to be found in the oscillatory character of the electric discharge. Clearly the rays due to subsequent oscillations would fall off in energy and be recorded on the photographic plate as groups of electrons moving with successively diminished velocities.

A number of photographs accompany the paper.

Dynamic Characteristics of the Wehnelt Rectifier.[1]

By R. A. Porter.

THE Wehnelt rectifier is a highly evacuated discharge tube, one electrode of which is glowing calcium oxide, while the other may be of any kind of conductor. In a series of articles Wehnelt[2] has described and explained many of the peculiarities of such a discharge tube. The voltage-current curves for voltages that have been applied long enough for equilibrium to become established, *i. e.*, the "static characteristics" of this tube, have been described by S. Marsh.[1] Although this tube is of special interest as a rectifier of alternating currents, there has been heretofore no investigation of the current-voltage curves for such a current, *i. e.*, of the "dynamic characteristics." Simon[2] has shown that in the case of other conductors these two sets of curves are not the same.

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[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] A. Wehnelt, Verh. d. D. Physik. Ges., 5, pp. 255–258, 1903; Ann. d. Phys. (4), 14, pp. 425–468, 1904; (4) 19, pp. 138–156, 1906; Physik. Zeitschr., 9, pp. 134–135, 1908; A. Wehnelt and F. Jentzsch, Verh. d. D. Physik. Ges., 10, pp. 605–615, 1908; Ann. d. Phys. (4), 28, pp. 537–552, 1909.

The dynamic characteristics were obtained from oscillograph records by measuring corresponding instantaneous values of current through the discharge tube and voltage between the terminals of the tube. From such records dynamic characteristics were plotted which show the effects produced respectively by variation in the applied effective voltage, the series resistance, the pressure in the tube, the temperature of the oxide electrode, the atmosphere in which the discharge occurs, and the frequency of the applied voltage.

The results may be summarized as follows: (1) The curve which shows the relation between applied voltage and current for the half-period during which luminous discharge takes place forms a loop (hysteresis). The luminous discharge begins at a certain voltage, called the ignition voltage; the characteristic then follows closely the path of the static characteristic of the glow discharge, returns from the point of maximum current at first along a line close to the curve for increasing current, but at smaller current values the voltage is considerably lower. (2) Within the range studied, the ignition voltage decreases as the pressure increases. (3) The ignition voltage increases with increase of the frequency of the applied A. C. voltage. (4) At low pressures, 0.001–0.003 mm., the characteristics are similar, whether the tube filling be of air, hydrogen, or oxygen. (5) At this low pressure the presence of mercury vapor in the discharge tube lowers all voltages to about one third the values which they have when mercury is excluded. (6) At larger pressures, 0.03–0.05 mm. the characteristics for all four gases are approximately the same. (7) When a constant voltage sufficiently high to produce luminous discharge has been applied to the tube, a definite length of time must elapse before luminous discharge begins. The length of this delay depends on the voltage and pressure; it is independent of the temperature of the cathode. (8) Observations with iodine as an atmosphere show an anomalous effect, in that the hysteresis under certain conditions is negative, and when a constant direct-current voltage is applied a perfectly regularly interrupted current may be obtained.

The peak in the voltage curve and the hysteresis effect can be explained on the ground that a certain length of time must elapse before a new potential gradient in the tube can be established.

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[1] S. Marsh, Dissertation, Göttingen, 1909.

[2] H. Th. Simon, Physik. Zeitschr., 6, pp. 297–319, 1905; 7, pp. 433–445, 1906.

The phenomena of the delay in ignition and of the production of an interrupted current by iodine are being further investigated.

Syracuse University,
Syracuse, N. Y.,
February 4, 1913.

The Dimensions of Electric and Magnetic Quantities.[1]

By David L. Webster.

As it is well known, any attempt to assign to electric or magnetic quantities dimensions expressed in terms of mass, length, and time, leads to fractional exponents in the dimensional formulas. For example, electric charge has the dimensions $m^{\frac{1}{2}}l^{\frac{3}{2}}t^{-1}$ in the electrostatic system, or $m^{\frac{1}{2}}l^{\frac{1}{2}}$ in the electromagnetic system, the difference being "explained" by the unknown dimensions of the electric and magnetic susceptibilities.

To remove the difficulty of interpretation of these fractional exponents, it is proposed to abandon mass as a fundamental concept, replacing it by quantity of electricity (e), and taking electric susceptibility, as indicated by the electron theory, to be a pure number. From this assumption, with the inverse square law of electrostatics, one may readily obtain the following dimensions: electric force, el^{-2}; potential, el^{-1}; current et^{-1}; resistance, $l^{-1}t = v^{-1}$; etc. And with the aid of Biot and Savart's law one obtains: magnetic force, el^{-2}; pole strength e; etc. And introducing Newton's laws of motion, one obtains: mechanical force, e^2l^{-2}; mass, $e^2l^{-3}t^2 = e^2l^{-1}v^{-2}$; etc.

This last formula suggests at once the concept of electromagnetic mass, the idea that all mass is of that nature, and the proportionality of the mass of an electron, directly to the square of its charge, and inversely to its radius and the square of the velocity of light.

A further simplification is introduced if time is considered as merely the distance that light may travel, in which case each t in the formulas is replaced by l, and the v's are all omitted.

A Powerful Source of Ultraviolet Light for Photo-Electric Work.[1]

By Albert W. Hull and Ancel St. John.

The source of light used is a vacuum discharge tube of the general form used by Lyman in his vacuum spectrograph, containing hydrogen at a low pressure. Experiments were made to find how the photo-electric efficiency depended on (1) length of capillary; (2) diameter of capillary; (3) pressure of gas; (4) current density in tube; (5) potential and frequency applied to tube.

The results may be summarized as follows:

(1) The efficiency increases slowly with length of capillary (50 per cent. increase in efficiency for 300 per cent. increase in length) in accordance with

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[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

the known absorption of ionized hydrogen. (2) The diameter of capillary, between the limits of 3 and 13 mm. makes no difference if the current is kept constant. (3) Pressure of the gas, between 2 and 15 mm., makes no difference if current is constant. (4) A strictly linear relation between current (in discharge tube) and photo-electric effect was found. (5) Various forms of interrupters and a powerful Tesla coil gave comparatively small photo-electric effects, corresponding to the smallness of current which they furnished. A transformer on a 110-volt A. C. circuit gave currents of 0.2 amp. through the tube, and produced a photo-electric current of 1.3×10^{-6} amp. per cm.^2 on a platinum electrode in a vacuum of 1/1000 mm. For comparison, a mercury-in-quartz lamp was set up at a distance of 10 cm. from the plate, and when running steadily at 3.2 amp. gave a photo-current of 5.5×10^{-9} amp./cm.^2 The hydrogen tube, therefore (with fluorite window), gives a photo-electric effect 250 times as large as the mercury arc.

WORCESTER POLYTECHNIC INSTITUTE,
February 14, 1913.

NOTE ON THE AGING EFFECT AS AN INFLUENCE ON THE NUMBER AND THE VELOCITIES OF PHOTO-ELECTRONS FROM THIN CATHODE FILMS.[1]

BY OTTO STUHLMANN, JR.

WHILE investigating the photo-electric properties of thin films deposited from a platinum cathode, for difference in photo-electric effect caused by emergent (E) and incident (I) light,[2] the writer found that films whose thickness were comparable with 10^{-7} cm., showed a gradual increase in the ratio of the emergent to the incident effect (E/I) as time went on. In one case this ratio E/I for a particular platinum film successively changed from 1.08 to 1.10 to 1.12 in three succeeding days. It gave very erratic and unreliable results, averaging 1.07 on the day previous to the taking of the first of the above values. Under normal conditions, the ratio E/I increases to a constant value as the thickness of the film decreases. It therefore seems probable that the film undergoes some sort of change as time goes on. Just after sputtering it was seemingly in an unstable state, gradually decreasing in thickness. Three days afterward it reached a stable state, giving a continuous value for $E/I = 1.12$.

Dike[3] in a recent paper discussing the velocities of photo-electrons emitted from films of platinum, similarly deposited as above, found that the velocities increased as the thickness of the film decreased. He also found that upon testing a film of platinum for 24 consecutive hours or more after depositing, he attained with constant illumination, the unusual result of a continuous increase in the velocities of the emitted electrons, as time went on. Since an increase in velocity of the emitted electrons is accompanied by a decrease in thickness of the emitting film, it seems plausible that the aging of the film was accompanied by a decrease in its thickness. This would account for the resulting increase in velocity as time went on.

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[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] Phil. Mag., Aug., 1910, p. 331; Pt. II., Dec., 1911, p. 854.

[3] PHYS. REVIEW, XXXIV., p. 459, 1912.

Let us examine why the above explanations of decrease in thickness of cathode films with time is the most probable one. The films are obtained by particles of the metal striking the glass or quartz surface and adhering. These particles are more or less loosely connected and can be considered as more or less irregular spheres packed into a semitransparent layer. If the discharge is very rapid the film becomes very spongy and can easily be removed by wiping with a finger. A slow discharge, on the other hand, produces a more compact and dense surface. Patterson[1] found, while working on the specific resistance of similar metal films, that heating them for some time by an electric current reduced their resistance in some cases more than 50 per cent., although he found it impossible to reduce the resistance to that of ordinary platinum. He also showed that heating them in a current of dry air or allowing them to lie idle for some days produced the same effect but not in so marked a degree.

Evidently, by heating, the particles settle into a more and more compact mass and consequently their resistance decreases.

Beilly[2] found that heating metal films at a temperature much below the melting point, conferred sufficient freedom on the molecules to enable them to behave as the molecules of the liquid metal would do. They arrange themselves under the influence of surface tension either in films or under certain other conditions into drop-like granular forms.

Evidently, allowing the films to age gradually has the same effect but not to so marked a degree as aging them rapidly by passing a heating current through the films. Through aging the semi-transparent granular structure will gradually sink into a more and more homogeneous layer and gradually decrease in thickness as the aging continues.

Evidently aging decreases the thickness of the films.

The ratio of the emergent to the incident photo-electric effect increases with the aging of the film, hence the decrease in thickness of the film would account for the results quoted.

The velocity of the emitted electrons increases with aging. Here again, aging decreasing the thickness of the film accounts for the gradual increase in velocities of the electrons, as time goes on.

University of Pennsylvania,
Philadelphia, Pa.

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[1] Phil. Mag., Dec., 1902, p. 666.

[2] Pro. Roy. Soc., 72, 1903, p. 226.

TRIAL OF A METHOD FOR OBTAINING EQUALITY OF TEMPERATURE, AT HIGH TEMPERATURES.[1]

BY C. F. LORENZ.

IN the simplest case two filaments, say wires of tungsten and platinum drawn through the same die, are similarly mounted in a gas-chamber which communicates with a sensitive aneroid pressure-indicator. Resistances in circuit with the two filaments are adjusted until there is no alteration of pressure when the filaments are switched on alternately, and equality of pressure is taken to indicate equality of temperature. Though the loss of energy of a filament through radiation is proportional to the diameter of the filament, the loss of energy through the cooling action of the surrounding gas is not proportional to the diameter, but varies much more slowly than the latter; this fact makes relatively large percentage errors in equality of diameters permissible, which is especially important when one or the other of the substances compared is one that is not capable of being drawn through a die, as, for example, carbon.

A comparison of tungsten and platinum was made with the result that these substances are very closely at a color match when they are at indicated equality of temperature.

PHYSICAL LABORATORY,
NATIONAL ELECTRIC LAMP ASSOCIATION,
CLEVELAND, O.

THE SERIES SYSTEM IN THE SPECTRA OF CA, SR, AND BA.[1]

BY F. A. SAUNDERS.

IN each element there are systems of single-line series, of pair series, and of triplet series. In the two latter systems both wide and narrow groups occur.

In regard to the single-line series, recent photographs in Ca have shown that the series formerly given as $SL1$[2] can be obtained with heavy reversals throughout, like a true principal series. In fact, in one case, the four last lines on the photograph were shown *as reversals only*, the emitted light of the separate series lines having fused into a continuous spectrum. More accurate measurements have been obtained of these lines, and strong arguments can be advanced for including in this series the lines 4,226.91 and 2,721.76, the first of which is the well-known flame line. Similar remarks apply to Sr. Using the Ba flame line 5,535.69 as the first line of Ba $SL1$, the others appear to be at 3,071.71, 2,702.78, 2,597.26, 2.543.3. Three of these give another series (Ba $SL2$) by a wave-number shift of 16,781.7, whose wave-lengths are 6,341.88, 4,947.50, and 4,605.11, though further work is required on this series. The formulæ for all these series are complicated; the denominator contains a function of the variable integer which is certainly more complicated than in the Ritz formula.

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[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

[2] Astrophysical Journal, 32, 153, 1910.

The system of pairs has been enlarged by the discovery (by Lyman) of ultraviolet lines, which form narrow pair series. These have the line of greater wave-length the stronger, as do typical "subordinate" series; and the lines are easily reversed. The pairs have constant wave-number differences, whose values are 65 for Ca, 280 for Sr, and 575 for Ba. A new relation apparently holds between these narrow pairs (p), and the wide pairs (P). The convergence wave-number of the complex pair series ($P1$) *plus* the wave-number of the first term, equals the convergence wave-number of the narrow pair (p) series. This relation is true within the limits of accuracy of location of the convergence wave-numbers, though this accuracy is not yet very great. The form of the relation is similar to that for the narrow pair series in the alkalies, announced by Runge, except that the "plus" in the above statement becomes minus in that case. Furthermore, the wave-number-differences in the p series are equal to those occurring in the first term of the $P1$ series (which consists of three lines), between the two lines of *shorter* wave-length. This law is fulfilled to a satisfactory degree of accuracy, and is again similar to a relation found by Runge, excepting that in the alkalies (*e. g.*, in Cs) the separation of the narrow pairs is equal to that of the two lines of *longer* wave-length in the first term of wide pairs.

Grateful acknowledgment is due to Dr. H. M. Randall for permission to use some unpublished results on Ba. The last relation above was used to predict a Ba line, later found by him at λ 12,085, which is probably the line of greatest wave-length in the first term of Ba $P1$.

If we call an ordinary "principal series of pairs" a complex series of single lines (as we reasonably may), the Rydberg-Schuster law and the two laws above mentioned may be grouped in one general form, somewhat as follows: In all complex series, the convergence wave-number minus (or plus) the wave-number of the first term equals the convergence wave-number of a derived series, and the wave-number-difference of the lines of this derived series is constant and equal to that of the lines of the first term of the complex series (or certain ones of them). This applies then (*a*) to the ordinary sharp (second subordinate) series of pairs in the alkalies, etc., which are derived from the principal series (Rydberg-Schuster law); (*b*) to the narrow pair series in the alkalies, derived from the complex pair series (law of Runge, "Bergmann" series, etc.), and to the narrow triplet series in Ca and Sr (Ba still doubtful) derived from the complex triplet series; and, (*c*) finally, to the ultraviolet p series in Ca, Sr and Ba, derived from the complex pair series.

THE CONSTANTS OF SPECTRAL RADIATION OF A UNIFORMLY HEATED ENCLOSURE.[1]

BY W. W. COBLENTZ.

IN a previous communication to this society[2] a general description was given of the work which was completed prior to December, 1912. It was then proposed to observe several more sets of energy curves, at the highest operable temperatures with a vacuum furnace before publishing the exact numerical results. Owing to unavoidable delays in assembling all the apparatus, the intention of withholding all the data until the completion of this investigation has now been abandoned, and in the present paper is given a brief summary of the most reliable data now at hand.

The temperature scale used in this investigation, for temperature above 1400° C. is the optical scale, which is obtained by adding to the thermoelectric temperatures[3] certain values, which are the mean of several groups of thermocouples. In assembling the present data for publication it was observed that for temperatures above 1400° C. there was an abrupt rise in the value of the constant C which increased systematically to abnormally high values for temperatures taken above 1500°. It was therefore necessary to choose between the possibility that (1) this variation in C at high temperature is owing to the failure of Planck's law at high temperatures or (2) that, for the particular thermocouples used in this research the aforementioned correct factors for reduction to the optical scale are too large.[4] The writer chose the latter conclusion and the results obtained at high and at low temperatures are now in agreement.

The systematic errors, which formerly caused the mean values of the various sets of observations to differ by 0.2 to 0.5 per cent., are now reduced to less than 0.2 per cent. They were caused by the presence of a disproportionate number of high temperature energy curves, in some sets of data, which contained the aforementioned corrections for reduction to the optical temperature scale. This modification of the correction for reduction of the thermoelectric temperatures to the optical scale has no effect upon the data obtained in 1912, owing to the fact that, intentionally (to save the thermocouple calibration; the difficulty with the scale correction was not foreseen), no energy curves were observed at these high temperatures. The former unexplained disagreement between the sets of observations made in 1911 and those of 1912 is now reduced to an excellent agreement. This was to be expected, for there was no apparent difference in the methods of operation other than the addition

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[1] Abstract of a paper to be presented at the Washington meeting of the Physical Society, April 26, 1913.

[2] Coblentz, PHYS. REV., 1, p. 249, 1913.

[3] Waidner and Burgess, Bull. Bur. Standards, 3, p. 205, 1907.

[4] In the complete paper it will be shown that, for the particular thermocouples, the mean value correction factors for reducing the thermocouple temperatures to the optical scale are too large by five to seven degrees, depending, of course, upon the temperature, above 1400° C.

TABLE I.

Series.	Number of Energy Curves.	Mean Value, *C*.	Remarks.
1911			
I	26	14469	Unpainted Marquardt porcelain radiator.
1912			
I	7	14463	Unpainted radiator used in 1911. Apparatus newly adjusted to determine the reproducibility.
II	14	14432	Radiator painted with chromium oxide.
III	7	14466	New unpainted Marquardt radiator.
IV	9	14446	Radiator painted with a mixture of the oxides of cobalt and chromium.
V	14	14476	Radiator repainted; optical path readjusted.
VIII	8	14461	New thermocouples; other adjustments the same as for the preceding series.
IX	9	14436	New water-cooled shutter, rest of apparatus unmodified.

Mean value. $C = 14456 \pm 4$ (94 energy curves). $A = 2911 \pm 1$.

of a number of high-temperature, energy curves to the series of 1911, which required the optical scale corrections.

The series of 1911 is the most accurate as regards low humidity while the series of 1912 excel in having the most improved methods and the longest experience in the work. The results of the two years work are given in Table I. Each value of C is the mean of the number of isothermal energy curves (94 in all) given in column 2, computed by eq. (2) published in the previous communication to this journal. The mean value is

$$C = 14456 \pm 4,$$

$$A = 2911 \pm 1.$$

In the complete paper it will be shown that the values obtained by Lummer and Pringsheim, if computed by the present methods would give a mean value of $C = 14{,}465 \pm 40$, and that the values published by Paschen, if similarly corrected would give a mean value of $C = 14{,}458 \pm 25$.

The most recent communication from Warburg and his associates[1] gives a mean value of $C = 14{,}374 \pm 4$. The mean value of the four sets of investigations weighted as follows (L and $P = 1$, $P = 3$, $W = 10$ and $C = 10$) gives a mean value of

$$C = 14{,}420 \text{ mikron deg.}$$

$$A = 2{,}905 \text{ mikron deg.}$$

The mean value of the most recent work by Warburg and by the writer is $C = 14{,}415$.

The value $C = 14{,}420$ is a convenient one to use. The difference in the values obtained by Warburg and by the writer indicates an uncertainty of five to six degrees at 1750° C. It is rather remarkable that the older data (which were reduced by the present system of computation long before the inconsistencies among the various sets of the present observations were eliminated) are in close agreement with the present results.

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[1] Warburg, Leithaüser, Hupka, Müller, Sitzber. Akad. Wiss. Berlin, I., p. 35, 1913.

WASHINGTON, D. C.,
March 5, 1913.

THE ACTINODIELECTRIC EFFECT OF SULPHUR.[1]

BY CHESTER A. BUTMAN.

THE actinodielectric effect is effectively a change in conductivity due to the influence of light. This term was first used by Lenard and Saeland[2] to describe an effect discovered by them while experimenting on the phosphorescent alkaline earth sulphides. The effect was further examined in detail by Oeder, Ramsauer and Hausser[3] for phosphorescent Ca-S-Bi. These last investigators showed that not only did red light give the effect, but it was a maximium with yellow, a minimium with green, with a rapidly increasing value in the ultra-violet.

On account of the previously discovered similarity between the electrical properties of sulphur and of the phosphorescent alkaline earth ***sulphides***[4] it occurred to the writer to see whether sulphur was also actinodielectric. Hence the effect was discovered and reported upon in a brief note.[5] The apparatus originally used has been redesigned, and during a brief vacation was set up in the new Sloane Laboratory at Yale University. The previous results obtained were confirmed, *i. e.*, the deflection of the electrometer was in the direction of the applied field, an effect was obtained with red light, and that the magnitude, the effect depended to some extent on the intensity of the light. The light in every case was passed through glass before it entered the chamber. In addition it was found that with ***no field***, and a good ***vacuum***, a photoelectric effect of the sulphur was obtained. Hence, the apparatus affords a convenient means of investigating the relation between the photoelectric and the actinodielectric effects. Further experiments are in progress at M. A. C., Amherst, Mass.

It has recently come to my attention that Bates[6] has independently discovered an effect in sulphur which is the same as the actinodielectric effect. Bates found that with a field on the guard ring of an electroscope with sulphur insulation, that the leaves charged up with a sign depending on the sign of the field when light was thrown on the sulphur.

[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] Lenard and Saeland, Ann. des Phys., 28, pp. 476–502, 1909.

[3] Oeder, Ramsauer and Hausser, Ann. der Phys., pp. 445–454, 1911.

[4] C. A. Butman, Amer. Jour. Sci., 34, p. 139, 1912.

[5] *Loc. cit.*

[6] Le Radium, 8, 312, 1911.

Another experiment of interest is that of F. Schmidt.[1] Schmidt has shown that the dielectric constant of sulphur and of phosphorescent Ca-S-Bi vary in the same direction, and approximately by the *same amount*, with a change in temperature from − 140° C. to + 80° C.

Note.—In view of the interesting discussion of the foregoing paper by Professor Ernest Merritt and Professor W. S. Franklin it may be well to call attention to a paper by G. Szivessy and K. Schäfer, entitled "Über die Erhöhung des elektrischen Leitvermögens bei flüssigen Dielektrika durch Bestrahlung mit ultraviolettem Lichte," Ann. der Phys., p. 511, 1911. These experimenters found that the electrical conductivity of paraffine oil was increased by projecting ultra-violet light upon it.

The experiments of Stark and Steubing[2] are important in this connection. They showed that all organic substances which became *fluorescent* by the absorption of light were *photoelectric*, while those which absorbed light and were not fluorescent gave no photoelectric effect. See also Nichols and Merritt, "Studies in Luminescence," p. 162, Carnegie Institution of Washington, 1912.

A New Vacuum Gage of Extreme Sensitiveness.[3]

By Irving Langmuir.

At very low pressures the viscosity of gases is one of their most marked characteristics. This property is made use of in the new gage.

The gage consists of a rotating disk above which is suspended, by a quartz fiber, another disk carrying a mirror. The viscosity of the gas causes it to be set in motion by the lower disk and this motion produces a torque on the upper disk which can be measured in the usual way by a beam of light reflected from the mirror.

The rotating disk is made of thin aluminium and is attached to a steel or tungsten shaft mounted on jewel bearings and carrying a magnetic needle. The suspended disk is of very thin mica. The lower disk can be rotated easily at a speed of 10,000 r.p.m. by means of a rotating magnetic field produced outside of the bulb containing the two disks. This field is most conveniently obtained by a Gramme ring supplied with current at six points from a commutating device run by a motor. In this way the speed of the motor determines absolutely the speed of the disk, since the two revolve in synchronism. The speed of the disk may thus be varied at will from a few revolutions per minute up to 10,000 or more.

The sensitiveness of the gage is extremely great. At 1,000 r.p.m., with a scale at about 60 cm. distance, we obtain about 400 mm. deflection for .001 mm.

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[1] P. Lenard, Über Lichtsummen bei Phosphoren, p. 9; Sitz. d. Heidelberges Akad. d. Wiss. Math. nat. Kl., 1912.

[2] Phys. Zeit., 9, pp. 481, 661, 1908; also J. Stark, Prinzipien der Atomdynamik, II., p. 216, 1911.

[3] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

of air. We find the deflection exactly proportional to the pressure below about .01 mm., proportional to the speed of the revolving disk and practically independent of the distance between the two disks. For different gases at the same pressure the deflections are proportional to the square root of the molecular weight. All these facts are in accord with the kinetic theory. At 10,000 r.p.m., one mm. deflection corresponds to .00000025 mm. There should therefore be no difficulty in detecting pressures as low as 10^{-7} mm.

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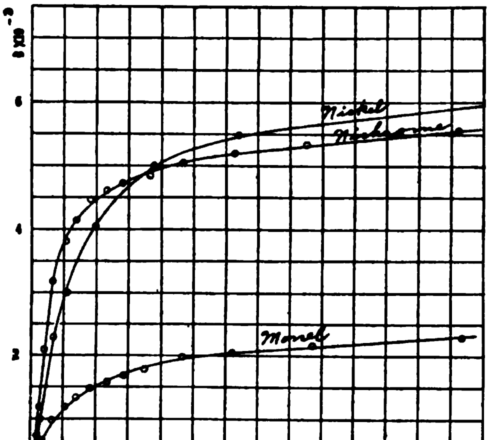
THE PHYSICAL REVIEW.

THE HALL EFFECT AND SOME ALLIED EFFECTS.

By Alpheus W. Smith.

The Hall Effect in Magnetic Alloys.

In an earlier paper[1] the author has examined the influence of temperature on the Hall effect in iron, nickel and cobalt. The results on these metals suggested a similar study of this effect in some magnetic alloys. For this purpose the two alloys, monel and nichrome, and three silicon steels were chosen. Monel is an alloy containing about 68 per cent. nickel, 1.5 per cent. iron, 1 per cent. manganese, and 29.5 per cent. copper. Nichrome is an alloy of nickel, iron, chromium and manganese.



Magnetisation Curves for Monel, Nichrome and Nickel.

Fig. 1.

The magnetization curves for these two alloys together with the magnetization curve for nickel are given in Fig. 1. The latter curve is taken from the work of Honda and Shimizu.[1] Under corresponding conditions the permeability of monel is about one half that of nickel. For fields less than 30 dynes the permeability of nichrome is greater than that of nickel but for greater fields the opposite is true. The three steels which were examined will be referred to as Steel I., Steel II. and Steel III. Steel I. and Steel II. were high silicon steels. The former contained 3.80 per cent. silicon, 0.025 per cent. carbon, 0.010 per cent. manganese, 0.06 per cent. phosphorus, 0.025 per cent. sulphur, and 0.05 per cent. aluminum. An analysis of Steel II. was not made, but it contained about the same amount of silicon as Steel I. Steel III. contained 0.25 per cent. silicon and the same amounts of the other elements which were present in Steel I. From these alloys plates of the usual form were cut for the study of the Hall effect. These plates were about 1.6 cm. wide and 3.5 cm. long. The thicknesses of the plates were as follows: Steel I., 0.0361 cm.; Steel II., 0.0332 cm.; Steel III., 0.0351 cm.; nichrome, 0.0830 cm.; and monel, 0.0625 cm.

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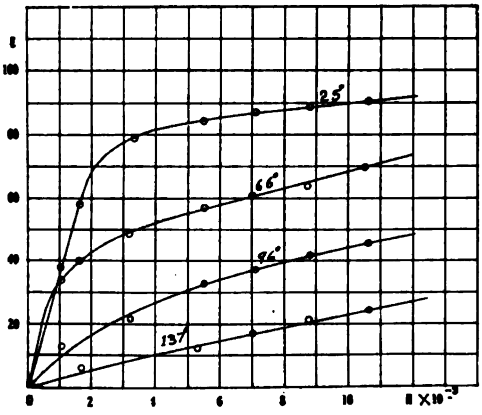
[1] Phys. Rev., 30, pp. 1–34, 1910.

The apparatus differed only in minor details from that used in the former paper and the method of procedure was essentially that used in the study of the Hall effect in nickel, iron and cobalt. For the details of the apparatus and the method of procedure reference is made to the paper already cited.



Hall Effect in Monel.

Fig. 2.

In Fig. 2 have been plotted the Hall electromotive forces in monel for different magnetic fields for four temperatures. Fig. 3 contains the corresponding results for nichrome at six different temperatures. Fig. 4 shows the behavior of the Hall electromotive force in the steels at room temperature. For the sake of comparison the case of iron and that of nichrome have been included. In each of these figures the Hall electromotive forces which have been plotted are the electromotive forces which would be observed if a current of one absolute unit flowed in a plate one centimeter in thickness. The magnetic fields are in absolute units and the temperatures in degrees Centigrade.

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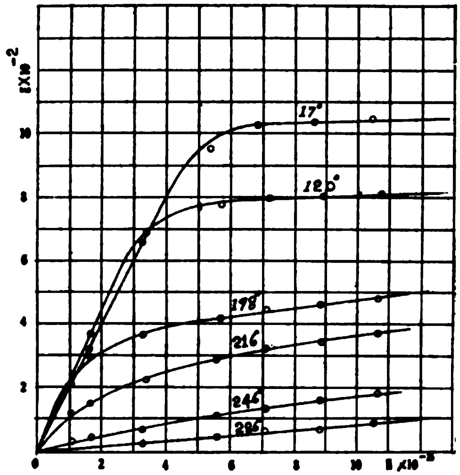
[1] Phil. Mag. (6), 10, p. 553, 1905.

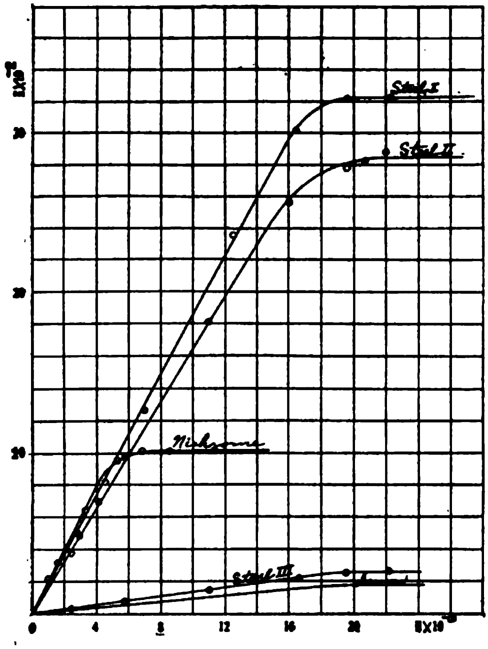
In either monel or nichrome at room temperature the Hall electromotive force is at first proportional to the intensity of the magnetic field. When saturation begins to be reached this proportionality fails. After the characteristic knee in the curve has been passed the Hall electromotive force increases slowly with further increase in the magnetic field. Saturation occurs earlier in monel than in nichrome. An increase in temperature causes a decrease in the Hall effect except that in monel for fields less than 1,000 C.G.S. units there is a small increase between 25° C. and 66° C. and in the case of nichrome there is for fields less than about 3,000 C.G.S. units a small increase between 17° C. and 120° C. When the temperature of the monel has reached 137° C. and that of the nichrome 286° C. the Hall electromotive force becomes proportional to the magnetic field as in the non-magnetic metals. At room temperature the Hall effect in monel is about three times its value in nickel and its direction is the same in the two cases. In nichrome it is about sixteen times as large as in nickel and its direction is opposite to that in nickel. Fig. 4 shows the influence of silicon on the Hall effect in iron. The curve for iron is taken from an earlier paper. Steel III., which contained 0.25



Hall Effect in Nichrome.

Fig. 3.

per cent. of silicon, has a Hall effect about 75 per cent. greater than that in electrolytic iron and about 10 per cent. greater than that in Kahlbaum iron. The further addition of silicon causes a rapid increase in the Hall effect so that Steel I. which contained 3.80 per cent. silicon gives a Hall effect about sixteen times that in Kahlbaum iron. On the other hand the addition of silicon to the iron causes a decrease in the temperature coefficient of the Hall effect. The direction of the Hall effect in each of these steels is the same as its direction in pure iron. Table I. gives the values of R calculated from the equation,



Hall Effect in Silicon Steels.

Fig. 4.

$$E = \frac{RHI}{d},$$

where E, H, and I are in absolute units and d in centimeters. In this table is also included the thermoelectromotive forces against copper. The first addition of silicon causes a decrease in the thermoelectromotive force and further addition of silicon causes a reversal. With this reversal of the thermoelectromotive force there is not associated a reversal of the Hall effect.

TABLE I.

Name.	Temp. ° C.	*H.*	$R \times 10^3$.	Thermo-electric Height 0°-100°.
Monel	25°	1,650	− 35.1	$+22.9 \times 10^{-6}$
	66°	1,650	24.2	
	96°	3,150	9.62	
	137°	5,300	2.38	
Nichrome	17°	3,300	+199	$+0.699 \times 10^{-6}$
	120°	3,300	209	
	198°	1,050	200	
	216°	1,050	113	
	246°	3,300	21.2	
	286°	5,550	8.6	
Steel I	19°	6,000	+152	$+12.4 \times 10^{-6}$
	130°	6,000	175	
	244°	6,000	208	
	310°	6,000	226	
Steel II	19°	6,000	+151	$+12.67 \times 10^{-6}$
	128°	6,000	171	
	247°	6,000	199	
	311°	6,000	218	
Steel III	18°	7,100	+ 11.5	-6.54×10^{-6}
	129°	7,100	23.4	
	235°	7,100	44.3	
	307°	7,100	62.3	
Kahlbaum iron	17°	6,000	+ 10.8	-9.71×10^{-6}
	132°	6,000	22.4	
	250°	6,000	42.4	
	321°	6,000	61.2	

NERNST EFFECT IN MAGNETIC ALLOYS.

A study of the variation of the Nernst effect with temperature has already been made in nickel and cobalt. It was also desirable to have data on the Nernst effect in the alloys which have been studied in the earlier part of this paper with respect to the Hall effect. The thickness of the monel was 0.148 cm. The other plates were those used in the study of the Hall effect. The apparatus was essentially that used for the determination of the Nernst effect in nickel and cobalt, so that reference will be made to that paper[1] for the details of the experiment.

In Fig. 5 the Nernst electromotive forces in monel have been plotted against the magnetic fields; in Fig. 6, the corresponding values for nichrome. In Fig. 7 have been given the Nernst electromotive forces in the silicon steels when one end of the plate was at about 20° C. and the other at the temperature of steam. The electromotive forces plotted for abscissæ in each of these figures have been reduced to the case that the flow of heat occurs in a plate one centimeter wide with a temperature gradient of 1° C. per centimeter in it. These curves are very similar to the corresponding curves for the Hall effect. There is at the low temperatures the initial proportionality between the electromotive force and the magnetic field with the characteristic bend in the region of saturation and then the gradual increase in the electromotive force with further increase in the magnetic field. Also a rise in temperature causes a decrease in Nernst effect except in the case of nichrome where for fields less than 3,000 C.G.S. units it increases slightly between 95° and 205° C. This is essentially the exception noted for the Hall effect. In the neighborhood of 167° C. for monel and 300° C. for nichrome these alloys seem to become non-magnetic and the Nernst electromotive force becomes proportional to the magnetic field. The direction of the effect in monel, nickel and nichrome is the same. From Fig. 7 it appears that steel containing 0.25 per cent. of silicon has a Nernst effect which is about one half of that in pure iron. The direction of the effects in the two cases is the same. The Nernst effect in steel containing 3.8 per cent. of silicon is opposite in direction to that in iron and about three and one half times as large. The addition of small amounts of silicon to the iron

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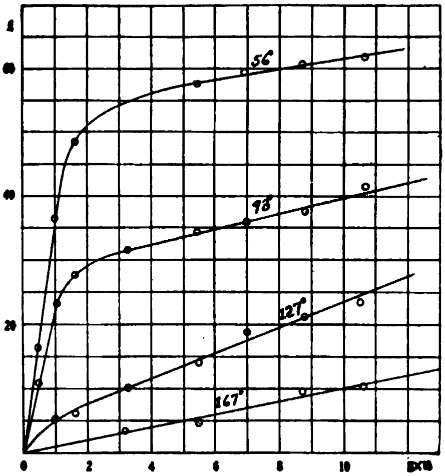
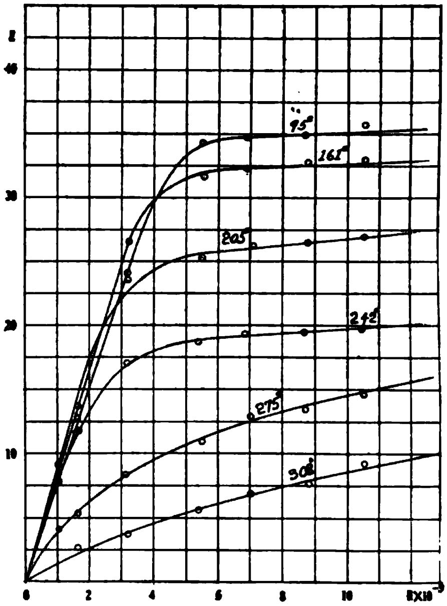
[1] PHYS. REV., 33, pp. 295–306, 1911.



Nernst Effect in Monel.

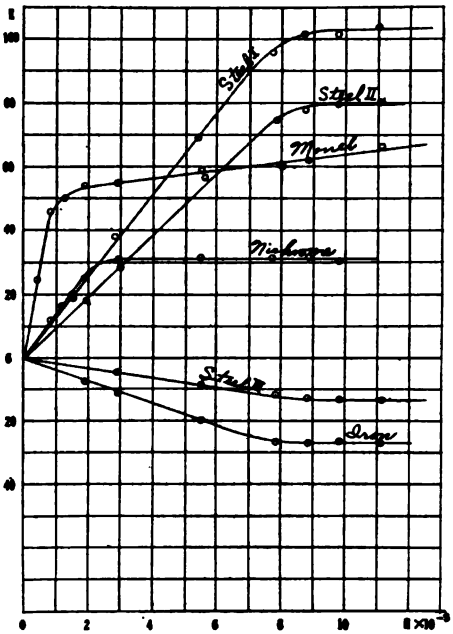
Fig. 5.



Nernst Effect in Nichrome.

Fig. 6.

causes the Nernst effect to decrease. The addition of larger amounts causes the effect to reverse its direction and to rise to a value much larger than that in pure iron. In the case of the Hall effect the addition of silicon to the iron caused an increase in the Hall effect but not a reversal of sign. The reversal of the thermoelectromotive forces occurs under the conditions under which this reversal of the Nernst effect occurs. If the Nernst effect is regarded as the rotation of the thermoelectromotive force in the plate the Nernst effect and the thermoelectromotive force should reverse simultaneously. This is exactly what happens in these alloys.



Nernst Effect in Silicon Steels.

Fig. 7.

Table II. contains the values of Q calculated from the equation,

$$E = Q\beta H\frac{\partial T}{\partial x},$$

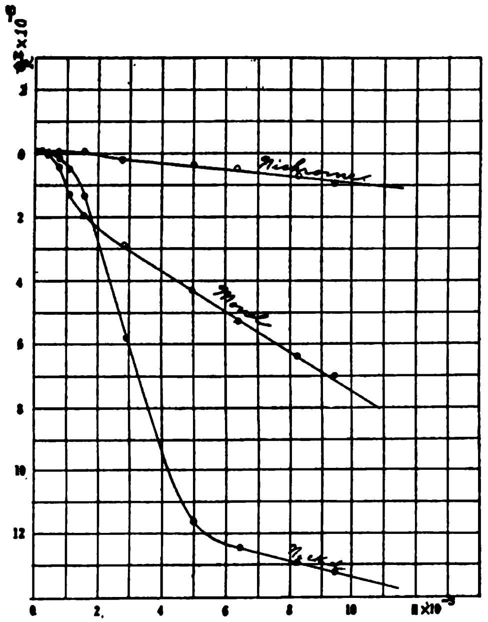
where E is the electromotive force in absolute units; H, the magnetic field in absolute units; β, the width of the plate in centimeters, and $\frac{\partial T}{\partial x}$, the temperature gradient in degrees Centigrade per centimeter.

TABLE II.

Name.	Temp. °C.	H.	$Q \times 10^8$.	Thermo-electric Heights 0°–100°.
Monel	56°	1,500	−32.3	+22.9×10⁻⁶
	93°	1,050	22.1	
	127°	1,000	5.3	
	167°	1,060	.97	
Nichrome	43°	2,500	− 7.08	+ 0.699×10⁻⁶
	95°	3,200	7.53	
	161°	3,250	8.12	
	205°	1,650	7.70	
	242°	1,650	7.00	
	275°	1,650	3.27	
	308°	7,000	.99	
Steel I	38°	15,400	− 6.19	+12.4×10⁻⁶
Steel II	33°	14,000	− 4.85	+12.67×10⁻⁶
Steel III	41°	15,600	+ 0.72	− 6.54×10⁻⁶
Iron	47°	15,600	+ 1.71	− 9.71×10⁻⁶

TRANSVERSE CHANGE OF RESISTANCE IN A MAGNETIC FIELD.

The wires were wound on thin sheets of mica as nearly as possible plane. The nickel wire which was from Hartmann and Braun was 0.030 mm. in diameter. The monel and nichrome wires were 0.075 mm. in



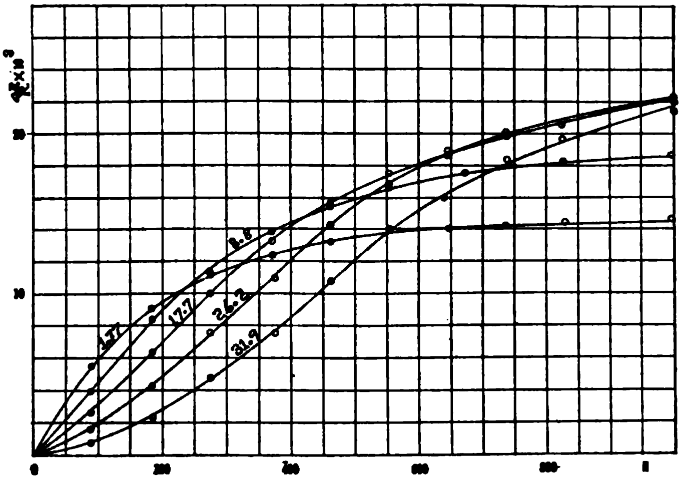
Transverse Change of Resistance in Monel, Nichrome and Nickel.

Fig. 8.

diameter. The coils were placed between the poles of a large electromagnet so that the lines of force were as nearly as possible normal to the plane of the coil. Necessary precautions were taken concerning temperature equilibrium. The observations on three coils at room temperature are recorded in Fig. 8. The slight increase in resistance for lower fields has been attributed to the failure to get the coil perfectly plane and exactly perpendicular to the lines of force so that there is superposed on the transverse change of resistance a longitudinal change arising from the component of the field parallel to the plane of the coil. The longitudinal change of resistance is an increase and for these small fields exceeds the transverse change. The transverse change of resistance in nichrome is small compared to that in nickel, and that in monel for fields greater than about 3,000 C.G.S. units is roughly one half the change in nickel.

INFLUENCE OF TENSION ON THE LONGITUDINAL CHANGE OF RESISTANCE.

It has been shown by Williams[1] that nickel wires sustaining different tensions give different values for the longitudinal change of resistance in a magnetic field. A similar series of observations has been made on



Longitudinal Change of Resistance in Nickel for Different Loads.

Fig. 9.

monel and for the sake of comparison nickel has been again studied. The monel wire was hung so as to coincide with the axis of a large vertical solenoid about one meter in length. The monel wire was about 80 cm. long and 0.0254 cm. in diameter. The solenoid was provided with a water jacket between the magnetizing coil and the space containing the wire to be studied. Further insulation from temperature changes was afforded by filling the space between the inner wall of the water jacket and the wire with cotton wool. The tensions were obtained by hanging different weights on the wire. The resistances of the wire at room temperature for different fields and different loads were measured. The wire was demagnetized after each observation. The tensions expressed in kilograms per square millimeter have been indicated on the curves in the figures. For the study of the nickel wire a more powerful solenoid was used. It was 46 cm. long. The nickel wire was .0425 cm. in diameter and 25 cm. long. In order to have a greater length of wire two

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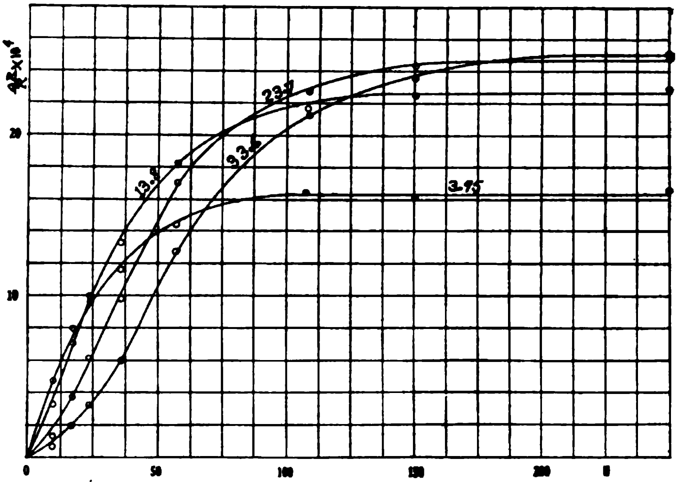
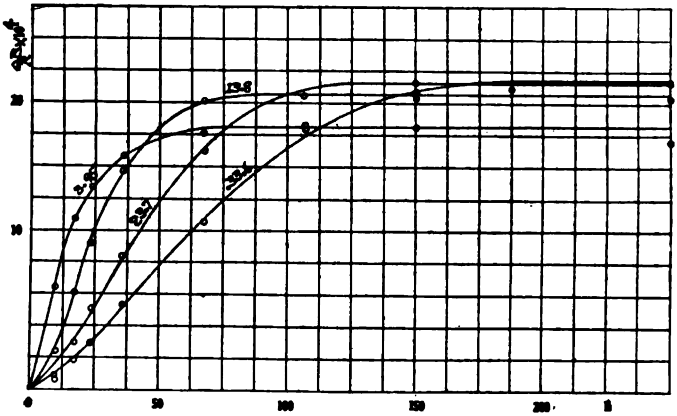
[1] Phil. Mag. (6), 6, p. 693, 1903.



Longitudinal Change of Resistance in Unannealed Monel for Different Loads.

Fig. 10.



Longitudinal Change of Resistance in Annealed Monel for Different Loads.

Fig. 11.

wires parallel to each other and near together were joined in series by soldering the lower end of each one to a heavy strip of copper to which the weights affording the tension were attached. These wires were then placed in the solenoid so that the axis of the solenoid was between them. Fig. 9 gives the results for an annealed nickel wire; Fig. 10, for the monel wire before annealing; and Fig. 11, for the monel wire after annealing. For small tensions the change of resistance rises rapidly to its limiting value beyond which it does not increase with further increase in the magnetic field. The increase in the tension makes this rise to the limiting value less rapid and at first increases the magnitude of the limiting value but for tensions exceeding a certain value of the change of resistance for the larger magnetic fields seems to be the same whatever the tension. Annealing the monel makes the influence of tension greater at lower fields and less at larger fields. The results for monel are exactly similar to those for nickel except that smaller fields are required to reach the limiting value in monel than in nickel.

INFLUENCE OF OCCLUDED HYDROGEN ON THE HALL EFFECT IN PALLADIUM.

It is well known that palladium when made the cathode is an electrolytic cell in which the positive current is carried by hydrogen occludes large quantities of hydrogen, so that when saturation has been reached the palladium is found to contain a quantity of hydrogen which at atmospheric pressure would be equal to more than one thousand times its own volume. A number of observers have shown that this occluded hydrogen causes the electrical resistance[1] to increase. This increase of resistance is approximately proportional to the quantity of hydrogen occluded. When saturation is reached the resistance is about 68 per cent. greater than that in palladium free from hydrogen.

In order to study the Hall effect in palladium which contained occluded hydrogen, plates 1.2 cm. wide and 2.5 cm. long were cut from a sheet of palladium 0.03 cm. thick. From the middle of either side of the plate projected arms to which were soldered or welded palladium wires which lead to the galvanometer leads. These Hall electrodes were adjusted so as to be nearly on the same equipotential. To the ends of the plates were soldered heavy strips of copper which served as primary electrodes. The plate was mounted on a sheet of mica from which a rectangular opening had been cut so that both sides of the plate were exposed. The leads to the plate were encased in small rubber tubing and the copper parts covered with wax to protect them from the acid. The plate was then mounted on a frame midway between two platinum plates which were parallel to it. These platinum plates were much larger than the palladium plate. They were joined in multiple and served as the anode of the electrolytic cell. The electrolyte was a seven per cent. water solution of sulphuric acid. When the palladium plate was in position in this cell the current flowed from the platinum plates to either side of the palladium plate so that hydrogen would be deposited at the same rate on either of the faces. Over the palladium plate in such a way as to collect the hydrogen evolved at it, was an inverted burette to which had been joined a funnel. In series with this electrolytic cell was an ordinary hydrogen voltameter with platinum electrodes. The difference between the quantity of hydrogen generated in this voltameter and the quantity generated at the palladium electrode gave the quantity of hydrogen absorbed by the palladium.

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[1] McElfresh, Proc. Amer. Acad., 39, p. 323, 1904.

The Hall effect was first determined in the plate in the usual way when the plate was free from hydrogen. The intensity of the magnetic field was 20,300 C.G.S. units and the observations were made at room temperature. The plate was then placed in the electrolytic cell and the current allowed to flow until the plate had absorbed about one thousand times its volume of hydrogen. The Hall effect in the plate was again determined. Two plates cut from the same sheet of palladium were examined. In the first of these, the palladium lead wires were soldered to the Hall electrodes; in the second plate they were welded. The second plate gave less trouble on account of thermoelectromotive forces, so that the results for it are more accurate than those for the first plate. Within the error of observation, which did not exceed two per cent. for the first plate and one per cent. for the second plate, there is no change in the Hall effect caused by the occlusion of the hydrogen. Table III. gives the results for the two plates.

Table III.

	Current in Amps.	Thickness in Cm.	R before Saturation with H.	R after Saturation with H.
Plate I.	4.50	0.0309	7.69×10^{-4}	7.54×10^{-4}
Plate II.	4.45	0.0305	7.54×10^{-4}	7.49×10^{-4}

In consideration of the change of electrical resistance under analogous conditions, this result was scarcely to be expected. Work on the Hall effect in alloys indicates that alloys with high electrical resistances have large Hall effects. The occlusion of hydrogen, however, does not change the electrical resistance and the Hall effect in the same manner. If the Hall effect depends on the mean free path of the electrons, the occlusion of such a large quantity of hydrogen would be expected to change its magnitude.

Hall Effect and Polymorphism of Tellurium.

From a study of the thermoelectromotive force and the electrical resistance of tellurium Haken[1] has concluded that tellurium exists in two crystalline forms. One of these forms seems to be stable above 350° C.; the other, stable below that temperature. It seemed that the study of the variation of the Hall effect with the temperature might give additional evidence of this polymorphism.

The tellurium was obtained from Kahlbaum. The plates were cast by heating the metal in a crucible and pouring it into a mould cut out of lavite. These plates were about 4 cm. long, 1.4 cm. wide and 0.3 cm. thick. From the middle of either side projected an arm about 1 cm. long and 0.4 cm. wide. These arms when fastened to the wires leading to the galvanometer served as the secondary or Hall electrodes. Either platinum or silver wires were fastened to the plate at the primary as well as at the secondary electrodes. The temperatures which were obtained by means of an electric furnace were measured with a platinum platinum-irridium thermal couple. The magnetic field had a strength of 8,800 absolute units. It was found that the way in which the plates were cooled and the heat treatment to which they were afterward subjected very largely changed the value of the Hall constant at room temperature. At higher temperatures the variations from this cause were not so large. The character of the curve showing the relation between the Hall constant and the temperature remained essentially the same whatever the heat treatment or the condition of cooling.

A number of plates were investigated. The values of the Hall constants obtained for two of these plates have been given in Table IV. and the same values have been plotted in Fig. 12. The plates for which these values are given were cast as nearly as possible in the same way by pouring the moulten tellurium into the mould at room temperature. The observations here tabulated are those made on the first heating of the plate from room temperature to the highest temperature indicated on the curve. The points on the curve marked with a circle refer to Plate I. and those marked with a circle and a cross refer to Plate I. A single curve has been drawn for both sets of points, although two curves, one lying a little above the other would better represent the facts. From this curve it is seen that with rising temperature the Hall effect in tellurium decreases very rapidly until 180° C. is reached. Between 180° and

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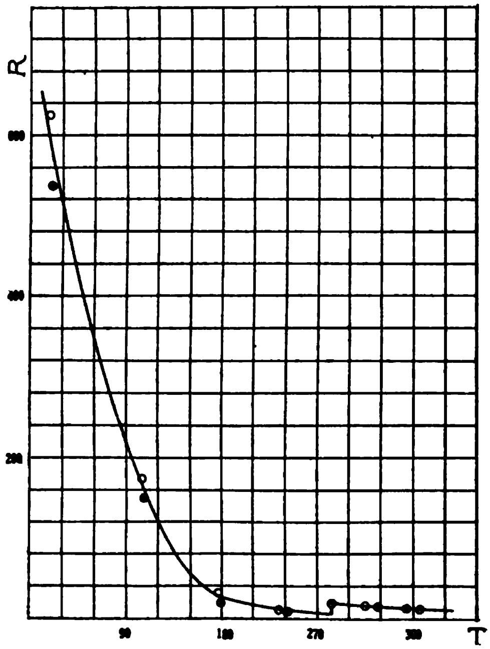
[1] Ann. d. Phys. (4), 32, p. 291, 1910.

TABLE IV.

Plate I.		Plate II.	
Temp. C °.	R	Temp. C °.	R
21.8	536	20.3	621
109	147	105	171
180	18.8	177	30.5
243	7.47	234	8.96
283	18.0	283	19.3
327	14.9	315	18.1
353	12.0	366	12.5



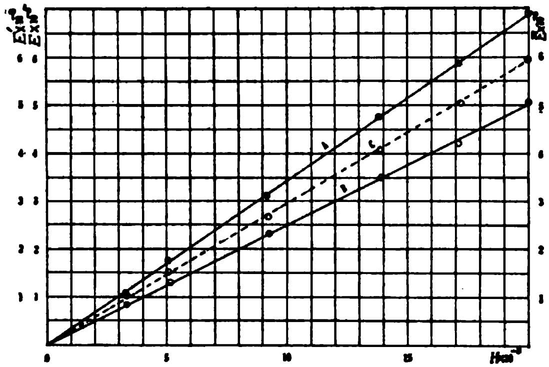
Hall Effect in Tellurium.

Fig. 12.

about 275° C. the effect decreases slowly. In the neighborhood of 275° C. a molecular transformation occurs in which the Hall constant is doubled. With further rise of temperature the constant decreases slowly as compared to the decrease at lower temperatures. From room temperature up to about 275° C. the tellurium is evidently a mixture of the two crystalline forms. One of these forms will be in unstable equilibrium. The relative concentrations of these crystalline forms will vary as the temperature is changed. In the neighborhood of 275° C. one of these crystalline forms goes over into the other and there remains a single crystalline form in which the Hall effect behaves as in non-magnetic metals. These results on the Hall effect in tellurium confirm the results of Haken that tellurium exists in two crystalline forms. The temperature at which this molecular transformation occurs as indicated by the Hall effect does not agree very well with the temperature at which Haken found the behavior of the thermoelectromotive force and the electrical resistance indicated a molecular transformation.

The Hall Effect in Iron Pyrites and Galena.

Very little is known concerning the Hall effect in crystals. This lack of information is largely due to the difficulty in securing suitable crystals from which to cut plates adapted to the study of this effect. It was, however, found possible to obtain a plate of iron pyrites and two plates of galena in which the Hall effect could be observed. The crystal of iron pyrites was a large unmodified cube from which was cut a plate in such a way that its plane was parallel to one of the faces of the cube. This plate was 2.2 cm. long, 1.1 cm. wide and 0.18 cm. thick. Galena also crystallizes in the isometric system and each of these plates was cut so that its plane was parallel to one of the faces of the crystal. One of these plates was 1.5 cm. long, 0.8 cm. wide and .30 cm. thick; the other was 2.0 cm. long, 0.8 cm. wide and 0.303 cm. thick. The ends of the plates were copper plated in order to make it possible to solder wires to them for the primary electrodes. In the middle of each side of the plates a small region was also electroplated. At these points were soldered the wires which served as the secondary electrodes. The observations were made in the usual way.



Hall Effect in Iron Pyrites and Galena.
Fig. 13.

Fig. 13 shows the results of these observations. In this figure the magnetic field in absolute units has been plotted for abcissæ and the electromotive force which would be observed in a plate one centimeter thick with a current of one absolute unit in it, has been plotted for ordinates.

Curve C is for iron pyrites. The ordinates for this curve are on the extreme left and have been denoted by E'. Curve A is for the first specimen of galena and curve B for the second specimen of galena. The ordinates of the former denoted by E are on the left; the ordinates of the latter on the right. These curves show that the Hall electromotive force for iron pyrites and galena is proportional to the magnetic field. At room temperature $R = -0.296$ for iron pyrites; $R = -34.2$ for first specimen of galena; and $R = -251$ for second specimen of galena. The negative sign means that the effect in these crystals has the same direction it has in bismuth. These crystals like bismuth are thermoelectrically negative with respect to copper. From the approximate relation between the thermoelectric series and the Hall effects, the Hall constant in these crystals would be expected to have the sign it has in bismuth.

SUMMARY.

1. An increase of temperature causes changes in the Hall effect and in the Nernst effect in monel and nichrome which are very similar to the changes produced in nickel, iron and cobalt under corresponding conditions.

2. The Nernst effect and the Hall effect in monel and in nichrome depend on the temperature in essentially the same way.

3. The addition of small quantities of silicon to iron causes a large increase in the Hall effect and a decrease in its temperature coefficient. Under similar conditions there is at first a decrease in the Nernst effect, then a reversal of its direction and an increase to a larger value in the opposite direction. Associated with this reversal of the Nernst effect is the reversal of the thermoelectromotive force against copper.

4. In a transverse magnetic field the resistance of monel and nichrome behaves like the resistance of nickel except that the changes under corresponding conditions are less than in nickel.

5. The influence of tension on the longitudinal change of resistance in a magnetic field in monel and in nickel is the same except for magnitude

6. Large quantities of occluded hydrogen do not change the Hall effect in palladium.

7. In the neighborhood of 275° C. there is a molecular transformation in tellurium. The Hall effect after this transformation is about double its value before this transformation.

8. The Hall effect has been determined in iron pyrites and galena. In these crystals the Hall electromotive force is proportional to the magnetic field.

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THE EFFECT OF DIELECTRICS ON UNIPOLAR INDUCTION.

By E. H. Kennard.

IN a recent article[1] Professor Barnett takes issue with the conclusions which I drew[2] from an experiment of my own on unipolar induction and which I should *a fortiori* draw from his results. In the Physikalische Zeitschrift I have already replied[3] to his chief criticism as presented in an earlier issue of that magazine,[4] but a further reply in connection with his later article may not be amiss.

Professor Barnett contends that neither my experiment nor his own decides the old question as to "motion of the force lines," but his argument appears to me to be fallacious in two particulars. The difficulty in both cases arises from the fact that on the moving force line theory e (the motional intensity) does not usually satisfy Laplace's equation.

In the first place, he assumes tacitly on p. 326 and explicitly on p. 327 that the electric density is proportional to div f. This was the assumption made by H. Hertz, who identified f (total intensity) and E (electric force) and rejected the motional intensity e. But on Lorentz's theory this assumption is not always allowable, and it may easily be shown to be incompatible with the moving line theory in the exact case mentioned by Professor Barnett on p. 327.

For suppose that the force lines there rotate with the magnet and let the surrounding dielectric be free ether. Then in the ether div E must vanish. But since $e = -[vB]$, where v = velocity of force lines,

$$\operatorname{div} e = -(B \operatorname{curl} v) + (v \operatorname{curl} B),^{5}$$

or since

$$\operatorname{curl} B = 0,$$

$$\operatorname{div} e = -2(BU), \tag{1}$$

where U = vector angular velocity of lines. Thus div e and therefore div f = div $(E + e)$ will not usually vanish even where there is no electrification. I see no escape from this conclusion except by assigning to the ether some very peculiar properties invented ad hoc.

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[1] Phys. Rev., Nov., 1912, p. 324.

[2] Phil. Mag., June, 1912, p. 937.

[3] P. ZS., Dec. 1, 1912, p. 1155; Mar. 15, 1913, p. 250.

[4] P. ZS., Sept. 1, 1912, p. 803.

[5] [ ] denotes the vector product, ( ) the scalar product.

In his latest article[1] on the subject Professor Barnett refers to discussions of unipolar induction by Poincaré and M. Abraham. Careful reading shows, however, that both of these authors approach the subject from the Hertzian standpoint and are thereby led virtually to make the same assumption as Professor Barnett concerning div. f. Furthermore, both authors dismiss the question as to whether the lines "move" or not with the observation that the lines are creatures of our imagination, so that the question has no sense. But neither author so much as mentions the substantial physical theory, apparently once a favorite in Germany, which employs the "moving lines" merely as a symbol, and which differs from Lorentz's theory chiefly in that in the equation

$$E' = E + 1/c[vB]$$

v is interpreted as velocity not relative to the ether but relative to the material system which is the source of the magnetic field. Hence for our purpose both of these references are really beside the point.

In the second place, in note (1) on p. 326 Professor Barnett does not state why $K - 1/K$ is to be replaced by unity. From the article there referred to, the reason seems to be that the moving lines are supposed to act on the ether (p. 433, top) in the same manner as on a material dielectric. But such an assumption is not a part of "current theory" as I know it, and I do not see how it is to be harmonized both with electrostatics and with the moving line theory. For simplicity, let us suppose that in the case cited above the magnet is electrically uncharged. Then if, as Professor Barnett assumes,

$$D = KE + Ke = Kf$$

we have everywhere div $D = 0$, curl $D =$ curl $e = 0$, and therefore D and f vanish at all points, together with the electricity density. Hence for electrostatic reasons div E should vanish, but it cannot do so because as shown above div e does not vanish. The only explanation in harmony with electrostatic theory would be the assumption that E itself contains a component due to the moving lines, but all such effects were included in e.

The real root of the difficulty appears to me to lie deeper yet. In the article to which the note refers Professor Barnett seems to treat the ether exactly like a material dielectric. But in electrostatics ether and matter are diametrically opposed: an outward displacement in matter (across a closed surface) leaves behind it in the matter a negative (free) charge, whereas an outward displacement in free ether leaves behind it a positive charge. And this difference is essential,—ether and matter are complementary: for if an outward displacement in matter is produced by any other cause than the insertion of a positive charge the ether must be free to take on a displacement backward corresponding to the change in E and serving to keep the total displacement unaltered.

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[1] P. ZS., March 15, 1913, p. 251.

Let us now combine with the moving line theory Lorentz's relation between D, E and e, and calculate the charge thus obtained on Professor Barnett's condenser. The result will likewise hold on this theory when the solenoid is at rest and the condenser rotates in the opposite direction, and in this case will also be true according to Lorentz's theory. We have in the short-circuiting wire $f = E + e = 0$, hence

$$E = -e = \omega B r,$$

where ω = angular velocity of lines (relative to condenser); and

$$\psi = \omega B \frac{r''^2 - r'^2}{2}, \tag{2}$$

where ψ = P.D. between inner and outer cylinders. In the dielectric

$$D = KE + (K - 1)e, \tag{3}$$

$$\therefore (rD)\frac{dr}{r} = KEdr - (K - 1)\omega B r dr,$$

and since (rD) is constant we have after integrating and applying (2)

$$D = \frac{\psi}{r \log \frac{r''}{r'}}. \tag{4}$$

Hence also

$$E = \frac{1}{K}\left\{\frac{\psi}{r \log \frac{r''}{r'}} + (K - 1)\omega B r\right\}, \tag{5}$$

$$f = \frac{1}{K}\left\{\frac{\psi}{r \log \frac{r''}{r'}} - \omega B r\right\}. \tag{6}$$

The last equation agrees (except with $-\omega$ for ω) with Professor Barnett's calculation on p. 326 if the factor $K - 1/K$ is there retained.

Finally if σ_1, q_1 are surface and linear charges on inner cylinder, by (4)

$$q_1 = \frac{2\pi\psi}{\log \frac{r''}{r'}}. \tag{7}$$

Since in the metal $f = 0$, $D = E = \omega Br$ and by (4)

$$\sigma_1 = \frac{\psi}{r' \log \frac{r''}{r'}} - \omega B r'.$$

If the inner cylinder is solid it contains a body charge $\rho_1 = \operatorname{div} D$ or

$$\rho_1 = 2\omega B. \tag{9}$$

The charge on the condenser is thus independent of K, as is stated also by Professor Barnett. This result may be generalized as follows: If a rigid system composed of conductors and a homogeneous dielectric surrounded by a conducting surface, rotates about an axis of symmetry of a magnetic field (or is "cut by moving lines" which thus rotate), the resulting distribution of electrification is independent of the dielectric constant.

For under these conditions curl $f = 0$ and the line integral of f around any closed path, say $ABCDA$, where ABC lies in the dielectric and CDA in a conductor, is zero. But in the conductor $f = 0$, hence the integral of f along ABC must also be zero. Now $D = E + (K - 1)f$, hence the line integral of D along ABC equals the line integral of E, *i. e.*, equals the P.D., ψ, between A and C. But since along CDA $f = 0$, ψ equals the induced E.M.F. in CDA. Now also in the whole system curl $D = 0$ and in the dielectric div $D = 0$. Hence D is everywhere determined in terms of the induced E.M.F. in the conductors, and the electrification on the conductors is independent of the dielectric coefficient. That is, e in the dielectric merely cancels the polarization produced by the charges on the conductors.

This general theorem evidently applies to my own experiment, where a magnet rotated inside a closed conductor and an induced charge was sought (in vain) on a cylinder surrounding the latter. I think therefore that both Professor Barnett's results and my own do invalidite at least the simplest and most natural form of the old moving line theory.

Elsewhere I have called attention to the fact that Professor Barnett's own conclusion as to relative motion rests in part upon an inference. He deduces the existence of a charge in his Case I. from known laws combined with results of Blondlot, H. A. Wilson and Barnett on the effect of dielectrics moving in a magnetic field. This seems not quite conclusive because in the latter experiments sliding contacts (or a similar device) and stationary connecting wires that did not lie along the axis of the system were necessary adjuncts, and the dielectric was naturally not enclosed in a conducting screen that rotated with it. It seems possible that such a screen might prevent the production of any charges inside it. Such an effect is indeed required by the old theory of Hertz, which is however in other respects not in accord with the results of these experiments. It would thus seem desirable that Professor Barnett's experiment, fundamental as it is, should be repeated in such a manner that both positive and negative results could be obtained with the same apparatus.

THE STABILITY OF RESIDUAL MAGNETISM.

By N. H. Williams.

A. Theoretical.

I. Analogies between Elastic and Magnetic Phenomena.

THE magnetization of iron under the influence of a magnetizing field is often considered as a phenomenon analogous to the distortion of an elastic body under stress. The elastic strain theory has been built upon this analogy. The intensity of a magnetizing field at a point is defined as the force per unit pole at the point. It is a vector quantity and in a homogeneous medium is a continuous function, hence it is permissible to think of the flux of the vector per unit area perpendicular to the vector as a measure of the field intensity; thus it becomes analogous to stress in a body which is in a condition of strain. The substance in which the magnetic field is established undergoes a change which is also a vector point function and its flux per unit area is called the magnetic induction at a point, and is analogous to the strain in a body under stress. The ratio of the magnetic induction to the intensity of the magnetizing field is called the permeability of the medium and is analogous to the reciprocal of an elastic modulus.

The deformation of an elastic body occurs in approximate accordance with Hooke's law; but beside this approximate proportionality of stress and strain, other effects are observed such as elastic after-effect or hysteresis, plastic yield, permanent set due to repeated application of the load, and the increase of elastic limit with over-strain.

A magnetic phenomenon which is analogous to elastic after-effect may be observed when a demagnetizing field is applied to magnetized iron or steel. After the first partial demagnetization due to the field, there is a further slight demagnetization which is too slow to produce an appreciable effect upon a ballistic galvanometer. There is a similar time lag when the field is in such a direction as to increase the induction, and the effect is much more apparent in soft iron than in steel. These facts have been mentioned by Lord Rayleigh[1] and by Ewing.[2]

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[1] Phil. Mag., March, 1887.

[2] Phil. Trans., 1885, p. 569.

Plastic yield and permanent set are analogous to the magnetic phenomena resulting from any change of magnetic induction exceeding a limit beyond which the specimen fails to return to its former state upon being removed from the field.

It was first shown by Fromme[1] and afterward by Ewing[2] that repeated applications of a magnetizing field produce a higher residual induction than a single application. This is analogous to the fact that repeated application of the load upon an elastic body may produce a permanent set provided the stress approaches the elastic limit.

II. *Definition of Terms.*

It will be convenient to use the term elastic limit as applying to magnetic phenomena and meaning the maximum intensity of field which may be applied to a specimen without a permanent change of its induction. A *partial* demagnetization will be designated merely as a demagnetization and the word recovery is here used to represent the change of flux which occurs when the demagnetizing field is removed.

III. *Purpose of the Work.*

The purpose of this paper is to present some experimental work which suggests further analogies between elastic and magnetic phenomena and especially to determine for different kinds of iron and steel the relation between the recovery and the maximum residual flux.

IV. *The Scope of the Work.*

The experiments show an approximate proportionality between magnetic stress and strain and that the limit beyond which the iron fails to regain its former condition after partial demagnetization may under favorable circumstances reach a variation of magnetic induction as high as 2,000 lines per sq. cm.

It is shown that for hard steel the hysteresis, for changes within the elastic limit, is quite small and the analogy between magnetic hysteresis and mechanical hysteresis is made evident by comparing this work with work recently done upon the elastic properties of steel tubes.

The effect of repeated application of a *demagnetizing* field is found to be similar to that found by Fromme and Ewing for a *magnetizing* field, *i. e.*, many repetitions of a magnetic stress, which is equal to the elastic limit, produce a change of induction that is equivalent to a permanent set.

An analogy is found for the increase of elastic limit with over-strain.

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[1] Pogg. Ann., 1875; Wied. Ann., IV., 1878.

[2] Phil. Trans., 1885, p. 570.

B. EXPERIMENTAL.

I. Madelung's Experiment.

Madelung[1] has published an account of an excellent piece of work on the magnetizing effects of high frequency currents. His method involved the use of the Braun tube in plotting variation of magnetic flux in steel wire with variation of field intensity caused by the discharge current from a condenser. Fig. 1 is a copy of one of his curves showing the demagnetizing effect of an aperiodic discharge. He plots what he calls the maximum hysteresis curve, *i. e.*, the curve resulting from carrying the magnetizing field to a very high value and then reducing it. It will be observed that the curve due to the discharge lies outside the maximum curve. This is due to the effect of eddy currents in the steel, which set up a counter magnetizing field and cause a shift in phase of the actual magnetizing field. The curve obtained from the instrument gives the relation of the magnetic induction to the magnetizing field as it would be if not modified by eddy currents. The three curves of Fig. 1 were taken with wires of different size, the curve taken with small wire being the lowest one. Since the eddy current effects decrease with diminished size of wire, this lowest curve would be expected to conform more nearly to the maximum hysteresis curve than the others, as it does.

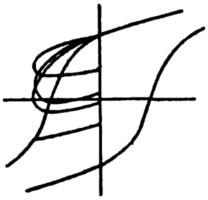
Fig. 1.

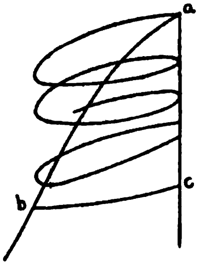

Fig. 1*a*.

Fig. 1*a* shows the effect of repeated application of the demagnetizing impulse. The curve falls to lower values of the induction as the impulse is repeated and finally reaches a stationary position where it becomes a closed curve. However, it never falls to a position where its extreme point lies on the hysteresis curve, but the point remains outside the area of the curve. Madelung points out that this is an error due to the eddy currents and suggests that if the current were of long duration and its time rate of variation at all times small, the curve traced would be the maximum hysteresis curve down to the point *b*, and from there it would be the line *bC*. This is undoubtedly true for the first application of the demagnetizing field but the question arises: what would be the result of many repetitions of the demagnetizing impulse? A part of the experimental work herein described deals with this question. An effort has been made to reduce the errors due to eddy currents and end effects to a negligible magnitude and furthermore to deal with the whole matter in a quantitative way so that we may know to just what extent demagnetization is followed by recovery and how much hysteresis occurs.

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[1] Annalen der Physik, 17, 1905, p. 861.

II. *General Method.*

The method adopted in the experimental work was an oscillographic method devised by the author and so far as he is aware it has not been used elsewhere. The specimen to be studied was of steel wire in the form of a ring wound with primary and secondary coils. The secondary of a transformer operating on a sixty cycle current was connected in series with a storage battery and adjustment made so that the maximum value of the alternating E.M.F. was equal to the storage battery E.M.F. This gave a unidirectional pulsating current fluctuating between the limits of zero and a maximum value. This current was made to pass through the primary coil on the magnetized core in such a direction as to produce a demagnetizing field. A portion of this current operated one loop or vibrator of the oscillograph while the other loop was connected to the secondary consisting of a few turns and also wound on the steel core. The inductance in the secondary was negligible and hence the ordinate of the curve obtained by the oscillograph from this coil is proportional to the rate of flux variation while the ordinates of the current curve represent the instantaneous value of the magnetizing field. This latter statement is only true provided the results of eddy currents and end effects are negligible.

Proper calibration is necessary in order to obtain actual values of field intensity and rate of flux variation from these curves.

In order to obtain the curve whose ordinates represent the change of flux due to the demagnetizing field, the curve whose ordinates represent the time rate of change of flux is integrated graphically, the process being as follows: The curves as photographed by the oscillograph are transferred by means of a pantagraph to cross-section paper, the copy being drawn to double scale. The paper used was divided into twentieths of an inch. At the instant when the ordinate of the curve representing rate of flux variation is zero and increasing in the positive direction, the flux is a maximum. Directly above this point *a*, Fig. 2, at the top of the sheet a line *bc* is drawn across the quarter inch in which this point lies parallel to the axis of abscissæ. Across the next quarter inch a line *cd* is drawn from the extremity of the preceding line and its slope is made equal to the negative of the ordinate *mn* of the curve representing time rate of flux variation at the middle point of this quarter inch. Where this line ends, another begins and crosses the next quarter inch with a negative slope equal to the ordinate of the flux rate curve at the middle of this space. The process is continued until the curve for the cycle is completed. If the work is carefully done the ordinate of the curve, measured downward, represents the flux variation with considerable accuracy. If the ordinary hysteresis curve is wanted an alternating current is used instead of the unidirectional pulsating current. The values of *H*, as computed from the ordinates of the current curve are plotted as abscissae and the values of the flux or the induction *B*, computed from the flux curve, are plotted as ordinates. Thus we have the hysteresis curve when the alternating current is used and the demagnetization and recovery curve when the pulsating current is used. A hysteresis curve obtained by this method is shown on Fig. 7, the specimen being piano wire in its normal condition.

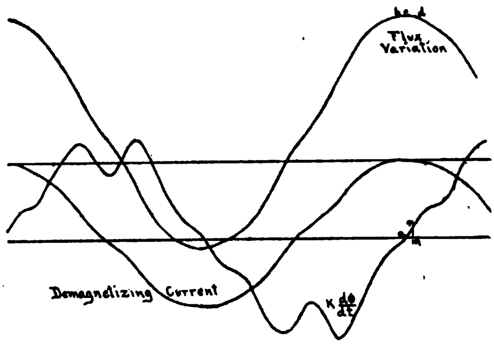
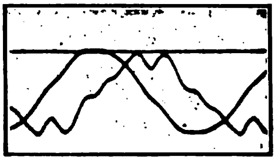


Fig. 2.



Oscillogram. The smooth curve represents the unidirectional primary current; the irregular curve the secondary current.

Fig. 2*a*.

The method has several advantages over the ballistic method for work involving small changes of *B* and *H*. It is possible to determine as many points on the curve as desired even when the curve represents very small changes. The apparatus is quite sensitive to small changes of flux while for large changes it is only necessary to reduce the number of secondary turns and put resistance into the circuit. For example, one curve was plotted in which the flux variation was only 114 lines per sq. cm. and it showed all the characteristics of the curves representing much larger changes; on the other hand hysteresis curves representing changes of magnetic induction of more than 30,000 lines per sq. cm. are easily obtained. The method also insures conditions giving a closed curve for the cycle since the impulse is repeated a large number of times before the photograph is taken and the cycle is traversed more than 1,000 times during the exposure of the sensitive paper.

The remaining details of the method were different for different specimens and will be mentioned in connection with the experiments to which they apply.

III. *Calibration.*

A ballistic galvanometer was used for a part of the work. It was calibrated by means of a current inductor in the usual way.

One mm. deflection was found to correspond to a flux variation of 32.6 lines of induction.

The same current inductor was also used in calibrating the oscillograph. A wire was used for the secondary coil of such a size that its resistance per turn was the same as that of the secondaries of the steel cores. This secondary was connected to the same loop of the oscillograph that was afterward used for the secondaries of the steel cores. The maximum flux through the coils was computed from the same formula as that used in calibrating the galvanometer. Calibration curves were taken with 25, 50, 75 and 125 turns respectively in the secondary. The unidirectional current was used here as in the work upon the steel cores. The flux curve in each case was obtained by graphic integration as previously described. The maximum ordinate of this curve corresponds to the flux as computed from the maximum current. The value in terms of lines of induction of one division of the cross-section paper is thus determined, *e. g.*, with 25 turns one division corresponds to 22.9 lines of induction. Since the maximum flux variation is used in this determination, it is evident that it must be computed from the maximum current rather than from the effective value.

Since a direct current instrument having its field furnished by a permanent magnet indicates the mean current flowing through it and since the mean value of an alternating current furnished by a transformer is zero, a direct current ammeter reads one half the maximum current when the current is unidirectional and fluctuates between zero and a maximum value. The instrument used was a Weston ammeter reading directly to .02 of an ampere and having large divisions so that one could estimate to .005 ampere with accuracy.

As an illustration of the accuracy of the process, two calibration curves for a secondary of 75 turns were taken at different times, one about two weeks later than the other and under conditions giving very different wave forms. The flux value of one division of the paper was 8.104 in one case and 8.089 in the other, showing a difference of two tenths of one per cent.

A separate calibration was made for plotting the hysteresis curves of the specimens. This became desirable since a secondary of only one turn was used and some resistance was inserted to reduce the induced current in the secondary circuit to a value that could be used with the oscillograph.

The magnetic flux per division of the cross-section paper as given by the calibration curves was as follows, the numbers in the first column being the number of secondary turns.

125 turns,	6.65 lines.
75	8.09
50	11.94
25	22.90

IV. Experiments with Specimen A.

The first specimen to be studied was of piano wire 1.03 mm. in diameter. It was in the form of a ring of mean circumference 25 cm. and cross-sectional area 1.4 sq. cm. A gap was cut in the ring about one cm. long and a plug of soft iron wire was made to fit it. The ring was heated to a red heat in an electric furnace and then plunged into cold water. The core was next insulated with tape and a secondary coil consisting of five sections of twenty-five turns each was wound upon it. The primary of 350 turns was wound outside the secondary.

The gap was made in the ring in order that it might be possible to determine at any time the residual flux in the core. This determination was made by means of the galvanometer and test coil previously calibrated. With the coil encircling the ring on the side opposite the gap, the plug was removed and the throw of the galvanometer noted. The coil was then quickly slipped off the ring and a second deflection observed. Since the calibration showed 32.62 lines of induction per mm. deflection, the total flux was found by multiplying 32.62 by the sum of the two deflections.

A series of photographs was taken for the glass-hard steel with maximum demagnetizing field ranging from 1.65 gauss to 34.5 gauss. The variation of induction in the first case was 114 lines of induction per sq. cm. while in the latter it was 1,960. In this latter case the remanence was 140 lines per sq. cm. and the application of the current caused a reversal of the magnetism, producing a negative induction of 1,820 lines per sq. cm. As the current was reduced to zero the magnetic flux returned to its former positive value. With somewhat smaller demagnetizing currents the magnetism was not reversed and the recovery on reducing the field to zero was from a value of the induction which was still positive to a higher positive value. For example, when the current was 1.56 amperes, giving a field of 27.4 gauss, the induction fell from 1630 to 60 and returned to 1,630 when the current became zero. The magnitude of these recoveries is appreciated when they are compared with the maximum residual magnetism observed in the specimen. This latter value was 8,036 lines per sq. cm.; the recovery of 1,960 is therefore 24.5 per cent. of the maximum residual induction.

The curves for this specimen were taken in order beginning with small demagnetizing current. The first application of the current produced a certain amount of permanent demagnetization. After many applications of the current the magnetism reached a stable condition in which the recovery was just equal to the demagnetization. Then the photograph was taken. Next a larger current was applied which produced a further permanent demagnetization and a new condition of stability. Then another photograph was taken. The method of determining the residual flux after each photograph was taken has been described. The removal of the plug before each observation introduced a demagnetizing field due to the ends and caused a marked decrease of the residual magnetism when the flux was at its high values. This effect decreases as lower values of the remanence are reached and vanishes when the flux becomes zero. These curves were plotted with magnetic flux as ordinates rather than the flux density or induction while the figures quoted above are values of flux density. The area of the specimen was 1.4 sq. cm. hence to obtain variation of induction from the curves the ordinates should be divided by 1.4.

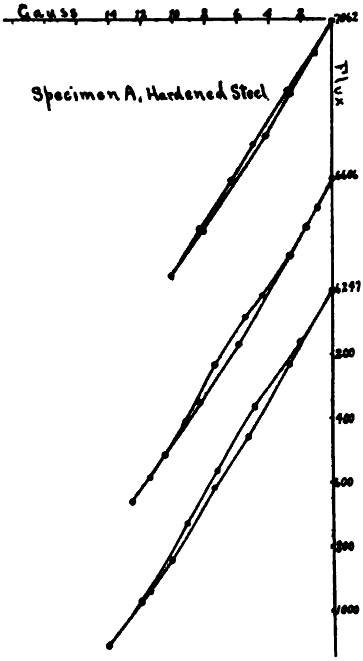


Fig. 3.

Figs. 2 and 3 are two of the series obtained from this specimen. The curves were all transferred to a single sheet (see Fig. 4) and properly placed with reference to the axes according to the residual flux determined after each was taken.

Data for Demag. and Recovery Curves of Specimen A.

Current.	No. of Secondary Turns.	Def. on Removing Plug.	Def. Due to Remaining Flux.	Remanent Flux.	Maximum *H*.	Gauss per Division.
.094	125	24.0	207	7,535	1.6537	.0438
.48	75	21.0	198	7,144	8.4446	.1206
.576	50	20.5	196	7,062	10.1336	.1842
.72	50	19.5	183	6,606	12.667	.2112
.80	50	18.5	173	6,247	14.074	.2346
.92	50	17.5	159	5,757	16.185	.234
1.00	25	16.0	146	5,284	17.593	.3824
1.18	25	14.0	122	4,436	20.76	.3844
1.32	25	12.0	99	3,621	23.222	.3776
1.40	25	10.0	88	3,197	24.63	.4561
1.56	25	7.0	63	2,283	27.445	.524
1.68	25	6.0	49	1,794	29.556	.5185
1.80	25	3.5	32	1,158	31.667	.5278
1.96	25	0	6	197	34.482	.6000

Data for Demagnetization and Recovery Curves Derived from the Current Curve and the Flux Variation Curve.

H = field intensity. ϕ = magnetic flux.

H	ϕ	H	ϕ	H	ϕ	H	ϕ
.24	22.3	.75	70	1.105	107	1.02	95
.48	49.9	2.08	168	2.763	233	2.95	238
.85	81.8	3.98	308	4.973	399	4.50	368
1.40	136.0	5.18	401	6.263	513	5.60	440
1.65	159.0	6.51	522	8.289	668	7.50	584
1.36	133.0	8.44	656	10.133	812	11.55	938
.70	68.0	6.93	559	8.289	668	12.67	1015
.48	46.2	5.43	437	6.080	501	10.60	862
		3.74	308	4.140	370	7.50	582
		2.65	219	2.763	239	5.58	438
		1.03	93			4.50	365
						1.89	155
2.111	167	1.872	191	1.912	160	1.53	130
4.93	370	4.446	406	4.589	389	3.84	321
7.27	573	7.956	668	8.413	664	7.88	641
9.15	740	10.070	859	10.707	870	11.92	916
11.96	979	13.104	1134	14.149	1164	14.61	1214
14.07	1110	16.180	1432	16.443	1374	18.45	1540
12.43	1014	13.104	1183	17.59	1466	20.76	1740
10.09	847	9.600	919	16.44	1374	18.07	1557
7.51	621	6.790	692	14.53	1213	15.00	1305
5.40	465	4.446	441	11.472	962	11.15	985
2.81	239	14.04	167	7.839	733	7.88	733
				5.162	504	4.23	378
				1.912	183	.87	114

H	ϕ	H	ϕ	H	ϕ
3.77	321	1.82	137	1.57	160
8.31	664	4.11	343	4.19	400
12.46	1054	8.66	710	9.43	836
15.86	1374	12.77	1053	14.67	1150
20.01	1740	16.41	1397	18.34	1534
23.26	1992	20.75	1786	23.31	1946
20.39	1786	24.86	2038	27.45	2198
16.61	1511	21.89	1866	23.80	1969
12.08	1145	18.24	1580	19.38	1649
8.68	847	13.68	1191	18.34	1237
4.91	458	9.12	893	10.18	962
		5.93	470	5.67	481
		1.82	149	1.05	137
1.80	183	2.64	251	4.5	378
4.67	492	7.38	618	9.9	801
10.37	939	13.20	1100	16.8	1340
16.07	1340	18.50	1511	22.8	1763
20.48	1800	23.75	2015	27.3	2255
25.93	2233	29.46	2427	34.5	2748
29.55	2496	31.67	2611	27.3	2358
25.93	2233	30.08	2473	22.2	1981
21.26	1855	26.64	2267	15.3	1488
15.54	1397	21.00	1855	9.9	1008
10.89	1031	14.98	1397	4.5	423
5.70	527	10.55	973		
1.39	137	4.22	424		

It will be observed that the extreme points of the demagnetization curves lie on a smooth curve. The determination of the relation of this curve to the maximum hysteresis curve was next undertaken. This portion of the hysteresis curve was determined ballistically and was found to lie outside the locus of the extreme points and as would be expected deviated widely from it near the upper end. This deviation was due to the end effects established when the plug was removed as previously explained. A second ballistic curve was traced under the same conditions that existed when the work on the oscillographic curves began, *i. e.*, the core was highly magnetized, then the plug was removed and replaced, thus the remanence was brought down to the same value at which it stood when the oscillographic work began. This second ballistic curve follows quite closely the contour of the locus of the extreme point of the demagnetization curve as is seen on Fig. 4. However, it lies outside of this locus as does the maximum hysteresis curve.

That the relation of these two curves is not due to error is shown by further work by the ballistic method. The specimen was first highly

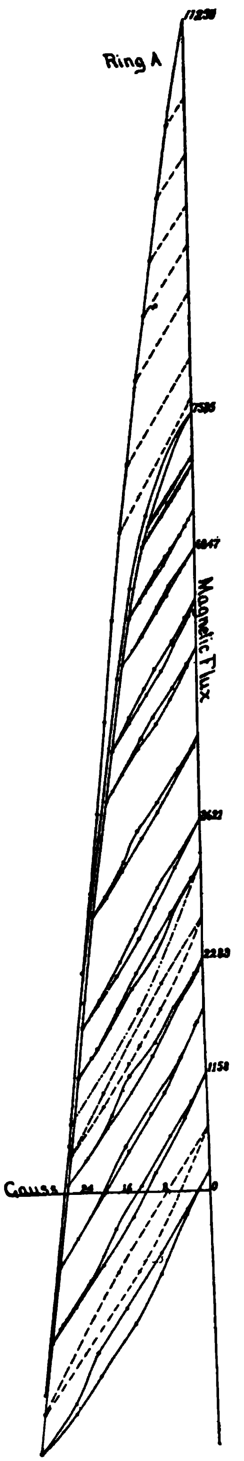


Fig. 4.

magnetized, then a certain demagnetizing current applied sufficient to bring the residual magnetism down to the region in which the oscillographic curves were obtained. Conditions would then be represented by some point on the maximum hysteresis curve. From this point the curve of recovery was traced ballistically, as many points being determined as seemed expedient. This curve was found to differ from that obtained by the oscillograph for the same demagnetizing current both in position and slope. The conditions differed in the two cases only in this: the ballistic curve was obtained the first time the steel was put through the cycle while the oscillographic curve was taken after several thousand repetitions of the cycle. It was next found that after opening and closing the circuit through the primary, a second ballistic curve approached more nearly the oscillographic curve. When the key in the primary circuit was opened and closed about a hundred times and the ballistic curve then traced, it was found that its extreme point fell almost exactly upon the locus of the extreme points of the oscillographic curve, and furthermore the slope and shape of the new curve agreed with those of the oscillographic curve. A number of cycles of demagnetization and recovery were traced ballistically. Some of them are shown on Fig. 4. in their relation to the other curves.

In plotting the hysteresis curve by the ballistic method, an interesting fact appeared regarding end effects. The residual induction after the application of a strong magnetizing field and before the plug was taken out was about 8,000 lines of induction per sq. cm. When the plug was removed the end effects caused it to fall to a value of 4,940 lines per sq. cm. The core was then remagnetized with the plug in place and a demagnetizing current applied. This current might be of any strength provided it did not reduce the induction below 4,940. The circuit carrying this current was then opened and a certain recovery ensued. Then the plug was removed and it was found that the residual induction was again 4,940. This constancy of the residual magnetism on open magnetic circuit persisted until a demagnetizing filed of 12.8 gauss had been applied and then suddenly the residual magnetism, with the magnetic circuit open, fell to lower values. The point at which it breaks down is the point where the recovery on removing this demagnetizing field of 12.8 gauss is equal to the demagnetization due to the ends when the plug is removed. Fig. 5 shows these relations in graphic form. Curve *A* represents the maximum hysteresis curve. The inercept between *A* and *B* is the recovery upon breaking the circuit. The intercept between *B* and *C* is the demagnetization following the removal of the plug. At the right of point *P* the demagnetization due to the ends exceeds the recovery while at the left of *P*, the recovery is greater. Intercept *PQ* represents both demagnetization due to the ends and recovery after removal of the field and it is at this point that the residual magnetism with the gap open ceases to be constant. The interpretation of this horizontal portion of curve *C* seems to be this: that there is a welldefined upper limit beyond which the remanence of a steel core with exposed ends cannot be made to pass.

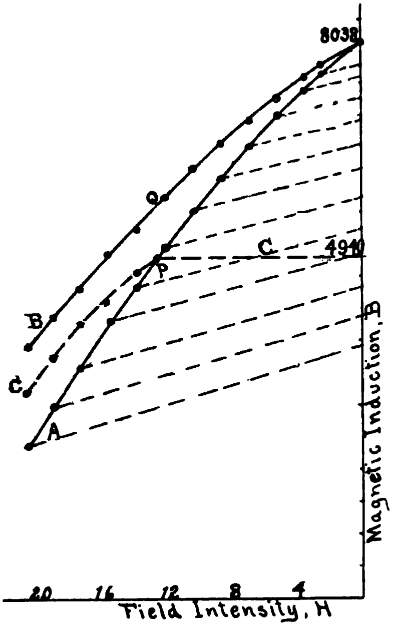


Fig. 5.

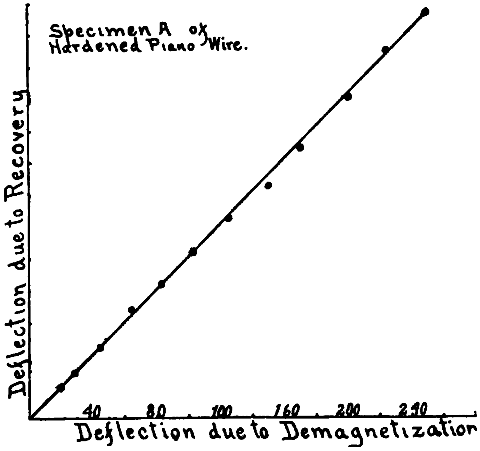


Fig. 6.

As has been mentioned, the recovery curves for a given specimen are practically parallel. For very hard steel the portion of the hysteresis curve with which we are concerned does not differ much from a straight line. If it were exactly a straight line, it would follow that the recovery would be strictly proportional to the difference between the maximum residual flux and the residual flux corresponding to the recovery curve. The line of Fig. 6 is presented to show how nearly this statement of proportionality approximates the truth.

In designing magnetic circuits, so much attention is usually given to reducing the magnetic reluctance of the joints that it seems desirable to find out if the reluctance of the joints had any influence upon the preceding experiments. A demagnetization and recovery curve was obtained by the oscillograph with the plug out of the circuit; thus the magnetic circuit contained an air gap about one centimeter long. The only marked difference between this curve and the others was a decrease in slope amounting to about thirteen per cent. This would lead to the conclusion that when the gap was filled with soft iron, the effect of the joints, upon the form or slope of the demagnetization and recovery curves would be insignificant.

V. *Experiments with Specimen B.*

The second specimen studied was a ring of no. 4 piano wire in its normal condition, *i. e.*, without heat treatment. It consisted of 1,215 wires each having a cross-sectional area of .0012 sq. cm., the aggregate area being 1.458 sq. cm. The mean length of the ring was 32.67 cm. The secondary coil consisted of five sections of 25 turns each. Outside the secondary, the primary coil of 350 turns was wound. The intensity of magnetic field within the metal was $\frac{4\pi \cdot 350\ i}{10 \cdot 32.67}$ gauss, i being the current in amperes. When a test coil of fourteen turns was used, one mm. deflection of the ballistic galvanometer indicated a change of flux of 32.62 lines or a change of flux density (B) of $32.62/1.458 = 22.37$. In dealing with softer specimens, which are much more susceptible to end effects, it seemed best to try to do the work without making any air gap in the magnetic circuit, hence this ring and two others of soft iron which were used were not cut.

Plate 7 shows the hysteresis curve by the oscillographic method when the maximum field intensity is about 62 gauss. The residual induction is seen to be 13,665. As a check upon this curve a determination of the residual inductance was made by the ballistic method using the same maximum magnetizing field and it was found to be 13,670. Such close agreement is probably the result of a fortunate observation as it is beyond the limits of accuracy that could reasonably be expected.

A series of demagnetization and recovery curves was obtained for this specimen by the oscillographic method. They differ in slope from those of the former series but otherwise show very little difference.

Since there was no opening in the magnetic circuit, the method used in the former case of determining the position of the curves with reference to each other and to the residual induction could not be employed. In the case of ring *A* the curves were placed each with its highest point at the point representing the residual flux actually measured after the curve was photographed and the curve through their extreme lower points was drawn. The results of that work would warrant us in taking the reverse process this time. The locus of the extreme points lies within the maximum hysteresis curve. A hysteresis curve that approaches the maximum hysteresis curve very closely can be obtained by the ballistic method but just where the locus of the extreme point lies with reference to this curve is a matter to be determined. This was done by means of a Koepsel permeameter. A field of 134 gauss was applied to a specimen of this wire, this value of field intensity being chosen in order to reproduce the conditions under which the ballistic curve was taken. The field was then reduced to zero leaving a remanence of 15,400 lines per sq. cm. A small demagnetizing field was next applied and the value of the flux density, *B*, observed from the instrument. Then the circuit through the magnetizing coil was opened and closed 50 times and another reading of *B* taken. The key was then opened and closed 50 times more and the value of *B* again observed. Next a larger demagnetizing field was applied and the process repeated. Fourteen points on this portion of the hysteresis curve were thus found and also fourteen on a curve lying within the hysteresis curve. This latter curve, it was assumed would represent the locus of the extreme points of the recovery curves provided the hysteresis curve were accurate. In using the permeameter it is necessary to correct the observeed curve by adding to its abscissæ the abscissæ of a shearing curve. No shearing curve

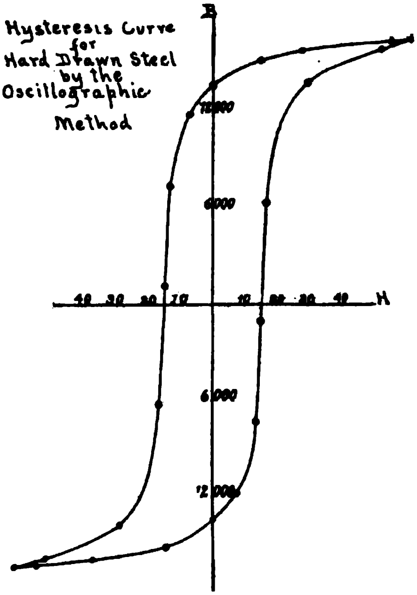


Fig. 7.

was at hand that could be assumed to be accurate for this specimen and so the ballistic curve shown in Fig. 8 was used. The curve of the extreme points of the recovery curves would lie within this hysteresis curve and it was assumed that the difference in ordinates of these two curves would be the same as the difference between the ordinates of the corresponding curves given by the permeameter. In this way the locus of the extreme points was determined. Both these curves are shown in Fig. 8. The oscillographic curves of demagnetization and recovery were now reduced to the same scale and placed upon the sheet in such a way that their extreme points fell upon the curve which lies within the hysteresis curve. The maximum recovery observed was 1,200 lines per sq. cm. and occurred in the region of zero residual flux while the maximum remanence was 14,900, thus the recovery is about 8 per cent. of the maximum remanence. The maximum recovery for the hardest steel was 24.5 per cent. of the maximum remanence. This large difference is due both to a smaller recovery and to a larger remanence in the second case.

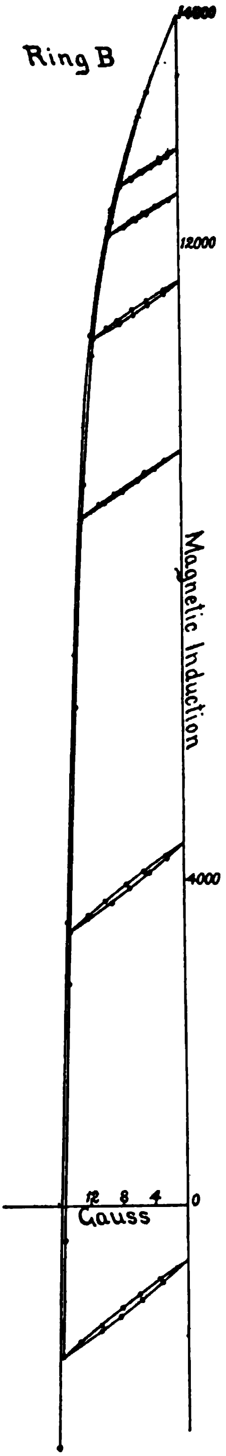


Fig. 8.

Attention has been drawn to a number of analogies between elastic and magnetic phenomena. Bridgman[1] has recently done a piece of work on "The Collapse of Thick Cylinders under High Hydrostatic Pressure" that has furnished curves bearing a striking resemblance to the curves of Figs. 4 and 8. Fig. 13 is a copy of one of Bridgman's figures inverted for the purpose of comparison with the magnetic curves. The specimen is a hollow steel cylinder and the strain is measured by the change of internal volume. Three points of resemblance are to be observed: (1) The hysteresis, (2) the increase of elastic limit with overstrain, (3) the effect of repeated application of the same stress as shown near the bottom of the figure. The actual change of elastic limit with over-strain is of course much less than the apparent change, a large part of that appearing in the figure being due to change of dimensions of the specimen.

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[1] Phys. Rev., January, 1912, p. 1.

The demagnetization and recovery discussed in the previous pages have taken place under conditions such that the metal was carried during each cycle to the limit of its elasticity for the particular remanence to which it returned. For lower values of remanence the elastic limit is higher. The question would naturally arise as to the effect of a demagnetizing field that did not approach the elastic limit. The shape of the curves already taken would lead one to expect to find the change of flux due to a given demagnetizing field very nearly independent of the residual flux. This would follow from the fact that magnetic stress and strain have been found approximately proportional. Lord Rayleigh[1] has shown that the change of induction resulting from a very small change of field intensity is independent of the magnetic state of the iron, or to quote his conclusion, "That the value of the susceptibility to small changes of force is approximately independent of the initial conditions as regards force and magnetization until the region of saturation is approached." He was dealing with field strengths ranging from .04 gauss to .000018 gauss. The oscillographic method was applied to this problem, using a much larger demagnetizing field. The results may be made clear by referring to two curves produced by the same field intensity. In one case the residual flux density was 12650 and in the other 500 lines of induction per sq. cm. The maximum field intensity was 7.27 gauss. The reduction in flux density from the higher remanence was 440 while it was 475 from the lower. This represents a difference of about 8 per cent., and means that the susceptibility to small changes of field intensity increases as the remanence decreases. Similar results were obtained by the ballistic method.

Data for Demagnetization and Recovery Curves of Specimen B.

$$\text{Max. } H = \frac{4\pi \cdot 350 \cdot i}{10 \cdot 32.67} = 13.46\, i.$$

i being the current.

Current.	No. Turns in Secondary.	Maximum H.	Gauss per Division.
.27	1.25	3.6342	.0512
.54	.50	7.2684	.1191
.54	.50	7.2684	.1483
.64	.75	8.616	.1325
.79	.50	10.6334	.1715
.92	.50	12.385	.1567
1.06	.50	14.2676	.1719
1.17	.25	15.75	.2045
1.13	.50	15.21	.1925

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[1] Phil. Mag., 23, 1887, p. 225.

Data for Demagnetization and Recovery Curves of Specimen B.

Value of one division in gausses taken from preceding table. Value of one division in lines of magnetic induction determined by the calibration. H = field intensity. B = variation of induction.

H	B	H	B	H	B
.427	25	.476	23	.89	41
1.254	77	1.071	82	2.14	139
2.330	160	2.50	156	3.40	213
3.020	212	3.69	225	4.59	286
3.630	265	4.78	303	5.77	368
2.690	219	5.97	368	7.27	475
1.970	164	7.27	440	5.77	384
.947	77	5.35	327	4.37	303
.358	32	3.94	254	3.18	238
		2.91	180	2.07	156
		1.67	98	1.03	74
		.70	35		
1.06	58	1.95	110	1.88	122
2.78	150	4.13	240	4.39	262
4.19	228	5.90	344	6.11	393
5.51	322	7.70	483	8.15	540
7.50	439	9.06	573	10.18	667
8.62	511	10.63	704	12.38	835
6.62	408	7.35	532	9.40	647
5.24	319	5.64	426	7.13	516
3.84	233	3.93	311	5.33	377
2.32	139	1.79	172	3.13	221
.99	60			1.25	82
2.15	156	1.02	71	1.16	49
4.99	328	3.58	235	1.93	98
7.23	491	6.14	416	5.77	266
9.92	695	8.28	585	8.08	385
12.04	880	11.04	806	9.72	491
14.27	1064	13.50	1015	12.80	638
12.04	901	15.75	1194	15.21	761
8.94	737	10.84	871	12.03	610
6.88	573	8.59	699	8.18	454
4.30	360	5.83	487	6.45	361
2.06	180	3.07	267	1.35	74

VI. *Experiments with Specimen C.*

Specimen C is a ring of transformer iron of mean length 37.93 cm. and cross-sectional area 2.07 sq. cm. The primary coil has 325 turns, hence the magnetizing field is

$$\frac{4\pi\cdot 325\, i}{10\cdot 37.93} = 10.767\, i.$$

One division deflection of the galvanometer corresponds to a change of induction of 15.76 lines per sq. cm. when there are fourteen turns in the test coil.

When the demagnetization and recovery curves for this specimen were plotted it was apparent that they were somewhat different from those obtained from the steel specimens. The hysteresis seemed much larger. A part of this was due to the scale to which the curves were plotted, but when allowance was made for that, they still seemed to have an abnormally large area. This peculiarity of the curves for this specimen is undoubtedly due to eddy currents in the iron. These currents are in opposition to the time rate of change of magnetic flux and hence lag behind the magnetizing current by an amount somewhat in excess of 90°. They produce a magnetic field one component of which is in direct opposition to the applied field while the other reacts to cause a lag in phase of the field behind the current. Since the curves are plotted on the assumption that the applied field is proportional to the ordinate of the current curve, it is evident that the lag of induction behind the field intensity as given by the curves would be greater than the true hysteretic lag. When there is hysteresis without eddy currents the induction shows a lag behind the current at some points along the curve but none whatever at points of maximum or minimum field intensity. A difference of phase between the two curves on the photographs at their highest points is therefore evidence of the disturbing influence of eddy currents. Furthermore, if a method of correcting for eddy currents can be found, the phase relation of the highest points will furnish the criterion for determining the magnitude of the correction. Eddy currents are produced by the variation of the flux, ϕ, and their magnitude could be expressed as $K'\ (d\phi/dt)$, where K' is a negative constant. The current in the secondary, which operates one loop of the oscillograph, is also proportional to the derivative of the flux with reference to the time and the curves are marked $K\ (d\phi/dt)$ on the figures. Some fraction of $K\ (d\phi/dt)$ will equal $K'\ (d\phi/dt)$, in other words some fraction of the ordinate of the secondary curve on the oscillographic record will constitute the correction that should be applied to the corresponding ordinate of the current curve to give the ordinate that is proportional to the actual field intensity. It is comparatively easy to tell by inspection of the curves what this fraction is; or at least it may be found after two or three trials in applying the correction to the current curve. The flux and its time rate of variation are behind the current in phase at the highest points of the curves. If the fraction chosen be too small, the flux will be behind the corrected curve while if the fraction chosen be too large the flux curve will fall ahead of the new curve in phase at the highest points. A value of the fraction is chosen which places the corrected curve in phase at the highest point with the flux curve and then the ordinates of this new curve represent the resultant or actual field intensity applied to the iron. Fig. 9 shows the corrected curve with points indicated by the small circles. The correction applied to each ordinate of the curve was 13 per cent. of the corresponding ordinate of the secondary curve. The hysteresis for this specimen, as shown by the corrected curves, is found to differ very little from that shown by the other specimens for an equal change of flux.

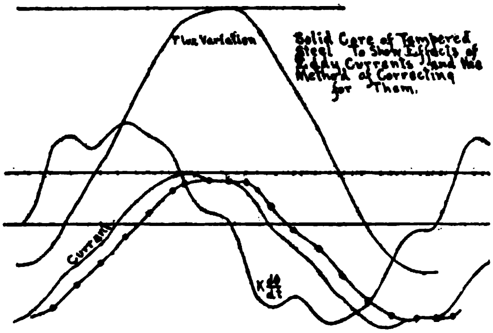

Fig. 9.

In order to further test this method of correcting for eddy currents, a solid magnet of tempered steel was used. Eddy currents would certainly be large in this case, but it was arranged to reduce end effects to a negligible magnitude by filling the small air gap with a carefully fitted block of iron. Fig. 10 shows the results. The curve plotted from the corrected data shows no more hysteresis than was found for the wire ring.

To obtain further data on the relative magnitude of the hysteresis in steel and soft iron for the kind of cycle here considered, a ring of soft iron wire was used. Wire having a diameter of .96 mm. was chosen. The wire of specimen *A* had a diameter 1.03 mm. If size of wire were all that determined the eddy currents effect, we should be assured of less disturbance from them with this new ring than was experienced with specimen *A*. However, in getting the total hysteresis curve this is not the case. With soft iron the hysteresis curve is nearly parallel to the axis of ordinates for a long way and hence the time rate of change of flux along this part of the curve is much greater than it is for hard steel where the slope of the curve is much less, hence in plotting the total hysteresis curve of soft iron, we should expect greater errors due to eddy currents than would arise in the case of hard steel. This argument, however, does not apply to demagnetization and recovery curves since the slope does not vary greatly from point to point along the curve and the time required for demagnetization or recovery is 1/120 of a second irrespective of the magnitude of the change. Hence for a given flux variation and for a given size of wire, effects due to eddy currents for soft iron ought not to differ widely from those in steel. Curve *D* of Fig. 12 was obtained from this new ring of soft iron wire. The phase difference at the maximum and minimum points of the current and flux curves was so slight that no attempt was made to correct for eddy currents. Apparently the hysteresis is somewhat larger than for the steel specimens. The conclusion is certainly justified that for cycles of the kind here considered the hysteresis of very hard steel is not greater than that of soft iron for an equal variation of flux density and it may be somewhat less.

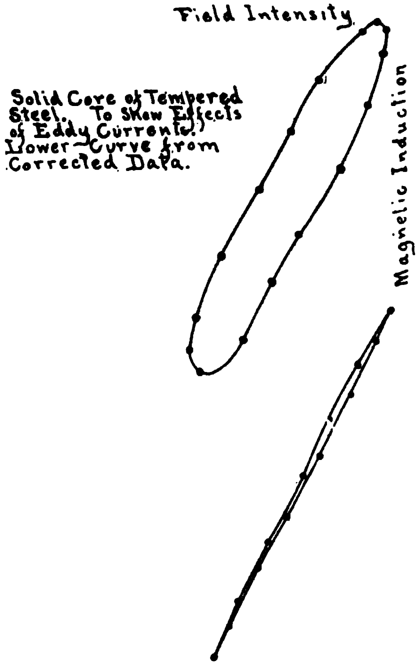

Fig. 10.

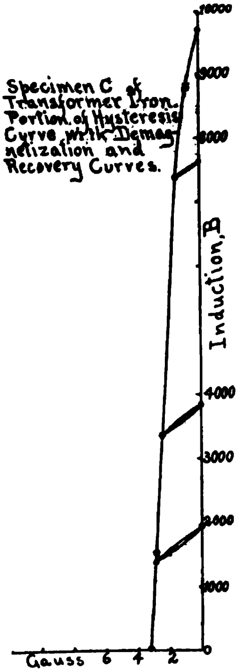

Fig. 11.

The hysteresis curve for ring *C* was plotted ballistically in Fig. 11 and the recovery curves drawn to the same scale as the hysteresis curve are also shown in their relation to the residual magnetic induction. The recovery in the region of small residual magnetism in this case is only about 6 per cent. of the maximum residual induction.

Fig. 12 is for the purpose of comparing the slopes of the curves for the different specimens.

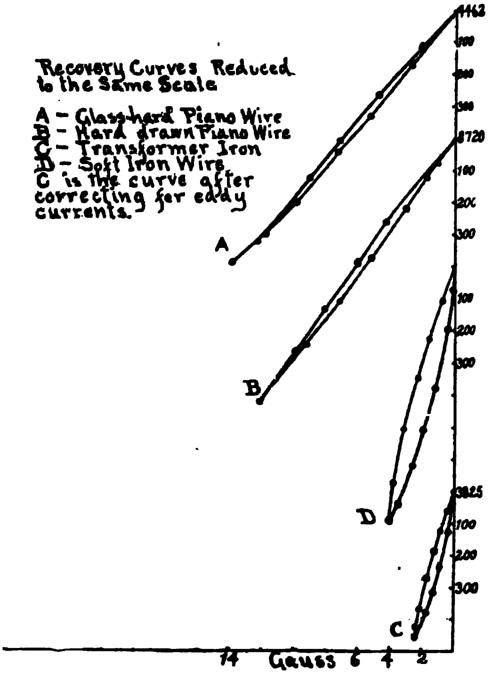

Fig. 12.

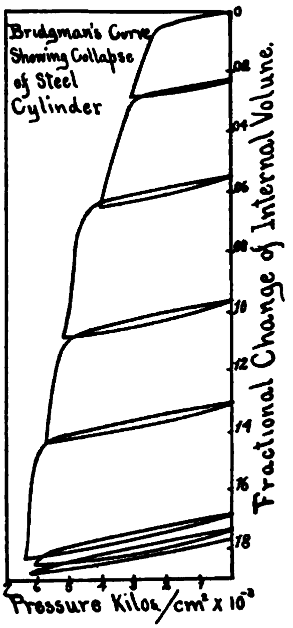

Fig. 13.

C. Conclusions.

1. An oscillographic method has been developed for plotting any kind of a magnetic cycle with an alternating current of 60 cycles provided eddy currents and end effects are negligible. The method is applicable when the flux variation is as small as 150 lines of induction and it may be used for any cycle in which the flux variation is greater than that amount.

2. It was shown that in a given specimen, for any value of residual magnetism, there is a definite elastic limit which is the maximum field that can be applied to the specimen without producing permanent demagnetization; and that if any field not exceeding the limiting field be applied and then removed, the specimen will return to its initial condition.

3. Within the elastic limit, magnetic stress and strain are approximately proportional, *i. e.*, the demagnetization is approximately proportional to the field intensity that produces it, provided the specimen returns to its original remanence when the field is reduced to zero.

4. The hysteresis as indicated by the area of the curves is small and is approximately the same for soft iron and hard steel.

5. The extreme points of the stable demagnetization and recovery curves lie inside the maximum hysteresis curve.

6. The susceptibility to small demagnetizing fields increases slightly with decrease of residual magnetism, the maximum difference observed being about eight per cent.

7. The recovery from the effect of a demagnetizing field is greater for low value of residual induction. For hardened steel it was found to be 1,960 lines of induction per sq. cm. or 24.5 per cent. of the maximum residual induction. For drawn piano wire it was 1,200 lines per sq. cm. or 8 per cent. of the maximum remanence and for soft iron it was 600 lines per sq. cm. or 6 per cent. of the maximum remanence. These numbers represent the elastic limits for low values of residual magnetism.

8. There is shown to be a well-defined upper limit beyond which the residual magnetism of an open magnetic circuit of steel can not be made to pass.

9. The slope of the demagnetization curves for hard steel is 57. For softer specimens it is larger and it reaches a value for very soft iron of about 180.

10. For hard steel the elastic limit is approximately proportional to the difference between the residual magnetism and the maximum residual magnetism.

The work was done in the physical laboratory of the University of Michigan under the direction of Professor K. E. Guthe. The author wishes to express his appreciation of Professor Guthe's interest in the work and his helpful suggestions while it was in progress.

University of Michigan,
Ann Arbor, June 1, 1912.

NOTE ON THE VELOCITY OF ELECTRONS LIBERATED BY PHOTOELECTRIC ACTION.

BY KARL T. COMPTON.

THE relation between the velocities of the electrons liberated by the action of light on metals and the wave-length of the incident light has been widely studied and with quite conflicting results. The superposition on the effect under investigation of secondary effects due to reflected light and reflected electrons, to variable contact difference of potential between the emitting and receiving electrodes, to electrostatic leakage, to impurity of the spectrum and to several other factors has proved of particular annoyance in this work. In recent work by Prof. O. W. Richardson and the writer[1] these sources of error seem to have been largely eliminated and quite consistent results were obtained supporting the view that the velocity of the liberated electrons is a linear function of the frequency of the incident light. By a quite independent method and at about the same time A. L. Hughes[2] reached similar conclusions. These results agree that the relation between the velocity and the frequency may be accurately expressed by an equation of the form

$$V = kn - w_0,$$

where V is the initial kinetic energy of the liberated electrons, expressed in equivalent volts, k and w_0 are constants and n is the frequency of the incident light. This is the type of equation which was first developed by Einstein[3] on the unitary theory of light and recently by O. W. Richardson[4] by thermodynamic and statistical methods which do not necessarily involve the unitary light theory. In it w_0 is the work, expressed in equivalent volts, which is done in carrying an electron from the inside to the outside of the metal and k equals $300h/e$, where h is Planck's constant and e is the charge on an electron. On these theories therefore

$$V = \frac{300h}{e} n - w_0.$$

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[1] Phil. Mag., Vol. 24, p. 576 (1912).

[2] Phil. Trans. of the Roy. Soc. of London, Series *A*, Vol. 212, p. 205 (1912).

[3] Ann. der Phys., Vol. 17, p. 146 (1905).

[4] PHYS. REV., Vol. 34, p. 146 (1912); Phil. Mag., Vol. 23, p. 615 (1912); ibid., Vol. 24, p. 570 (1912); Science, Vol. 36, p. 57 (1912).

The experimental results have accurately confirmed the linear relation between V and n, although they have led to values of h slightly smaller than the generally accepted value 6.55 $(10)^{-27}$ erg sec. The average value of k, for eight different metals, was found by Professor Richardson and the writer to lead to $h = 5.4\ (10)^{-27}$ erg sec., while Hughes' results for a number of metals lead to $h = 5.6\ (10)^{-27}$ erg sec.

A different formula has been derived by Jakob Kunz[1] on the theory that light consists of discrete vibrating Faraday tubes. He arrives at the conclusion that the energy of the liberated electrons varies as the square of the frequency and that the relation may be expressed by

$$V = sn^2 - w_0$$

where w_0 represents the work done by an electron in escaping from the metal and s is a constant.

In a recent paper Mr. David W. Cornelius[2] published results which claim to support Kunz's theory and to discredit the Einstein-Richardson formula. In this paper there are certain errors and inconsistencies which if corrected make the support of Kunz's theory much less convincing and which in fact seem to lend support rather to the theories of Einstein and Richardson. In view of the importance of settling this matter it has seemed to the writer worth while to briefly discuss the results of Mr. Cornelius and to compare the two types of theories in the light of evidence from direct experiment and from associated phenomena.

Mr. Cornelius experimented with potassium and cæsium in glass bulbs, prepared by melting or distilling *in vacuo*. He illuminated the metal by light from a carbon arc. The light was rendered parallel by a system of lenses and was then passed through a prism which directed the desired beam of light on to a slit in front of the photoelectric cell. Different wave-lengths of light were directed on the slit by rotating the prism.

With this arrangement Mr. Cornelius measured the maximum potential acquired by the receiving electrode when various wave-lengths of light were directed on the metal. He plotted these values of the potential against the corresponding wave-lengths and obtained a curve which is concave upwards. He then drew this conclusion: "The curve certainly is not a straight line as Planck's law would require." The fact is, however, that Planck's law (or Einstein's formula) requires a linear relation between the potential and the frequency and not between the potential and the wave-length. This last relation is represented by a hyperbola, as is shown in Fig. 1, and it is this curve with which Mr. Cornelius' curve should have been compared. The only justification for calling the curve a straight line would be in case only a small section of the spectrum were considered. Mr. Cornelius worked between wave-lengths 420 $\mu\mu$ and 750 $\mu\mu$, which represents the section *ab* of the curve in Fig. 1 and which has a distinct curvature. This test of the Einstein-Richardson formula is therefore misleading.

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[1] Phys. Rev., Vol. 29, p. 3 (1909); ibid., Vol. 30, p. 212 (1909).

[2] Phys. Rev., Vol. 1, series 2, p. 16 (1913).

The correct test of the two theories is given by plotting volts against frequency in the one case and against the square of the frequency in the other. I have done this for a typical set of Mr. Cornelius' readings for potassium in Fig. 2. Curve 1 represents the variation of the initial energy of the electrons with the frequency, Curve 2 represents this variation with the square of the frequency and Curve 3 represents the variation with the cube of the frequency. The units along the abscissa are 10^{15}, 10^{30} and 10^{45} respectively. According to Einstein's or Richardson's theories Curve 1 should be a straight line. According to Kunz's theory Curve 2 should be a straight line. As a matter of fact neither is a straight line and the experimental results fit most closely a variation with the cube of the frequency. It is true that the linear relation is rather more closely followed in Curve 2 than in Curve 1. However, this slight support of Kunz's theory is more than offset by the following evidence.

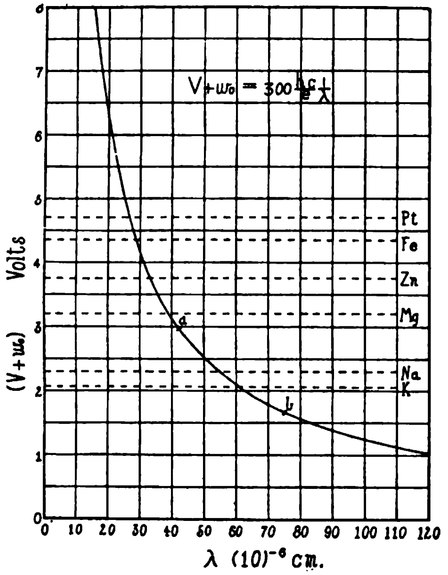

Fig. 1.

Assuming Kunz's theory to be correct, the slope of Curve 2 should give the constant *s*; and the point at which the curve, extended, cuts the axis of volts should give the constant w_0, or the work expressed in volts done by an electron in escaping from the metal. The position of intersection is rather uncertain owing to the curvature of the curve. But taking the most favorable possible case and drawing the straight line *ab* we find that it cuts the axis of volts at a point making $w_0 = 0.35$ volts. This is the largest value for the work done by an electron in escaping which we can obtain from the readings and it is about the same value which Kunz[1] calculates for several other metals according to his formula.

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[1] Phys. Rev., Vol. 31, p. 536 (1910).

But this value is impossibly small, being less than the average energy of thermal agitation at 23° C., which is equivalent to 0.38 volts. If it were correct it would require that in the course of their thermal agitation more than half of the electrons striking the surface of the metal perpendicularly during any interval of time would escape. Kunz's theory therefore leads to values of w_0 which are impossible and not even of the right order of magnitude.

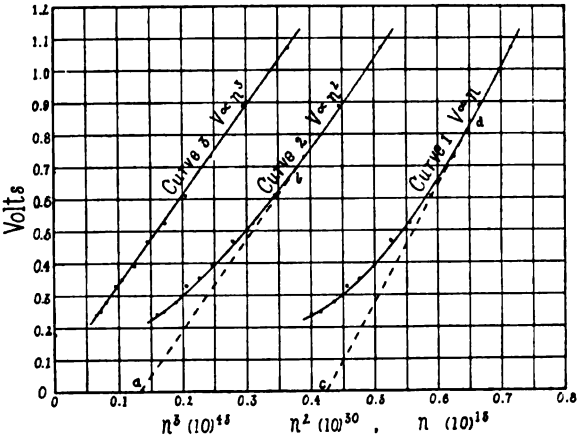

Fig. 2.

Consider now Curve 1 in relation to the Einstein-Richardson formula. Take the line *cd* to represent most favorably the linear relation between *V* and *n*. It will be shown later that the points on the lower part of the curve are less trustworthy than those above. The slope of this line is the constant *k* from which we should be able to derive Planck's constant $h = 6.55\ (10)^{-27}$ erg. sec. The actual slope leads to a value $h = 5.7\ (10)^{-27}$ erg sec., which is not a serious disagreement with the theory and which is very close to the values previously mentioned in connection with the results of O. W. Richardson and the writer and of A. L. Hughes. And the intersection with the axis of volts gives $w_0 = 1.6$ volts, which is not far from the value of the work done by an electron in escaping from potassium as indicated by other methods.

As a matter of fact measurements of w_0 have been made by three independent methods for a number of metals including platinum and the similar metal osmium.[1] Probably osmium gave the most reliable value as about 4.7 volts, and w_0 for platinum is certainly very near to this value. A calculation of w_0 for platinum from photoelectric measurements by a process similar to that followed in the preceding paragraph gave $w_0 = 4.4$ volts.[1] Thus for platinum, the only metal for which we have very reliable measurements of w_0 to use in testing the theories, the Einstein-Richardson formula is verified.

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[1] O. W. Richardson, Phil. Trans., *A*, Vol. 201, p. 497 (1903); Vol. 207, p. 23 (1906); H. A. Wilson, ibid., Vol. 202, p. 243 (1903); F. Deininger, Ann. der Physik, Vol. 25, p. 296 (1908); O. W. Richardson and H. L. Cooke, Phil. Mag., Vol. 20, p. 173 (1901); H. L. Cooke and O. W. Richardson, Phys. Rev., series 2, Vol. 1, p. 71 (1913).

It is possible however to estimate w_0 for potassium and other metals indirectly from the contact difference of potential between platinum and the metal considered, for O. W. Richardson[2] has shown that for two metals s and p

$$w_s - w_p = e\left\{V_p - V_s - \theta\frac{\partial}{\partial\theta}\left(V_s - V_p\right)\right\},$$

where $V_p - V_s$ is the contact difference of potential between the metals and e is the charge on an electron. The last term which represents the Peltier effect may be neglected and therefore, knowing $V_p - V_s$ and w_p, the work in ergs necessary to withdraw an electron from platinum, we are able to calculate w_s for the metal considered. In fact the difference between the works required to extract an electron from each of two metals, expressed in equivalent volts, is simply equal to the contact difference of potential between the metals. There is abundant qualitative support for this and it is in harmony with the facts of thermionic emission. If we determine w_0 for potassium in this way we find that $w_0 = 2.05$ volts, which is not far from the value found from Curve 1.[3]

Later in his paper Mr. Cornelius calculates theoretically the work done in the escape of an electron moving to infinity against the electrostatic attraction of the parent atom, from a distance equal to the radius of the atom. It has long been known that such a calculation gives values for w_0 of the right order of magnitude. The amount of work which must be supplied by the light to expel an electron from potassium is thus calculated by Mr. Cornelius to be $4.57\ (10)^{-12}$ ergs. The interesting fact is that if this value is reduced to equivalent volts it comes out equal to 2.45 volts, which is in good agreement with the requirements of the Einstein-Richardson formula and is very far from being consistent with Kunz's theory.

Thus we see that we are led to inconsistent and impossible conclusions if we accept Mr. Cornelius' results as a support of Kunz's theory. If, on the other hand, we apply these results to test Einstein's and Richardson's theories we find them fairly consistent with the theory and with related facts of thermionic emission and contact difference of potential.

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[1] O. W. Richardson and K. T. Compton, Phil. Mag., Vol. 24, p. 576 (1912).

[2] Phil. Mag., Vol. 23, p. 264 (1912); Vol. 23, p. 615 (1912).

[3] In this connection Mr. J. W. Woodrow (PHYS. REV., Vol. 35, p. 203 (1912)) in discussing the contact difference of potential between platinum and potassium takes a value from Winkelmann's Handbuch der Physik which is valid only for the metals in contact with electrolytes and is probably not applicable to the case under consideration.

The fact remains however that, while Curve 1 has the better of the argument, it is not in as good accord with the Einstein-Richardson formula as might be expected if this formula is the correct one. In the first place it is not a straight line as the theory demands. In the second place it is quite improbable that there is really any photoelectric emission from potassium at a wave-length as long as 750 $\mu\mu$, which is the longest wave-length used in the measurements under discussion. In fact it is probable that photoelectric emission from potassium takes place only for wave-lengths shorter than about 600 $\mu\mu$. The evidence for this is found in the paper by O. W. Richardson and the writer referred to previously and cannot be gone into fully here. Suffice it to say that the behavior of eight metals, sodium, magnesium, aluminum, zinc, tin, bismuth, copper and platinum indicated that there is a certain critical frequency characteristic of each metal below which there is no photoelectric emission. In the notation already used this critical frequency is accurately given by

$$\frac{300h}{e} n = w_0$$

and is seen, on Einstein's or Richardson's theories, to be that frequency at which the electron acquires just sufficient energy to enable it to escape from the metal. Also results by R. Pohl and P. Pringsheim,[1] who have done very careful work with metals of the alkali group including potassium, bear out this relation, although they were not interested in this particular point. In order to completely remove the argument against the Einstein-Richardson formula it is necessary to account for these two discrepancies.

These variations of the experimental results from the theory might be explained by supposing a thermionic emission superimposed on the photoelectric emission. It is well known that the very electropositive metals like potassium and cæsium with fresh clean surfaces emit electrons in the dark at ordinary room temperatures. It would seem possible that far in the red end of the spectrum where the radiation energy is large, the illuminated surface might be locally heated sufficiently to produce very distinct thermionic currents. A quantitative estimate of the effect possible from this cause may be obtained as follows. In the theory of current-flow in an infinite conductor it has been shown that the resistance offered to the flow of electricity (or heat) from an area on the surface of the infinite conductor into its interior is[1]

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[1] Verh. d. D. Phys. Ges., 12, p. 215 (1910); 12, p. 349 (1910); 13, p. 474 (1911); 14, p. 46 (1912).

$$R = \frac{1}{k}\frac{0.82a}{\pi a^2},$$

where k is the specific conductivity and a is the radius of the area. In our problem we may consider the main body of the potassium to be an infinite conductor with its boundaries at a temperature T_0 and consider the flow of energy across the illuminated area πa^2. If E is the amount of radiant energy incident on unit area of the surface in a unit time and if T is the temperature to which the surface is thereby raised we have obviously in the steady state

$$\frac{dQ}{dt} = \frac{T - T_0}{R} = \frac{T - T_0}{\frac{1}{k}\frac{0.82a}{\pi a^2}} = E\pi a^2,$$

whence

$$T - T_0 = \frac{0.82Ea}{k}$$

gives the rise in temperature at the area of the surface considered. Taking values of a and of E calculated from the probable energy in the arc, the area of the slits in the spectrometer and so on, and neglecting all absorption or reflection by lenses and prism, I calculated the most favorable case I could reasonably construct and found a temperature increase of 5° C. In any actual case it is probably very much smaller than this and this is too small to account adequately for the failure of the curve to conform accurately to the Einstein-Richardson formula.

A much more probable explanation lies in the experimental arrangement described by Mr. Cornelius as follows: "The light [from the arc] passes through a slit and a system of lenses which gives a beam of parallel light upon the prism P, which is capable of rotation, thus providing for an intense source of light, which after passing through the prism, falls upon the slit S of a light-tight box containing the photoelectric cell." The point is that a beam of parallel light strikes the prism and is refracted thence to the photoelectric cell *without being brought to a focus on the slit* in front of the cell. Both in his description and his diagram he describes this arrangement. It is obvious that without the lines of the spectrum being brought to a focus there will be great impurity in the light falling upon the slit. He calibrates his instrument by means of the sodium lines.

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[1] Lord Rayleigh, Theory of Sound, Vol. 2, Appendix *A*.

But if the light from the lenses were parallel for the yellow light it would be slightly divergent in the red part of the spectrum unless an achromatic lens system were used to render the light parallel. This divergence, together with the relatively low dispersion in the red spectrum, would still further increase the impurity of the light in the region of the long wavelengths. The value of the wave-length recorded would correspond to the setting of the instrument and to the average wave-length falling upon the slit. But the maximum potential would be a function only of the shortest wave-length present, which would be quite different from the wave-length recorded. I am not sure but what the experimental arrangement used by Jakob Kunz[1] is not to a smaller extent open to this same criticism. At any rate the error due to impurity in the spectrum would have exactly the effect of changing an Einstein-Richardson curve into one that would more or less fit Kunz's theory, and it seems certain to have played a part in the results of the paper under discussion.

In view of these facts one should not expect more than a very rough agreement between Mr. Cornelius's results and the correct theory and one should expect the agreement to be poorest in the region of long wavelengths. Taking everything into consideration the agreement with the Einstein-Richardson formula is all that could be expected. But from the evidence given earlier in the paper it is impossible to find in the results of Mr. Cornelius any valid support for the theory of Kunz.

Mr. Cornelius and several others who have been working along this line recently with the alkali metals have been bothered by the fact that frequently a freshly prepared photoelectric cell gives no photoelectric effect until some time after it is made. Then the size of the currents and the maximum potential increase to a maximum and reach a steady state in the course of a few days. The steady state is usually reached more quickly if the cell is heated. This is probably due to a varying contact difference of potential between the surface of the potassium and the platinum receiving electrode. It has been shown[2] that when both the electrodes are earthed there is an effective field between their surfaces which is equal, in the case of potassium and platinum, to about 2.75 volts. The greatest initial kinetic energy found among the electrons liberated from potassium by light in the visible spectrum does not exceed about one volt. Therefore, unless an external field is applied to aid their escape, no electrons will succeed in reaching the receiving electrode. Mr. Cornelius and the others do not seem to have taken this into account. But potassium has a vapor pressure and will slowly distill over on to the platinum at ordinary temperatures, there forming an alloy or a surface layer of potassium and thus decreasing the contact difference of potential until the electrons are able to escape, and finally reaching the state where this field between the electrodes has practically vanished. Where an external field has been applied to neutralize the contact difference of potential all the evidence shows that the fresher and cleaner the surface the larger the photoelectric currents.

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[1] Phys. Rev., Vol. 30, p. 212 (1909).

[2] K. T. Compton, Phil. Mag., Vol. 23, p. 579 (1912).

The relation between frequency, velocity and metal as given by the Einstein-Richardson formula may be most clearly understood by an examination of Fig. 3, which represents all the requirements and implications of the formula. As abscissæ are plotted frequencies, with corresponding wave-lengths indicated below. As ordinates are plotted the values of the total energy, expressed in equivalent volts, acquired by the electrons, which in any case consists of an amount w_0 used in getting away from the metal plus an amount V which represents the initial kinetic energy with which they leave the metal. The diagonal line has a slope $300h/e$. All measurements of electron velocities for any metal or frequency should fall on this line provided the contact difference of potential has been allowed for and the correct value of w_0 is used.

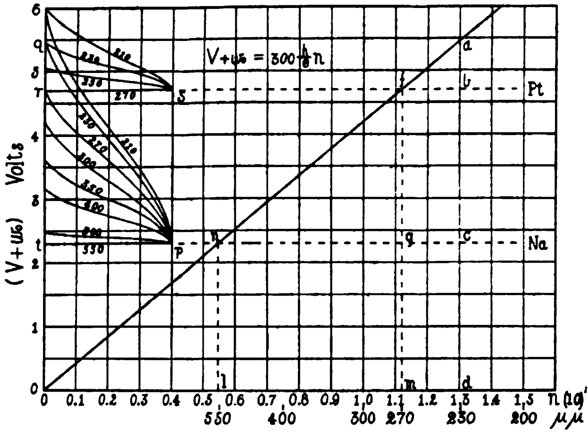

Fig. 3.

Take for instance the case of platinum for which, as we have seen, we may take $w_0 = 4.7$ volts. The horizontal line marked *Pt* represents $w_0 = 4.7$ volts. Thus for any wave-length, such as 230 $\mu\mu$, the ordinate *ad* represents the total energy acquired by an electron, of which an amount *bd* is lost before the electron gets free of the platinum; whence *ab*, or 0.75 volts, gives the maximum initial kinetic energy with which an electron leaves platinum illuminated with light of wave-length 230 $\mu\mu$.

In a similar manner the electron velocities due to any other wave-length on platinum may at once be predicted. In particular, at a wave-length 270 $\mu\mu$ the energy *fm* is just sufficient to take the electron from the surface of the metal and no more; for longer wave-lengths no electrons can escape and the metal appears photoelectrically inactive. This point is a strong support of Richardson's theory, since he gets as solutions for his theoretical equations[1]

$$V = \frac{300h}{e} n - w_0 \quad \text{when} \quad w_0 < \frac{300\,h}{e} < \infty$$

and

$$N = 0 \quad \text{when} \quad 0 < \frac{300\,h}{e} n < w_0;$$

showing that the number N of emitted electrons is zero for all values of frequency less that the critical value

$$n = \frac{w_0 e}{300\,h}.$$

Take again the case of sodium. The contact difference of potential between platinum and sodium is 2.4 volts. The line marked Na is drawn at a distance 2.4 volts below the line Pt and its height 2.3 volts above the axis represents w_0 for sodium. We see therefore that the line *ac*, or 3.15 volts, represents the maximum initial kinetic energy from sodium for a wave-length 230 $\mu\mu$; *fg*, or 2.4 volts, is the energy for 270 $\mu\mu$; and the critical wave-length is 550 $\mu\mu$. The properties of photoelectric emission from other metals may similarly be represented by drawing corresponding lines, some of which are shown in Fig. 1.

This diagram also fixes the terminal points of the distribution of velocity curves for any given metal and wave-length. As examples I have indicated several of these curves in the upper left-hand corner of the diagram. To be interpreted, *rs* and *tp* represent, in the cases of platinum and sodium respectively, the axis along which is plotted the number of emitted electrons; while the distances from *r* and *t* respectively to the feet of the various curves represent the maximum initial energies. The location of the foot of each curve is determined from the main part of the diagram. For instance, for platinum and wave-length 230 $\mu\mu$, *rq* is taken equal to *ab*, and so on. If Fig. 3 is held with the left side down these distribution of velocity curves appear in their ordinary form.

This figure represents probably as clearly and compactly as can be done the various requirements and implications of the Einstein-Richardson formula. These points have all received quite conclusive support, especially in the work of A. L. Hughes and of O. W. Richardson and the writer mentioned at the beginning of this paper. The predicted values of many of the critical wave-lengths receive additional support in the many papers by R. Pohl and P. Pringsheim[1] and others.

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[1] Phil. Mag., Vol. 24, p. 570 (1912); Science, Vol. 36, p. 57 (1912).

These considerations apply to the relations between frequency, metal and velocity. In addition to giving these relations, Richardson's theory leads to an expression for the number of electrons emitted by any given metal under the influence of a unit amount of light energy of a given wave-length. Experiments on this relation between frequency, metal and number are now in progress at this laboratory. It is interesting to notice that Richardson's theory requires a maximum rate of emission at a wave-length characteristic of each metal and is to this extent supported by the experiments of Pohl and Pringsheim.

It is the hope of the writer that this discussion, by calling attention to various points that must be considered, to the effect of certain systematic errors and to some of the results which have been obtained and the various evidence which supports them, may lead to a clearer understanding of the problem and thence to more accurate results than have hitherto been obtained.

Palmer Physical Laboratory,
Princeton, N. J.

[1] Op. cit.

THE VARIATION OF THE ALPHA-RAY IONIZATION OF RADIOACTIVE SOLIDS WITH THE THICKNESS OF THE LAYER.

By Herbert N. McCoy.

THE subject indicated by the title of this paper was first studied by Rutherford and McClung,[1] who assumed an exponential absorption law. Later the writer[2] studied the matter extensively after having devised a method of making very thin layers or films of uranium compounds of quite uniform thickness. The results were in satisfactory agreement with an exponential absorption law. After Bragg's discovery that an α particle has a definite range, it became evident that there could be no true "absorption" of the α rays and therefore that an exponential law could not serve as a logical explanation of the facts. Bragg,[3] St. Meyer and v. Schweidler[4] and others have studied the closely related problems of the activity of a thick or thin active layer covered with one or more thin foils.

The work of Bragg and Kleeman[5] and also McClung[6] supplemented by the more recent work of Geiger[7] and Taylor[8] shows the true shape of the ionization curve. Geiger found that the ionization at any point in the path of a particle is inversely proportional to the cube-root of the remainder of its range. This leads to the conclusion[9] that the total ionization of an α particle of range R is proportional to $R^{2/3}$. Although it is very probable that this empirical law is not absolutely exact it is at least a close approximation and as such is very useful in several ways. In a paper just published with C. H. Viol,[10] it was shown that the relative activities of the α-ray products of thorium are very closely proportional to the $2/3$ powers of the ranges of their respective α-rays. I have also found a similar result in a still more recent, unpublished, study of radium and its products.

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[1] Phil. Trans., A, *196*, 25 (1901).

[2] Jour. Amer. Chem. Soc., *27*, 391 (1905).

[3] Phil. Mag., *11*, 754 (1906).

[4] Wien Sitzungsberichte, *115*, II. *a*, 736 (1906).

[5] Phil. Mag., *8*, 726 (1904); *10*, 318 (1905).

[6] *Ibid.*, *11*, 131 (1906).

[7] Proc. Roy. Soc., A, *82*, 486 (1909).

[8] Amer. Journ. Sci., *28*, 357 (1909); Phil. Mag., *21*, 571 (1911); *23*, 670 (1912).

[9] Taylor, Phil. Mag., *21*, 571 (1911).

[10] Phil. Mag., *25*, 333 (1913).

If we assume that the total number of ions produced in air by any α particle is proportional to the 2/3 power of the distance it travels in air after leaving a radioactive layer, we may readily get an expression for the activity of such a layer as a function of its thickness. Let $n =$ the number of α particles per second from 1 g. of the substance. The number shot upward from 1 sq. cm. of a layer of thickness $dl = \frac{1}{2} n\rho dl$, where ρ is the density. These go equally to every element of a circumscribed hemisphere of surface $2\pi a^2$ ($a =$ radius). Those rays which emerge between the angles θ and $\theta + d\theta$ to a normal strike a zone of area $2\pi a^2 \sin\theta d\theta$ and therefore constitute the fraction $\sin\theta d\theta$ of the rays shot upward; so that the number of rays reaching the zone is $\frac{1}{2} n\rho \sin\theta d\theta dl$. Now suppose the current due to a single α ray is $Cr^{2/3}$ where C is a factor which is constant for the α rays of all substances and r is the effective air range. If R is the range when there is no retardation in the solid, it is readily seen that $(R - kl/\cos\theta) = r$ is the effective range of the emerging ray, if l is the depth from which the α particle comes and k is the ratio of R to the "range" l_0 of the α particle in the solid; that is $k = R/l_0$. The current due to rays between θ and $\theta + d\theta$ from an infinitesimal thickness dl at a depth l from the upper surface is $\frac{1}{2} n\rho C(R - kl/\cos\theta)^{2/3} \sin\theta d\theta dl$ and for the whole of the rays is

$$dA = \tfrac{1}{2} n\rho C \int_0^{\theta} \left(R - \frac{kl}{\cos\theta}\right)^{2/3} \sin\theta d\theta dl,$$

where the upper limit of $\theta = \cos^{-1} kl/R$; since if $\theta > \cos^{-1} kl/R$ the α rays from the lower surface do not get out of the film. If l_0 is the "range" in the solid and $l/l_0 = x$; x is the thickness of a film in terms of the thickness just necessary to give maximum activity. Since $k = R/l_0$; $kl/R = l/l_0 = x$ and therefore

$$dA = K dx \int_0^{\cos^{-1}x} \left(1 - \frac{x}{\cos\theta}\right)^{2/3} \sin\theta d\theta,$$

where $K = \frac{1}{2} n\rho C R^{2/3} l_0$, a constant for a given substance. Therefore the ionization current of a film of thickness x is

$$A = K \int_0^{x} dx \int_0^{\cos^{-1}x} \left(1 - \frac{x}{\cos\theta}\right)^{2/3} \sin\theta d\theta.$$

If the indicated integral of function θ is written $f(x)$ then

$$A = K \int_0^{x} f(x) dx.$$

As the θ integration gave a very complex expression which presented difficulties for the x integration, the ordinary method was abandoned and the integrations accomplished without difficulty, by graphic methods. The results are shown in Table I., in which A is expressed in terms of the maximum activity as unity, the numbers in the last column being proportional to those in the third column.

TABLE I.

x	$f(x)$	$\int f(x)dx$	A
0.0	57.30	0.00	0.000
0.1	41.58	4.87	0.284
0.2	31.85	8.52	0.497
0.3	24.06	11.31	0.659
0.4	17.67	13.39	0.780
0.5	12.61	14.90	0.869
0.6	8.44	15.95	0.930
0.7	5.01	16.59	0.967
0.8	2.44	16.96	0.989
0.9	0.76	17.12	0.998
1.0	0.00	17.16	1.000

The graph of A as a function of x is practically identical with an exponential curve up to about $A = 0.5$; while for larger values of x it lies only 1 to 3 per cent. higher. Since my earlier work on uranium compounds and also that of Goettsch[1] had shown agreement with the exponential law within the experimental error of less than 1 per cent., excepting for films of nearly maximum activity where the deviation was a little greater, it follows at once that the facts are also in good agreement with the formula developed above. However the earlier experiments did not conform strictly to the conditions assumed in the derivation of the formula in that the films were made in shallow tins, having rims 0.8 cm. high instead of on flat plates; also, the uranium used contained UX the β rays of which produced a measurable part of the observed activity and finally uranium itself gives two sets of α rays of different ranges.[2]

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[1] Jour. Amer. Chem. Soc., *28*, 1540, 1906.

[2] The suggestion that uranium gives two sets of α rays has occasionally been credited to Boltwood, Amer. Jour. Sci., *25*, 298 (1908). I should like to point out that the first announcement of this idea, as far as I can find, was that by Dr. Ross and myself at the Chicago meeting of the Amer. Phys. Soc., Dec. 1, 1906. The abstract of this paper dated November 23, 1906, appeared in the PHYS. REV., January, 1907. In this paper as a result of our measurements of the relative activities of uranium and radium in minerals we concluded that "Since uranium is about twice as active as the equilibrium amount of radium the former must produce two α particles for every one produced by the equilibrium amount of the latter. This indicates that each atom of uranium produces upon disintegration two alpha particles or that there is an alpha-ray product not yet isolated between uranium and uranium X." Dr. Boltwood,

The first two conditions could easily be corrected experimentally, while the effect of two sets of α rays could be treated theoretically.

For an infinitely thin film $f(x) = 57.30$; while for $x = 1, \int f(x)dx = 17.16$; Table I. The ratio $\frac{17.16}{57.30} = 0.2994$ is the activity of a film of just sufficient thickness, l_0, to have maximum activity, in terms of the activity of the same mass of material spread into an infinitely thin film on a flat surface. This ratio may be taken practically as 0.3. The activity of a thick film of a uranium compound, say U_3O_8, is the sum of the partial activities of the two sets of α rays. If we call the maximum partial activities of 1 sq. cm. A_1 and A_2 respectively, we may easily find theoretically the relative magnitudes of A_1 and A_2. Let W_1 be the weight of 1 sq. cm. of a film just thick enough to have maximum activity for the U_1 rays (those of uranium itself) and W_2 be the weight for U_2 rays. Let a_1 be the activity due to U_1 rays of 1 g. in an infinitely thin film on a flat plate and a_2 be the activity for the U_2 rays. Then $A_1 = 0.3 W_1 a_1$ and $A_2 = 0.3 W_2 a_2$ but $W_1/W_2 = R_1/R_2$ where R_1 and R_2 are the respective ranges and

$$\frac{a_1}{a_2} = \left(\frac{R_1}{R_2}\right)^{\frac{2}{3}}.$$

Therefore

$$\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^{\frac{5}{3}}.$$

For uranium Geiger[1] has found $R_1 = 2.37$ and $R_2 = 2.75$ at 0° and 760 mm. Therefore $A_1 = 0.781\, A_2$ and $a_1 = 0.906 a_2$; also

$$A_1 + A_2 = 0.3 \times 0.934 (a_1 + a_2) W_2.$$

If a film is thick enough to have maximum activity for the more penetrating U_2 rays the activity due to the rays of uranium itself (U_1) should be 0.781 of the activity due to U_2. Maximum activity for U_1 rays would be reached for a thickness $R_1/R_2 = 0.862$ as great as is required for the U_2 rays. By plotting a curve with x and A as coördinates and adding

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whose first communication on this subject is in a letter dated December 17, 1906, to Nature, January 3, 1907, wrote: "It will be noted in the above that the activity of uranium is about twice that of the radium present, which is in good agreement with the conclusion of Moore and Schlundt that there are two alpha ray changes in uranium if it is assumed that the average range of the two uranium particles is about 3.5 cm." Dr. Boltwood's conclusion was based, therefore, on the same facts (obtained in independent experiments) as our own. While it is true that Moore and Schlundt did think that UX gives alpha rays, thus suggesting a second alpha-ray change in uranium; this was disproved later and so has nothing to do with the matter in question. See also McCoy and Ross, Jour. Amer. Chem. Soc., *29*, 1708 (1907).

[1] Phil. Mag., *24*, 653 (1912).

to it a curve of the same form, but having its maximum ordinate 0.781 of that of the first curve while its maximum abscissa is 0.862 we get the theoretical curve for the total activity of a film of any uranium compound as a function of its thickness, where the thickness is expressed as a fraction of that required for maximum activity of the U$_2$ rays. (See Fig. 1.) Table II. gives the total activities for various values of x. It is seen by comparison with Table I. that the total activities thus found are, for each value of x, a little greater than for a substance giving but one set of α rays.

Table II.

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Activity	0.300	0.522	0.686	0.810	0.894	0.945	0.977	0.993	0.999	1.000

To test the accuracy of these deductions as carefully as possible new experiments were made with films of U$_3$O$_8$. Pure uranium nitrate was completely freed from UX and also any possible trace of radium by means of barium sulphate in the usual way. The nitrate was decomposed by heat and the oxide heated to constant weight at 700° in an electric muffle.[1] The product so obtained was pure U$_3$O$_8$. This material was made into films on flat circular brass or copper plates in the manner previously described,[2] alcohol being used as the suspending liquid. The films were dried at 105°; those that showed any unevenness or imperfection were discarded. The activity measurements were made in the gold-leaf electroscope described by McCoy and Ashman. The errors of measurement were not over 0.2 to 0.3 per cent.

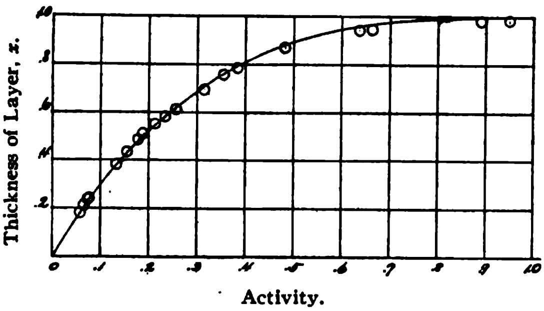

Fig. 1.

Theoretical curve with experimentally determined points.

Two series of films were made; the first on copper plates 7.00 cm. in diameter; the second on brass plates 6.914 cm. in diameter. The results are given in Table III. and plotted in Fig. 1, where the points are seen to fall very close to the theoretical curve.

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[1] McCoy and Ashman, Amer. Jour. Sci., *26*, 523 (1908).

[2] McCoy and Ross, Jour. Amer. Chem. Soc., *29*, 1699, 1907.

TABLE III.

First Series.			Second Series.		
Weight in Mg. per. Sq. Cm.	Observed Activity.	Calculated Activity.	Weight in Mg. per Sq. Cm.	Observed Activity.	Calculated Activity.
0.544	0.179	0.165	0.775	0.240	0.224
0.683	0.217	0.200	0.812	0.243	0.232
1.385	0.392	0.375	1.878	0.485	0.478
1.637	0.437	0.430	1.950	0.498	0.491
1.996	0.512	0.500	2.291	0.569	0.552
2.258	0.551	0.545	3.780	0.766	0.760
2.481	0.590	0.594	5.119	0.867	0.879
2.715	0.612	0.616	9.458	0.980	0.997
3.339	0.693	0.706	10.09	0.985	0.999
3.768	0.759	0.758	12.83	0.991	1.000
4.074	0.783	0.794	13.57	1.000	1.000
6.764	0.941	0.959	16.37	1.000	1.000
7.033	0.946	0.966			
19.87	1.000	1.000			

In order to calculate the theoretical activities it is first necessary to find a factor which converts the weight into the fraction which this weight is of that necessary to give maximum activity. This factor will be a constant for all films of U_3O_8; it may be found from any one determination or the mean of several. This factor proved to be 94; so that $x = 94 W$ where W is the weight in g. per sq. cm. The values given as "calculated activity" are those corresponding to the value of $x = 94 W$ indicated by the theoretical curve (Fig. 1). We see that the agreement between facts and theory is quite close; not, perhaps, within the experimental error. But this is not to be expected, as some of the assumptions made in deriving the formula are not absolutely exact. Thus, the assumption that the factor k is a constant for a given substance is not strictly in accord with the fact that the stopping power of a metallic foil is somewhat dependent upon the velocity of the α particle.[1] The error caused by taking k constant is not large and probably would not greatly change the theoretical curve.

By means of an equation given in a foregoing paragraph it is easy to get a second test of the accuracy of the theoretical assumptions made in this paper. It was shown that

$$A_1 + A_2 = 0.3 \times 0.934\,(a_1 + a_2)W_2.$$

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[1] Bragg, Phil. Mag., *13*, 507 (1907); Taylor, Amer. Jour. Sci., *28*, 357 (1909); Phil. Mag., *18*, 604 (1909).

If we take the α-ray activity of 1 sq. cm. of a thick film of U_3O_8 as the unit of activity, then $A_1 + A_2 = 1$ and

$$a_1 + a_2 = \frac{1}{0.3 \times 0.934 W_2}$$

$a_1 + a_2$ is the activity that 1 g. of U_3O_8 would have in the form of an infinitely thin film on a flat surface. The value of $a_1 + a_2$ is readily found from the weight and observed activity of films as given in Table III. The ratio of activity to weight increases with the thinness of the film and reaches a maximum for a vanishingly thin film. By a short and quite definite extrapolation of this ratio, as determined by the values given in Table III., it was found that $a_1 + a_2 = 335$.[1] Since $x = 94W$ if $x = 1$, $W_2 = 1/94 = 0.01064$, the weight per sq. cm. just necessary to give the maximum U_2 activity. By substituting this value in the above equation $a_1 + a_2 = 336$. The close agreement between the values found in the two ways is further proof of the correctness of assumptions made in the theoretical discussion.

The "range" of the α rays in U_3O_8 is easily calculated from the above data. If ρ, the density of U_3O_8, is taken as 7.31,[2] l_0, the range of the U_2 rays $= 0.01064/7.31 = 0.00145$ cm. The range of the U_1 rays is $R_1/R_2 \times 0.00145 = 0.00125$ cm. Then the constant $k = R_2/l_0 = 1890$ for both sets of rays. By substituting for the ranges in air and U_3O_8 the weights per sq. cm., which an α ray can penetrate we get a constant into which a knowledge of the density of U_3O_8 does not enter. The mass of a column of air 1 sq. cm. in section and 2.75 cm. in length is 0.00355 g.; while $W_2 = 0.01064$ g. The ratio of the latter weight to the former is 3.01. This is the ratio of the weights of U_3O_8 and air which produce the same retardation of an α particle; it is the same for both sets of rays of U_3O_8.

The relative "absorption" or, properly speaking, retardation of the α rays in passing through matter of different kinds has been studied extensively. Bragg and Kleeman[3] found that the stopping power of a substance was an atomic property and for equal masses per sq. cm. nearly inversely proportional to the square-root of the atomic weight. I arrived at similar conclusions[4] in a somewhat different way from that of Bragg and Kleeman. The conclusions reached in the preceding paragraph may easily be compared with the results expected by the square-root law. By this law we calculate that 59.1 per cent. of the retardation that an α particle experiences in passing through a layer of U_3O_8 is caused by the uranium and 40.9 per cent. by the oxygen. Since 0.01064 g. of U_3O_8 containing 84.8 per cent. of uranium, will completely stop a U_2 particle; the same result would be produced by 0.0153 g. of uranium per sq. cm. or by 0.00400 g. of oxygen. The corresponding calculated values for nitrogen and air are 0.00374 g. and 0.00377 g. The ratio of 0.01064 to 0.00377 is 2.82; while the experimental result is 3.01. The calculated ranges in air at 0° and 76 cm. are 2.52 cm. and 2.92 cm. for U_1 and U_2 instead of 2.37 cm. and 2.75 cm. as found by Geiger. The differences are probably largely due to inexactness of the square-root law.

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[1] The value, 335, is only half of the total activity, as half of the rays are absorbed in the plate on which the film is made. The total activity is therefore $2 \times 335 = 670$. Since U_3O_8 contains 84.8 per cent. of uranium, this corresponds to an activity of 790 for 1 g. of uranium. This is exactly the value I found in the earliest determination of this constant: Jour. Amer. Chem. Soc., *27*, 391 (1905); while with Dr. Ross, later, *Ibid.*, loc. cit., the value found was 796.

[2] Ebelman, Ann. Chim. phys. (3), *5*, 189 (1842).

[3] Phil. Mag., *10*, 318 (1905).

[4] Jour. Amer. Chem. Soc., *28*, 1555 (1906).

SUMMARY.

1. A formula has been developed for the α-ray activity of a layer of a radio-active solid as a function of its thickness. It was assumed that the air range of an α-ray is diminished in its passage through the solid by an amount directly proportional to the distance it travels in the solid before emerging and that the ionization produced is proportional to the 2/3 power of the distance the α-ray then travels in air. The theoretical curve for the rise of activity with increasing thickness of the layer is almost the same as an exponential curve, although the fundamental equations are entirely different.

2. It has been shown that the activity of a film of any α-ray substance just thick enough to give maximum activity is 0.3 of the activity that the same material would have if spread on a flat surface in an infinitely thin film.

3. It has been shown that the partial activities of a thick film which gives equal numbers of α-rays of ranges R_1 and R_2 are in the ratio $(R_1/R_2)^{\frac{5}{3}}$.

4. It has been found experimentally that the activities of films of U_3O_8 were in good agreement with those calculated by theory.

5. The weights per sq. cm. of U_3O_8 and of air which produce equal retardation of an α-ray are as 3.01 to 1. This ratio is approximately that calculated by the law, first stated by Bragg and Kleeman, that the stopping power of matter is approximately inversely proportional to the square-root of the atomic weight of the material through which an α-ray passes.

In the experimental part of this work I have been ably assisted by Mr. Edwin D. Leman, to whom I wish here to express my sincere thanks.

KENT CHEMICAL LABORATORY,
UNIVERSITY OF CHICAGO, February 10, 1913.

THE PERIODS OF TRANSFORMATION OF URANIUM AND THORIUM.

By Herbert N. McCoy.

THE period of uranium has been determined in two ways, first, by counting the scintillations per second from 1 g. of this element and second, by determining the proportion of radium to uranium in minerals, the period of radium being also known. These methods gave 4.8×10^9 and 5.6×10^9 years respectively. A third possibility depends on the linear relation between the logarithms of ranges and decay constants; but this is very inexact as a means of determining periods. In the case of thorium the methods depending on counting the scintillations of the α rays is the only one applicable with any certainty. In the present paper it is shown that the period of a radio-element can be calculated from its range and the ionization current of unit mass. The ranges of all radio-bodies are now known with a fair degree of accuracy; while the ionization currents of uranium and thorium have been carefully measured in connection with other investigations carried out in this laboratory.

It now appears to be well established[1] that the number of ions, N, produced by any α ray of range R is closely proportional to $R^{\frac{2}{3}}$ or $N = aR^{\frac{2}{3}}$. The constant a may be calculated from the work of Geiger and also of Taylor[2] who have determined independently the number of ions formed by an α particle of polonium. The former found 162,000, the latter 164,000. Since $R = 3.58$ cm. for polonium at 0° and 76 cm.,[3]

$$N = 6.97 \times 10^4 R^{\frac{2}{3}}. \tag{1}$$

If I is the total ionization current in amperes of 1 g. of a radioactive element of atomic weight M and decay constant λ, it is easy to show that

$$\lambda\ (\text{sec.}) = \frac{IM}{NF} = \frac{IM}{6.97 \times 10^4 R^{\frac{2}{3}} F}, \tag{2}$$

where $F = 96{,}600$ coulombs. If the unit of time is the year

$$\lambda\ (\text{year}) = 4.68 \times 10^{-3} \frac{IM}{R^{\frac{2}{3}}}$$

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[1] Geiger, Proc. Roy. Soc., A, *82*, 486, 1909. Taylor, Am. Jour. Sci., *28*, 357, 1909, Phil. Mag., *21*, 571, 1911. McCoy and Viol, Phil. Mag. *25*, 333, 1913. McCoy, preceding paper.

[2] Phil. Mag., *23*, 670, 1912.

[3] Geiger, Phil. Mag., *24*, 653, 1912.

and the half-period of decay is

$$T = \frac{\log_e 2}{\lambda} = \frac{148\ R^{\frac{2}{3}}}{IM}. \tag{3}$$

Since the ionization currents or activities of the equilibrium amounts of the members of a given series are proportional to the two thirds powers of the respective ranges,[1] we may also write

$$T = \frac{148 \Sigma R^{\frac{2}{3}}}{I'M}, \tag{4}$$

where I' is the ionization current of 1 g. of the mother element, together with the equilibrium amounts of those products the ranges of which are summed.

The periods of uranium and thorium may be calculated by the aid of the formula 4. The two α rays of uranium have ranges of 2.37 cm. and 2.75 cm. at 0° and 76 cm.[2] and 1 g. of uranium gives an α ray ionization current of 4.61×10^{-10} amperes,[3] therefore

$$T = \frac{148 \times 3.741}{4.61 \times 10^{-10} \times 238.5} = 5.04 \times 10^9 \text{ years.}$$

This result may be compared with that obtained by the scintillation method. The number of particles produced per second was found by Geiger and Rutherford[4] to be 2.37×10^4 for 1 g. of U; taking the value of e as known, the period of uranium was found to be 4.8×10^9 years. Closely agreeing results were obtained by counting the scintillations of a uranium mineral. Brown[5] found only about three fourths as many scintillations as Rutherford, his figures indicating a period of 6×10^9 years. Boltwood and Rutherford's determination[6] of the weight of radium in 1 g. of uranium together with Boltwood's[7] value for the period of radium, 2,000 years, lead to a value of 5.6×10^9 years as the period of uranium. Finally by means of the logarithmic law relating ranges and decay constants,[8] the period of uranium is found to be about 7×10^9 years.

It might be thought that equation (3) which gives the period as a function of the ionization current and the range would be less useful than one obtainable from the law of Geiger connecting ranges and decay constants, which would give the period as a function of the range alone. But the superiority of equation (3) is apparent if we estimate the effect on the calculated period of a small error in R; by equation (3), a decrease of 0.1 per cent. in R decreases the period by less than 0.1 per cent., while by Geiger's law it doubles the calculated period. However, Geiger's law, in addition to its theoretical significance, is very valuable as a means of estimating the periods of such substances as uranium-two and ionium.

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[1] McCoy and Viol, *loc. cit.*

[2] Geiger, Phil. Mag., *24*, 653, 1912.

[3] McCoy and Ashman, Amer. Jour. Sci., *26*, 528, 1908.

[4] Phil. Mag., *20*, 691, 1910.

[5] Proc. Roy. Soc., A, *84*, 151, 1910.

[6] Amer. Jour. Sci., *22*, 1, 1906. Boltwood, *ibid.*, *25*, 296, 1908.

[7] Amer. Jour. Sci., *25*, 493, 1908.

[8] Rutherford, Phil. Mag., *13*, 110, 1907. Geiger and Nuttall, *ibid.*, *22*, 613, 1911; *23*, 438, 1912.

The period of thorium may be calculated from data furnished by earlier work done in this laboratory together with a knowledge of the ranges of thorium and its radio-products. The total activities of 1 g. of uranium + U_2 and 1 g. of thorium + all products are 796 and 1,009 respectively, the unit of activity being that of 1 sq. cm. of a thick film of U_3O_8.[1] Since the total ionization current of 1 g. of uranium + U_2 is 4.61×10^{-10} ampere,[2] that of 1 g. of thorium + products is 5.84×10^{-10} ampere. The ranges of the thorium series have recently been redetermined by Geiger and Nuttall,[3] who give the following for 0° and 76 cm. Th, 2.58; Rt, 3.67; ThX, 4.08; Em, 4.74; A, 5.40; C_1, 8.16; C_2, 4.55. For the whole series $\Sigma R^{\frac{2}{3}} = 16.36$, if but 65 per cent. of $R^{\frac{2}{3}}$ for C_1 and 35 per cent. of $R^{\frac{2}{3}}$ for C_2 be taken in the summation,[4] in accord with the fact that C_1 and C_2 give respectively 65 and 35 per cent. as many α particles per second as each of the other members of the series. By means of the above data the calculated period is found to be 1.78×10^{10} years.

A second determination may be made from the activity of thorium alone, which Ashman[5] found to be 11 per cent. of that of thorium + all its products; this result combined with the range of thorium gives a calculated period of 1.86×10^{10} years.

Geiger and Rutherford[6] found that 1 g. of thorium + its products emit 2.7×10^4 α particles per second and calculate therefrom that the period of thorium is 1.31×10^{10} years. Although there is still a wide difference between the latter value and the one here found, which is about 1.8×10^{10} years, yet both are much smaller than the value 3×10^{10} years which had been assumed earlier.[7]

It is obvious that the period of radium might be calculated by formula

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[1] McCoy and Ross, Journ. Amer. Chem. Soc., *29*, 1699, 1710, 1907.

[2] McCoy and Ashman, *loc. cit.*

[3] Phil. Mag., *24*, 653, 1912.

[4] Marsden and Barrett, Proc. Phys. Soc., *24*, 50, 1911. Barratt, Le Rod., *9*, 81, 1912.

[5] Ashman, Amer. Journ. Sci., *27*, 65, 1909.

[6] Geiger and Rutherford, Phil. Mag., *20*, 691, 1910.

[7] See Mme. Curie, Die Radioaktivität II., 544.

(3) if the ionization current of unit mass were known with sufficient exactness. Apparently up to the present time this constant has not been determined directly with much accuracy.

SUMMARY.

The period of a radioactive element can be calculated with considerable accuracy from its range and the ionization current of unit weight. The periods so found of uranium and thorium are 5.0 × 10^9 and 1.8 × 10^{10} years respectively.

KENT CHEMICAL LABORATORY,
UNIVERSITY OF CHICAGO,
February 12, 1913.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

ON A PROGRESSIVE DEVELOPMENT OF THE FUNDAMENTAL PRINCIPLES OF MECHANICS.[1]

BY H. M. DADOURIAN.

THE object of this paper is to present in a compact form the salient points of a method of treatment of mechanics which I have followed in a recent book[2] on the subject.

Briefly speaking the following is the general plan of the treatment: (1) The fundamental principle of mechanics is stated in the form of a single law, and in such a manner as to bring it, at least in its simpler aspects, within the experience and mental grasp of the beginner. (2) The interpretation of the law is gradually enlarged as the subject is developed. By this treatment the presentation of the subject is given a high degree of unity and coherence, while its development is made gradual and progressive.

The following is the law upon which the entire subject is based: The sum of all external actions to which a body or a part of a body is subject at any instant vanishes:

$$\Sigma \mathbf{A} = 0.^{3}$$

The meaning of the term 'action,' in other words, the interpretation of the law is widened and developed in four stages, in connection with the application of the law to four different classes of problems.

(*a*) *Equilibrium of a Particle.*—Before applying the law to problems under this head the idea of force is introduced, and the latter is defined as a vector magnitude which represents the action of one particle upon another. Then it is shown that forces are the only type of actions to which a particle is subject, when in equilibrium; consequently the law states that the sum of all the forces to which a particle is subject at any instant vanishes. Therefore the following conditions are obtained for the equilibrium of a particle:

$$\Sigma \mathbf{F} = 0, \quad \text{or} \quad \begin{aligned} \Sigma X &= 0, \\ \Sigma Y &= 0, \\ \Sigma Z &= 0. \end{aligned}$$

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[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] Analytical Mechanics, for students of Physics and Engineering, D. Van Nostrand Co., New York, 1913.

[3] Letters in the black type denote vector magnitudes.

(*b*) *Equilibrium of a Rigid Body.*—Here it is shown that a rigid body is capable of having two distinct and independent types of motion, namely, motion of translation and motion of rotation. Consequently it is concluded that a rigid body is capable of being subjected to two types of action, one of which is related to motion of translation, while the other is related to motion of rotation. These are called ***linear action*** and ***angular action***, respectively. Then the law is put in the following form

$$\Sigma(\mathbf{A}_l + \mathbf{A}_a) = 0,$$

where $\mathbf{A}_l$ denotes linear actions and $\mathbf{A}_a$ angular actions. But since $\mathbf{A}_l$ and $\mathbf{A}_a$ are independent of each other the law may be split into the following two sections:

I. The sum of all the linear actions to which a body or a part of a body is subject at any instant vanishes:

$$\Sigma\mathbf{A}_l = 0.$$

II. The sum of all the angular actions to which a body or a part of a body is subject at any instant vanishes:

$$\Sigma\mathbf{A}_a = 0.$$

In the case of equilibrium, forces are the only type of linear actions and torques the only type of angular actions. Consequently the following well-known conditions of equilibrium are obtained from the two sections of the law:

$$\Sigma\mathbf{F} = 0, \quad \text{or} \quad \begin{aligned} \Sigma X &= 0, \\ \Sigma Y &= 0, \\ \Sigma Z &= 0, \end{aligned}$$

and

$$\Sigma\mathbf{G} = 0, \quad \text{or} \quad \begin{aligned} \Sigma G_x &= 0, \\ \Sigma G_y &= 0, \\ \Sigma G_z &= 0. \end{aligned}$$

(*c*) *Motion of Translation.*—Here the concept of ***linear kinetic reaction*** is introduced as a new type of action which appears whenever a body is given a linear acceleration. The quantitative definitions, or the measures, of kinetic reaction and mass are then illustrated by two ideal experiments. In one of the experiments the particle is supposed to have a longitudinal acceleration while in the other a transverse acceleration.

An accelerated particle, therefore, is subject to two types of linear actions, namely, forces and the kinetic reaction. Hence the law states:

$$\Sigma\mathbf{F}_i + \mathbf{K}_l = 0,$$

or

$$\Sigma\mathbf{F}_i = -\mathbf{K}_l,$$

or

$$\mathbf{F} = m\dot{\mathbf{v}},$$

where $\mathbf{F}$ denotes the sum, or the resultant, of the forces, while $-m\dot{\mathbf{v}}$ is the kinetic reaction. The last relation, which is called the *force-equation*, is then split into two component equations which are related to the tangential and normal directions: Thus

$$F_\tau = m\dot{v}$$

and

$$F_n = -m\frac{v^2}{\rho}.$$

Therefore, in general the magnitude of the resultant force is given by

$$m\sqrt{\dot{v}^2 + \frac{v^4}{\rho^2}}.$$

On this view the "centripetal force" is the real force while the "centrifugal force" is the transverse kinetic reaction. The latter is no more a force than the longitudinal kinetic reaction.[1]

(*d*) *Motion of Rotation.*—Here the *angular kinetic reaction* and the moment of inertia (angular mass) are introduced in a manner similar to the introductions of their linear analogues. Then applying the second section of the law the following relations are obtained:

$$\Sigma\mathbf{G}_i + \mathbf{K}_a = 0,$$

or

$$\Sigma\mathbf{G}_i = -\mathbf{K}_a,$$

or

$$\mathbf{G} = I\dot{\omega},$$

where $\mathbf{G}$ denotes the resultant torque and $-I\dot{\omega}$ the angular kinetic reaction. The last equation is the analogue of the force-equation and is called the torque-equation.

The principles of the conservation of dynamical (kinetic and potential) energy, of the conservation of linear momentum and of the conservation of angular momentum are derived from the law. The following examples show the elegance and ease with which the form of the statement of the law lends itself to proving important dynamical theorems.

Let $\mathbf{F}_i$, $\mathbf{F}_i'$, and $\mathbf{F}_i''$ denote, respectively, the resultant force acting upon a particle in a system, the force due to its connections with the rest of the system, and the force due to external bodies: then applying the law to the particle we obtain

$$\mathbf{F}_i - m_i\dot{\mathbf{v}}_i = 0,$$

or

$$\mathbf{F}_i' + \mathbf{F}_i'' - m_i\dot{\mathbf{v}}_i = 0.$$

Therefore, taking the sum over all the particles of the system we have

$$\Sigma(\mathbf{F}_i' + \mathbf{F}_i'' - m_i\dot{\mathbf{v}}_i) = 0.$$

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[1] In order to emphasize the difference between force and kinetic reaction the latter may be defined as the action of the æther (or whatever takes its place in future physical theories) upon a particle of ponderable matter, while force is defined as the action of one particle of ponderable matter upon another.

But, applying the law directly to the system, we get

$$\Sigma(\mathbf{F}_i'' - m_i\dot{\mathbf{v}}_i) = 0.$$

It is evident from the last two equations that

$$\Sigma\mathbf{F}_i' = 0,$$

which states that the sum of the internal forces vanishes.

The principle of the conservation of linear momentum may be proved in the following manner.

$$\Sigma(\mathbf{F}_i'' - m_i\dot{\mathbf{v}}_i) = 0.$$

Therefore

$$\Sigma\mathbf{F}_i'' = \Sigma(m_i\mathbf{v}_i)$$

$$= \frac{d}{dt} Z(m_i\dot{\mathbf{v}}_i).$$

Therefore when the sum of all the external forces acting upon the system vanishes

$$\frac{d}{dt}\Sigma(m_i\mathbf{v}_i) = \Sigma\mathbf{F}_i'' = 0,$$

$$\Sigma(m_i\mathbf{v}_i) = \text{constant}.$$

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THE IONIZATION IN UNSTRIATED DISCHARGE.[1]

BY C. D. CHILD.

A DIFFICULTY which must be met in explaining either the unstriated discharge or the gaseous part of the arc is one concerning the rate of recombination of the positive and negative ions. This rate is proportional to the number of positive ions times that of the negative ones. Both the positive and the negative ions are here proportional to the current, so that the rate of recombination is apparently proportional to the square of the current. Since the rate at which ions are formed must equal the rate at which they recombine, this rate would also appear to be proportional to the square of the current. But there are valid reasons for believing that neither of these conclusions is possible.

This difficulty can be explained, if we remember that there are two kinds of negative ions, one of electronic size and one of molecular size. Approximately all of the current is carried by the first, because of their greater velocity, while a large part of the electrostatic effect is produced by the second kind.

A formula for the relation between current, density of the gas, and the electric force can be derived by making the following assumptions: first, that the ionization is caused by the impact of the more rapidly moving electrons on the molecules, and secondly that but a small percentage of the collisions between the electrons and the molecules result in combination of the two, while with the great majority of such collisions the electrons and molecules separate after collision without any permanent change in either.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

These assumptions lead to the conclusion that the electric force approaches infinity as the density of the gas approaches zero, that it rapidly decreases as the density increases, reaching a minimum when the pressure is a small fraction of a millimeter, and then increases with increasing density. This result agrees with the measurements which have been made on the relation between electric force and density of the gas in these forms of discharge.

This explanation was also found to agree with measurements which were made on the electric force and the temperature and pressure of the vapor in the mercury arc.

By making the further assumption that the light from electric discharge comes from the molecules which have been ionized, this explanation also leads to a formula for the efficiency of the discharge when considered as a source of light. This formula was found to agree with the measurements which have been made on this relation.

On Special Conditions in the Electric Furnace which Produce a Spectrum Similar to that of the Spark.[1]

By Arthur S. King.

THE enhanced lines of metals, a class of lines which appear much stronger in the spark than in the arc, are in general difficult to produce in the electric furnace. However, by operating a tube resistance furnace at a temperature exceeding 2600° C. the writer has recently obtained all of the stronger enhanced lines of titanium from λ 4200 to λ 4600. They are very faint in the furnace as compared to the lines characteristic of the arc spectrum. Special tubes whose walls were made quite thin for a portion of the length gave higher temperatures, with a slight increase in the relative strength of the enhanced lines. Phenomena of special interest appeared when these tubes burned through, forming an arc around the wall of the tube in which 20 K.W. or more were consumed at a potential of about 30 volts. The conditions given by this high-current arc resulted in the enhanced lines appearing very strong, their intensity compared to the arc lines being such as is obtained usually only with a powerful spark.

In order to test whether the condition of the vapor at the center of the tube was different from that near the wall under these circumstances, the spectrum was photographed with a long slit passing across the diameter of the image of the tube's interior. The middle of the spectrum line then registered the condition of the vapor at the center of the tube, while the ends of the line were produced by the vapor near the arcing wall.

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[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

Photographs made in this manner showed that the arc lines of titanium and the carbon bands were much stronger near the wall of the tube, while the enhanced lines of titanium were as strong in the center of the tube as at the wall, and the spark line of carbon λ 4267 appeared very much stronger in the center than at the wall. This latter line does not appear in the carbon arc burning with continuous current, usually requiring the condensed spark with cold electrodes for its production. The behavior of the arc lines, as well as general considerations, indicate that both temperature and potential gradient are lower in the center of the tube, but it is in this region that the spark lines appear relatively strong in this source.

Another remarkable feature is that the arc lines show a one-sided structure when produced by the vapor at the center of the broken tube, the red side of the line being strongest, while the lines produced near the wall of the tube are nearly or quite symmetrical. This is another point of similarity between the center of the broken tube and the condensed spark, the latter source showing a tendency to give lines apparently displaced toward the red. The regular furnace spectrum taken with long slit, when the tube does not burn through, shows no difference in intensity or structure between center and ends of the line.

Mount Wilson Solar Observatory,
December 9, 1912.

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THE

PHYSICAL REVIEW.

MULTIPLE REFLECTION OF SHORT ELECTRIC WAVES FROM SCREENS OF METALLIC RESONATORS.

By W. L. Severinghaus and W. S. Nelms.

THE problem of electrical resonance has been taken up in a great many different forms by many investigators. The oscillator used as a source has usually emitted an uncertain wave-length and been the cause of more or less difficulty. The present work offers a solution to some of these difficulties and a means of getting a short electric wave of any definite length which is much less damped than that of the Righi vibrator. The apparatus is in general similar to that used for selective reflection in the infra-red spectrum, namely, an exciting source, reflecting screens or surfaces, an interferometer and a non-selective receiver.

Apparatus.

The apparatus is most conveniently discussed under three heads, (1) the oscillating system, (2) the reflecting system, (3) the receiving system.

1. *The Oscillating System.*—The vibrator is the ordinary form of Righi vibrator. It consists of two steel spheres set in sockets ground in wood fiber supports which are so arranged that the distance between the spheres can be accurately controlled and made as small as desirable. The supports are devised to allow of quick and easy adjustment to the focus of the mirror for spheres of different diameters, whenever a change in the wave-length is desired. A steady stream of clean paraffin oil flows over the balls and fills the gap between them. Lead wires are brought from the secondary of the induction coil, which is directly under the vibrator, making side sparks in air about a half centimeter. The oil gap is perhaps not more than 0.1 mm., being adjusted to give maximum energy. This vibrator is placed at the focus of a spherical parabolic mirror, of 50 cm. focal length, in order to get sufficient energy, and to

obtain a plane wave front. The vibrator is in general the same as that described in a paper by Woodman and Webb.[1]

The parabolic mirrors used had to be made in the laboratory, since sufficiently large ones could not be bought, and they proved so satisfactory that a brief statement as to their construction may be of interest. Soft plaster of paris was poured into a shallow circular box made by fastening a ring of sheet brass on a flat wooden base. A parabolic templet was rotated about a vertical axis through the center of the box until the plaster had hardened. The parabolic surface thus formed was covered with heavy lead foil making an excellent reflector of the desired focal length. The diameter of the mirror was 83 cm. and the focal length was 50 cm.

One of these mirrors was held in a rack which had freedom of adjustment so that either its height from the floor or the angle it made with the vertical or horizontal could be controlled. The vibrator was placed at the focus of the mirror so that the electric vector of the oscillation was in the horizontal plane (see Fig. 1). The vertical as well as the horizontal distribution of energy in the beam on both sides of the principal axis of this mirror is shown in the curves of Fig. 2. The energy falls to half

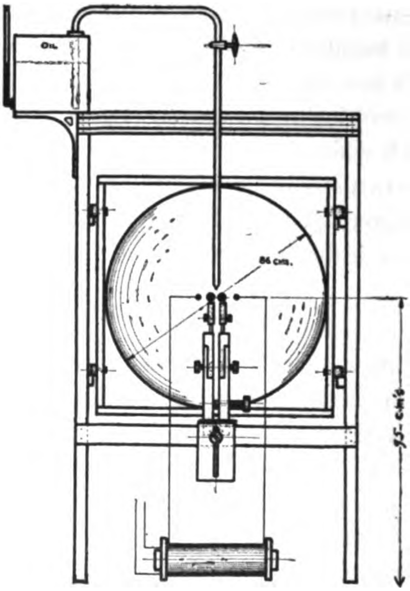

Fig. 1.

Oscillating system.

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[1] Woodman and Webb, PHYS. REV., Vol. XXX., p. 563, 1910.

value at about 30 cm. distance from the central axis of the beam, whereas with 12 cm. mirrors it fell to half value at 15 cm. from the axis. This added width of the beam was quite a help in some of the later work with resonator screens.

The mirror is known to affect in some way the character and wave-length of radiation obtained from the oscillator. This is one of the great difficulties in using the Righi vibrator as a direct source of energy. It seems to set a limit for the possible wave-length to be otherwise obtained by increasing the size of the spheres. At any rate it makes the increase in energy and in wave-length beyond certain values not at all proportional to the increase in the size of the oscillator. An attempt was made to find the reason for this effect by making some plaster casts on 12 cm. focal length mirrors and covering these forms with lead foil. It was thought that the energy reflected from the mirror back on to the vibrator might cause the trouble. The central portion of the lead foil was therefore removed, so that none of the reflected energy came to the vibrator, but the phenomenon was the same as before. The foil was cut up into strips and the radiation again investigated, but no satisfactory information was obtained as to the cause of the trouble. That the energy and wave-length will increase with the dimensions of the vibrator, when no reflector or a reflector of large focal length is used, was shown by Webb and Woodman, who also performed some experiments on this "mirror action."[1] The effect seems to grow less as the focal length becomes large compared to the wave-length.

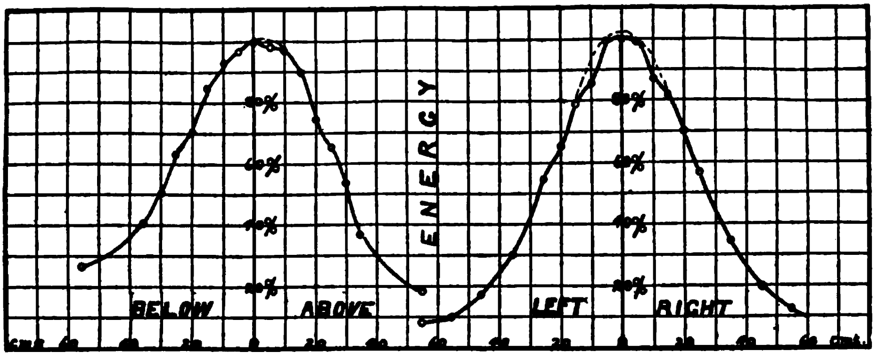

Fig. 2.

Distribution of energy in the beam.

It cannot be said that even the large mirror used in this work did not have an effect on the radiation from the oscillator. It did, however, allow of sufficient variation in the wave-length for our purposes, and that is all we needed as we were not interested in the effect of the dimensions of the oscillating system on the wave-length emitted by it. It is barely possible that the induction coil and lead wires have some effect on the wave-length and damping. For this reason we prefer to speak of the *oscillating system*, with all that may be involved, and merely assume that the wave-length measured by the Boltzmann mirrors is the true wave-length of the emitted energy.

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[1] Webb and Woodman, Phys. Rev., Vol. XXIX., p. 100, 1909.

2. *The Reflecting Sytsem.*—The reflecting system consisted first of two rigid wooden stands upon which any desired surfaces could be mounted at S_1 and S_2 (see Fig. 3). After reflection at S_1 and S_2 the energy fell on

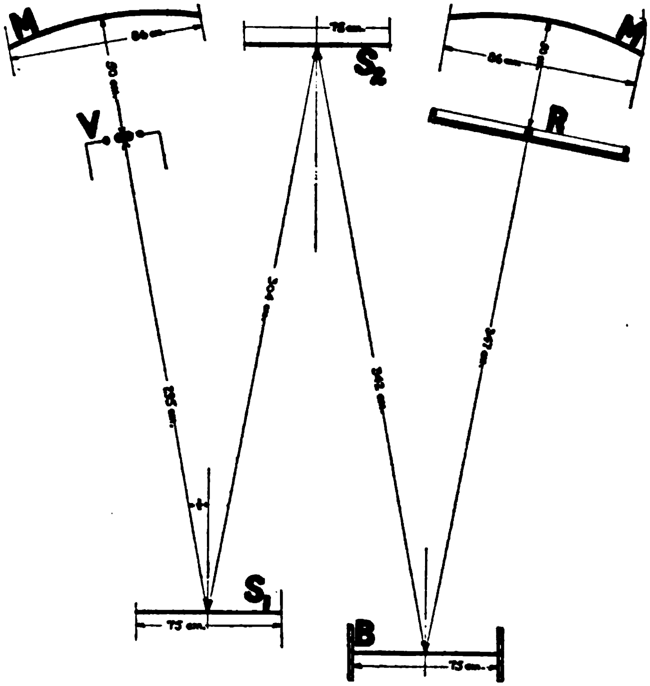

Fig. 3.

the Boltzmann mirrors (B) which consisted of two pieces of plate glass (76 × 38.5 cm.), the upper and the lower half, covered with tin foil. Total reflectors to be used at S_1 or S_2, when desired, were made by covering heavy plate glass with tinfoil and mounting them in wooden frames to fit the stands. The selective resonator screens were in every case made by pasting the tinfoil resonators on tracing cloth. The tracing cloth was thoroughly wetted, stretched over a wooden frame, tacked and dried before the resonators were put on. This gave a very plane surface which reflected none of the energy and had been found by others to be the same as suspending the resonators in air. These screens as also the total reflectors were 85 × 85 cm., a trifle larger than the area of the vibrator mirror aperture. The angle of incidence of the beam on the screens was 11° and the total length of path from vibrator to receiver was 14.9 meters.

3. *The Receiving System.*—The energy was reflected from the Boltzmann mirrors to a parabolic mirror, exactly similar to the one used with the vibrator, and brought to a focus on a *non-selective* Klemenčič receiver. Observations indicated that this mirror had little or no effect on the wave-length measurements. It seems certain at any rate that the receiver mirror does not exert so large an influence as the vibrator mirror.

The receiver in its final form was made by stretching copper wings (*a*, *a*, Fig. 4) over a wooden frame. These wings were fastened at the outside ends with sealing wax, while at the center they were sewed on to a base of hard rubber. The thermo-junction consisted of fine platinum and constantan wires about 0.0001 inch in diameter, soldered on to the wings and welded together at the center by a spark from a static machine. The wires leading to the galvanometer were soldered to the under side of the wings at the junction and led out immediately without passing through or along any dielectric. The total width of dielectric in contact with the wings at the center was not more than 4 millimeters, while the total length of the receiver was about 200 centimeters, which was from 12 to 35 times the length of the waves measured. The receiver is for the most part the same as the one perfected by Webb and Woodman[1] and described in their paper.

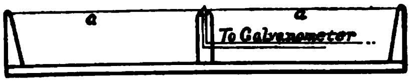

Fig. 4.
Non-selective receiver.

The reliability of results in wave-length measurements depends very largely on the character of the receiver. It has been shown by a number of investigators[2] that the interference curves obtained with selective receivers are a result of the combination of two frequencies, namely, that of the vibrator and that of the receiver itself. The apparent damping of the oscillation is also greatly affected by the tuned receiver, so that an oscillation which, measured with a non-selective receiver, shows high damping, shows very little damping when measured with a tuned receiver. It is then extremely important in the work which is here presented that the receiver be strictly dead beat or non-selective. The receiver described above meets these conditions.

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[1] Webb and Woodman, l. c., p. 93.

[2] Hull, Blake and Fountain, Ives, Willard and Woodman.

The general plan and dimensions of the apparatus can be seen in Fig. 3. The figure explains itself and throughout the discussion the letters used in the drawing shall be the symbols for the respective parts. Fig. 1 shows the front elevation of the oscillating system. The alignment of the apparatus was made by testing the electrical intensity at various points, and then adjusting the angle of the oscillator and screens until the distribution of energy was symmetrical about the centers of the two screens, Boltzmann mirrors and receiver mirror. With total reflectors at S_1 and S_2 a watch held at the focus of one of the mirrors could be heard very distinctly with the ear at the focus of the other mirror.

METHOD OF WAVE-LENGTH DETERMINATIONS.

All of the work was done with two observers, one reading the galvanometer connected with the main receiver and the other the galvanometer connected with a check receiver. The check receiver was also non-selective and was placed just above the vibrator outside the beam reflected from the oscillator mirror. The use of the check receiver helped to eliminate errors which would otherwise have arisen from a deterioration of the vibrator.

Before proceeding with the wave-length measurement the energy in the beam had to be tested and the vibrator or receiver so adjusted, by slightly raising or lowering, that the energy received from the upper half of the Boltzmann mirrors was equal to that received from the lower half. This "balancing of the beam" was continued until the two halves agreed within one or two per cent. A failure to do this would tend to displace the positions of the maxima and minima in the curves.

The Boltzmann mirrors were set for no difference of path, the deflections on the main and on the check receiver were read and the ratio of main to check recorded. The mirrors were then separated by some amount and the ratio of main to check again recorded; then the mirrors set for zero difference of path and the ratio of main to check recorded. The second ratio was divided by the average of the first and third, expressed in per cent. and taken as one reading for the given difference of path. This was repeated for any point as often as necessary to be reasonably certain of the value. In this way readings were taken for as many different settings of the Boltzmann mirrors as desired, the values always being expressed in per cent. of energy received for zero difference of path. A curve was plotted with distance between Boltzmann mirrors as abscissæ and per cent. of energy as ordinates, from which the wave-length was then determined, it being twice the average distance between successive maxima or successive minima.

There is always a certain amount of energy, which arrives at the receiver by diffuse reflection from the walls of the room, which we may call "stray energy." It is small compared with the energy in the reflected beam and is constant in amount regardless of the setting of the interference apparatus. It will therefore not affect the distance between maxima and minima. This stray quantity must, however, always be subtracted before plotting curves which are to be used in studying the damping of the oscillation.

Wave-Length Interference Curves.

With S_1 and S_2 total reflectors the wave-length measured is that of the oscillating system itself. Figures 5, *A*, and 6, *A*, are typical curves obtained under these conditions. The wave-length being in each case about 76 mm., which could be determined within two per cent. The maxima and minima are fairly evenly spaced (see Fig. 6, *A*) with the first minimum about 12 per cent. of the total energy, the first maximum 68 per cent., the second minimum 33 per cent. and the second maximum 47 per cent. of the total energy at zero difference of path. For wider separations of the Boltzmann mirrors the periodic interference ceased and the energy received was nearly constant. That is to say, when the distance between the mirrors is one wave-length, or when the total difference of path is two wave-lengths, interference no longer takes place because of the high damping. If the curve is carried out farther it is seen to lie along the 45 per cent. axis. It was found for these curves that the value for this axis was in every case approximately equal to the sum of the percentages of energy received from the two halves of the mirror separately. Each half would accordingly produce an effect of about 23 per cent. If the two effects are impressed on the receiver in such time relations as not to produce interference of the oscillation in the receiver then the total energy is simply additive. But if they be added so as to produce double the amplitude, the energy being proportional to the square of the amplitude, the deflections will be four times as great as those produced by each half alone. This is the reinforcement obtained when the two mirrors are set for zero difference of path. The fact that the axis of symmetry is not the 50 per cent. axis may be due to diffraction effects produced by the adjacent edges of the two halves of the Boltzmann mirrors.

Fig. 5, *B*, shows the results when S_1 was made a screen of resonators and S_2 was a total reflector, the oscillator being the same as for 5, *A* (λ = 76 mm.). S_1 acted as a selective screen for the highly damped energy falling upon it and reflected a new wave-length determined more by the dimensions and distribution of the resonators on the screen than by the incident wave-length. The wave-length was increased to 99 mm. and the damping considerably diminished, as can be best seen by comparing the second and third maxima and minima in the two curves *A* and *B*.

Fig. 5, *C*, was the result when S_2 was made a screen of resonators exactly similar to S_1, the vibrator being the same as before. S_2 was being excited by energy which was the result of a combination of vibrator and one resonator screen. The damping was again reduced and the wave-length increased to 110 mm. with the maxima and minima at very evenly spaced intervals. This was the wave-length due to the natural frequency of the resonators themselves, as was later shown by tuning the vibrator to the resonators and measuring the resultant wave-length. Starting with a highly damped radiation from the oscillating system of 76 mm. wave-length we have obtained after two reflections from resonating screens a wave of 110 mm. in which the damping is surprisingly small. If we regard the highly damped radiation from the oscillator as a con-

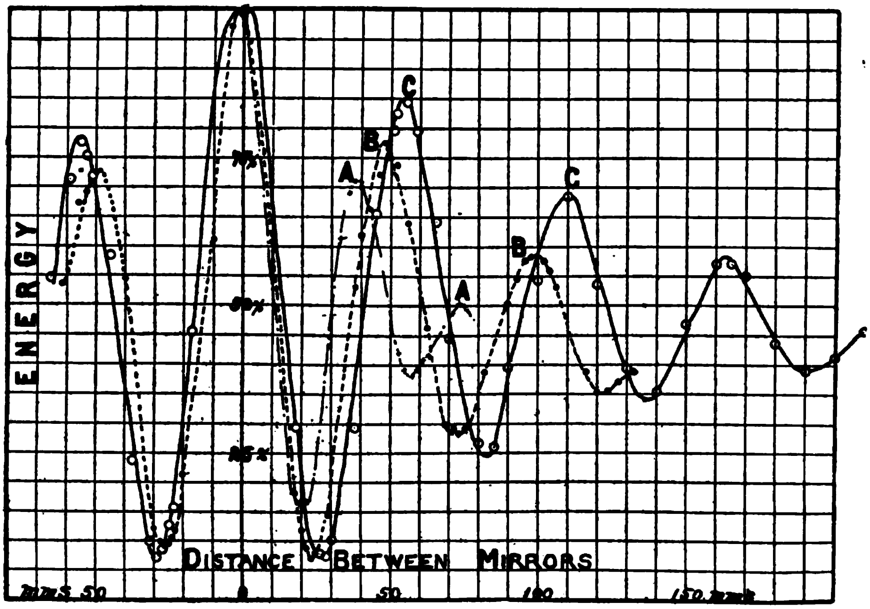

Fig. 5.

Interference curves.

tinuous spectrum then the analogy to the case of *reststrahlen* is very striking.

It is to be expected that the purest or least damped radiation will be obtained, with but two reflections, if the screens and vibrator are very nearly in tune. These conditions were obtained with two screens made of resonators 36 mm. long and 0.3 mm. wide. The resonators were mere threads of tinfoil with an open space of 54 mm. between them from side to side and 10 mm. between them from end to end. The exciting source, the vibrator, was sending out a wave of 76 mm. length while the wave-length after two reflections was only 84 mm., so that the *second* screen could be considered excited by energy in tune with its resonators. Fig. 6, *B*, shows the results of these conditions. They meet the expectation.

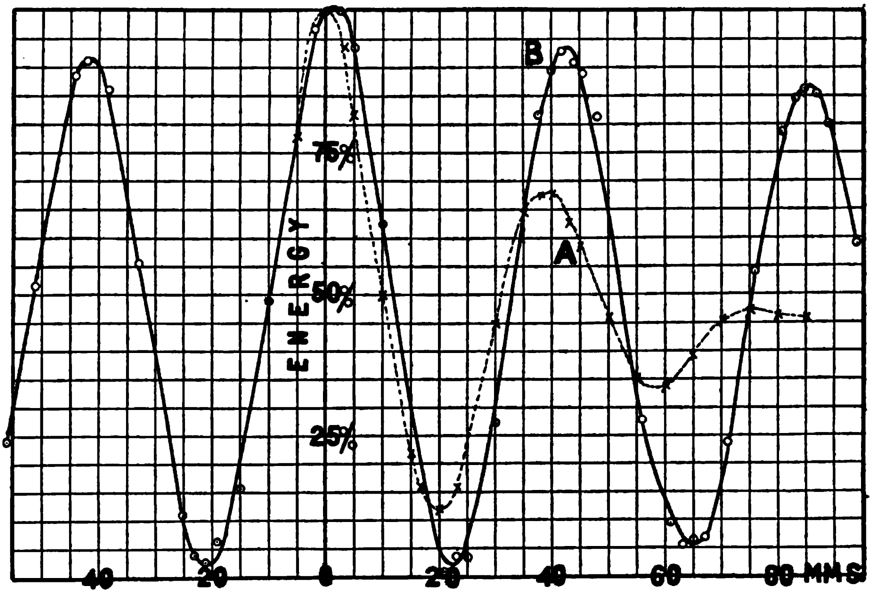

Fig. 6.

Interference curves.

The oscillator curve and the curve after two reflections are plotted together in Fig. 6 for purposes of comparison. The energy at the first minimum in *A* is 12 per cent. while that in *B* is only 2 per cent.; at the first maximum in the one it is 68 per cent. and in the other 93 per cent., showing the much sharper interference obtained in the case of the resonator radiation. The stray energy is so large a part of the total energy received when the mirrors are set for greatest interference that the exact value of the energy at the minimum of the resonator curve cannot be determined. One of the points of this interference curve which has been corrected for stray energy will be seen to lie almost on the zero energy line.

With a double reflection, then, from linear resonators in tune with the oscillator, energy of very little damping and of very definite wave-length is to be obtained. The properties of the resonator radiation are those of a "monoperiodic" oscillation to a much higher degree than the radiation of the oscillator. These screens may therefore be used to advantage as a *new source* in investigations with short electric waves.

REFLECTION COEFFICIENTS.

While the smallness of the damping is about all that could be desired for certain uses of the electrical oscillation, yet difficulties arise due to the lack of energy at one's disposal. A large part of the incident energy passes on through the screen and must be reflected out of the beam by a deflecting mirror located a short distance behind the resonators. The amount of energy transmitted depends on the number of resonators in the field and the closeness of their tuning to the energy which falls upon them. A numerical study of these quantities in a particular case will be of interest.

The screens used for this test were those used to obtain the curve in Fig. 6, *B*. The resonators were 36 mm. long and 0.3 mm. wide with an open space of 54 mm. between the horizontal rows. There were seventeen columns with one cm. between the ends of resonators. Their natural wave-length was 84 mm. In Table I. the first column gives the wave-length of the highly damped oscillating system.

TABLE I.

λ, Mm.	$\frac{e_1}{E}$, Per Cent.	$\frac{e_2}{e_1}$, Per Cent.
75	14	36
85	16	34
95	15	36
120	13	31

E is the energy arriving at the receiver when S_1 and S_2 (Fig. 3) are total reflectors; e_1 is the energy when S_1 is one of the above-mentioned screens of resonators and S_2 is a total reflector; e_2 is the energy received when S_1 and S_2 are both screens of resonators. $e_1/E \cdot 100$ gives the per cent. of incident energy reflected by the first screen and $e_2/e_1 \cdot 100$ gives the percentage reflection of the second screen. It will be seen then that under the best conditions of tuning the energy after reflection from two resonator screens was only 34 per cent. of 16 per cent., or 5.4 per cent. of the initial energy from the oscillator when two total reflectors were used. The above resonator distribution was rather sparse and the energy might be greatly increased by increasing the number of resonators.

A comparison of the two columns e_1/E and e_2/e_1 is also of interest as a further evidence of the "purity" of resonator radiation. Energy relations of this kind are difficult to establish to any high degree of precision, but the table shows that in every case the ratio e_2/e_1 was more than twice as great as e_1/E. Even for the closest tuning the first screen reflected only 16 per cent. of the highly damped radiation falling upon it whereas the second screen, which was excited by the energy coming from the first screen, reflected 34 per cent. of the incident energy.

This increase in the reflection coefficient is not due to increased polarization of the oscillations, as the oscillator radiation has been shown to be almost entirely polarized. This can also be seen from Table II.

TABLE II.

λ, Mm.	$\frac{e_1}{E}$, Per Cent.	$\frac{e_2}{e_1}$, Per Cent.
75	10	14
85	12	16
95	16	22
120	17	23

For this test the screens consisted of continuous lengths of foil across the frame parallel to the electric vector. They were non-selective reflection gratings with the same distance between the lines, *i. e.*, 54 mm., as between the resonators of the former screens. The numbers in the two columns of Table II. increase about proportionately due to the fact that, as the incident wave-length increases, the amount reflected increases for any given open space in the grids. This is in accord with the results of Blake and Fountain.[1] They found that a wire grid would act as a total reflector if the distance between wires was as small as one fortieth of the incident wave-length.

DAMPING COEFFICIENTS.

Klemenčič and Czermak[2] have worked out an expression for the damping coefficient which can be applied to the interference curves obtained with the Boltzmann mirrors. By integrating and adding the effects produced by the two halves of the mirror they obtain the final form

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[1] Blake and Fountain, PHYS. REV., Vol. XXIII., 1906, pp. 274–276.

[2] Klemenčič u. Czermak, Annalen d. Physik u. Chemie, 1893, Vol. 50, p. 179.

$$\alpha = \frac{A^2T}{4\gamma}(1 - e^{-2\gamma x/\lambda} + C^2),$$

in which γ is the damping coefficient, T the period of the oscillation, x the difference in path of the two halves of the beam, λ the wave-length, A is a constant and C an expression involving x, γ and λ. α is proportional to the galvanometer deflection. Values of α are found when $x = 0$, $x = \lambda/2$, $x = \lambda$, $x = n\lambda/2$. If these values be called α_1, α_2, α_3 and α_n we get the expression

$$\frac{\alpha_1 - \alpha_2}{\alpha_3 - \alpha_2} = e^{\gamma/2},$$

whence

$$\gamma = 2 \text{ nat. log. } \frac{\alpha_1 - \alpha_2}{\alpha_3 - \alpha_2}.$$

This equation is in a form which can be applied to any two maxima and the minimum between them in an interference curve.

The conditions for which the above formula was developed were that the difference in path be small compared with the distance from the interference mirrors to the source, that the wave be a plane wave, and that the damping coefficient be small. Hull[1] showed later that the receiver affected the wave-length curves and that if the above formula is to be used the receiver must be dead beat or non-selective. All of these conditions have been met in the present work and the formula was used for the wave-length curves obtained. When applied to the curve showing least damping (Fig. 6, *B*) the value for γ comes out about 0.15, which is quite small and in good agreement with the values obtained by theoretical considerations of various writers. (For a general review of these results the reader is referred to J. A. Fleming's Principles of Electric Wave Telegraphy, pp. 189 to 198.) A coefficient of 0.15 indicates that the energy dies down to one per cent. in about fifteen complete oscillations. The same equation applied to the oscillator curve gives a value of about 1.4 for γ, which is in good agreement with Webb's[2] values. According to this coefficient the value of the energy falls to one per cent. after about two complete oscillations, which gives a good idea of the great difference in the damping of the energy from the two sources. The resonator damping is probably almost wholly due to radiation. It may be of interest to note that the ohmic resistance of the resonators to direct current is not more than five or six ohms in the extreme cases.

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[1] G. F. Hull, Phys. Rev., 1897, Vol. 5, p. 231.

[2] Webb and Woodman. Phys. Rev., 1909, Vol. XXIX., p. 116.

TUNING CURVES AND OVERTONES.

The properties of this resonator radiation are also brought out in tuning curves. The second screen of resonators (S_2) was varied while the oscillator and S_1 were kept constant. Balls of 7/8 inch diameter were used for the vibrator, the wave-length of which was very nearly the same as that of S_1, which consisted of resonators 2 × 60 mm. with a lateral separation of 42 mm. A set of screens with the resonators varying in length from 36 to 140 mm. was used in turn at S_2. These resonators were all placed on the screens with the same separation both laterally and end to end so that there was practically the same amount of metal in the field for all screens although the number of resonators on the various screens differed.

The result is seen in Fig. 7, *A*, where the ordinates are the energy and the abscissæ the ratio of the length of resonators at S_2 to the length at S_1. The energy received at the main receiver was taken as 100 per cent. when S_2 was a screen exactly like S_1, and the energy for other screens at S_2 was referred to this value. There is a very decided maximum for the ratio unity, *i. e.*, when the two screens are exactly alike, and another maximum, though much smaller, when the resonators at S_2 are just twice as long as those at S_1.

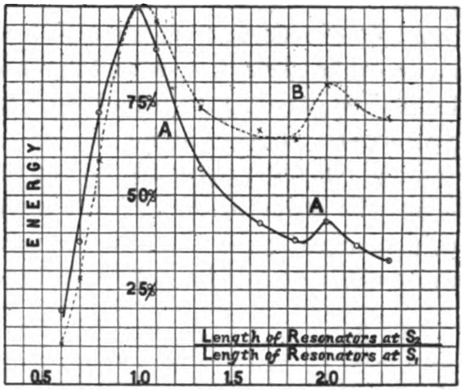

Fig. 7.

Resonator tuning curve.

This shows then, as was to be expected, that the maximum reflection is to be obtained when the excited screen is exactly in tune with the exciting source. It furthermore shows, what has not been heretofore conclusively shown experimentally (so far as the authors are aware) that a linear resonator may oscillate in an overtone with a wave-length equal to half its fundamental wave-length. There was resonance when the second screen was excited by a wave-length equal to half its fundamental. Furthermore the wave-length measured under this condition of resonance was found to be that of the overtone and not that of the fundamental. The damping and equal spacing of maxima and minima in the wave-length curve were similar to that obtained with resonators oscillating in their fundamental frequency.

This result is in agreement with some electromagnetic experiments performed by Rubens and Nichols[1] with heat radiation of great wave-length. Metallic resonators ruled on silvered glass were used as reflecting surfaces for heat waves of about 22.5 μ. Reflection maxima were found for lengths of 12.4 μ and again for 24.4 μ. Interpreted in the light of our Fig. 7 one would say the 12.4 μ resonators were oscillating in their fundamental frequency whereas the 24.4 μ were oscillating with the frequency of their first overtone. It is of interest to notice furthermore that they found even greater reflection when the surfaces were ruled into continuous lengths across the plate, *i. e.*, when the resonators were practically of infinite length compared to the wave-length falling upon them. In this case the plates were probably acting as non-selective reflection grids and the percentage reflection depended on the relation of the distance between the metallic strips to the incident wave-length. It was hardly a resonance phenomenon. If the strips had been close enough together the surface would have acted as a total reflector.

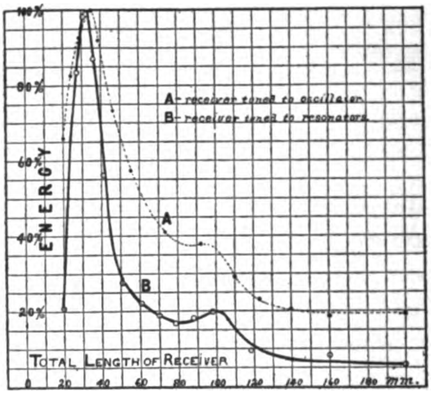

Fig. 8.

Receiver tuning curves.

We are at a loss to understand why Blake and Fountain[2] were not able to obtain this first overtone resonance in their reflection and transmission curves. There may have been some difficulties due to the fact that a selective receiver was used in all of their work. It should be mentioned that their method of tuning differed from the present method in that they started with long resonators and clipped off successive amounts; thus keeping the total number of resonators in the field constant, whereas in our work separate screens were made up for each length of resonator and enough resonators placed on the screen to maintain both lateral and end to end separation the same. The former method will surely bring about differences in energy due to great differences in the amount of surface covered by resonators. This would make the resonance to the fundamental much less striking but should not greatly affect the position of the maximum nor the appearance of the overtone resonance. Fig. 7, *B*, shows what should be expected in this case, *i. e.*, when the number of resonators on all screens was kept constant. This curve was obtained from Fig. 7, *A*, on the assumption that the reflected energy is directly proportional to the number of resonators in the field.

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[1] H. Rubens and E. F. Nichols, PHYS. REV., 1897, Vol. V., p. 164.

[2] L. c., pp. 269–273.

Receiver Tuning Curves.—The relatively small damping or increased "purity" of the resonator radiation was also seen when the Klemenčič receiver itself was tuned to the oscillator directly, and when tuned to the resonator radiation. The curves were obtained in the same way as those of Webb and Woodman. The galvanometer deflection was first taken when the receiver was long and again for each length as pieces were clipped

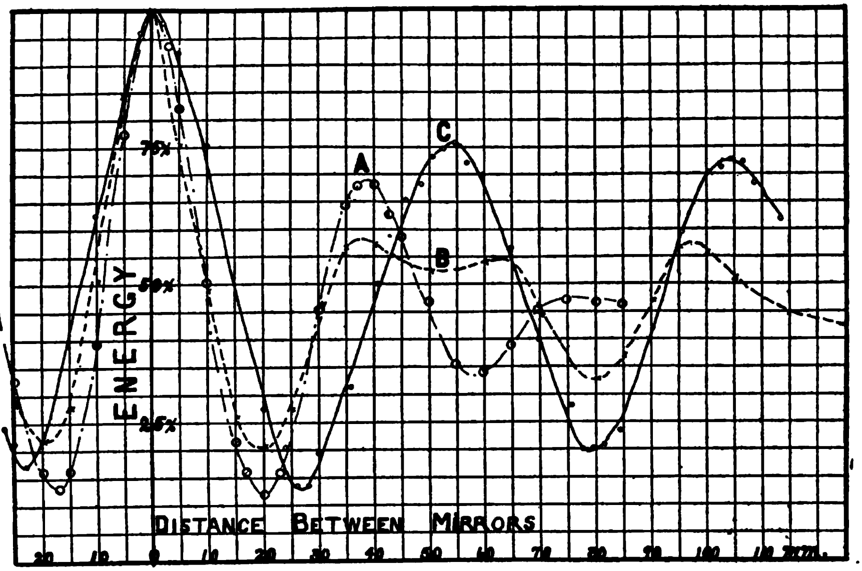
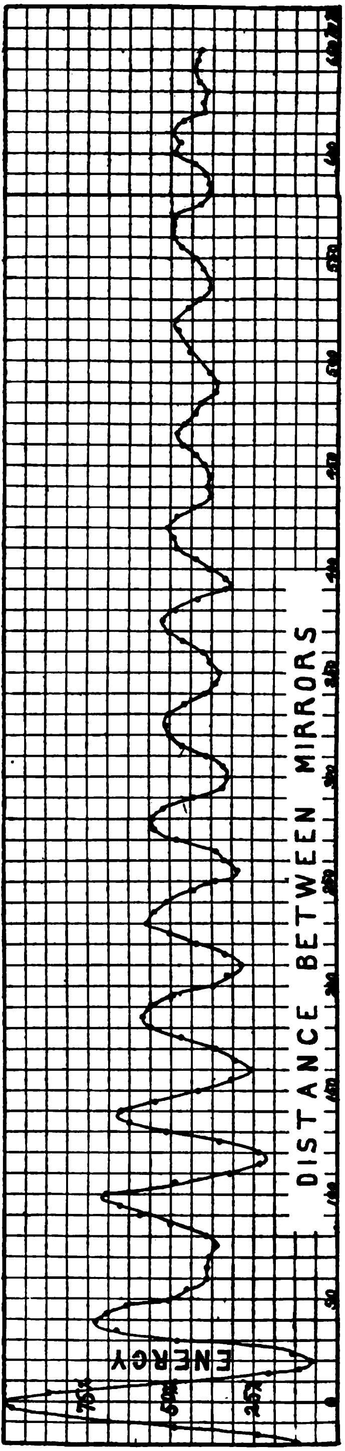


Fig. 9.

Interference curves.



Interference curve.

Fig. 10.

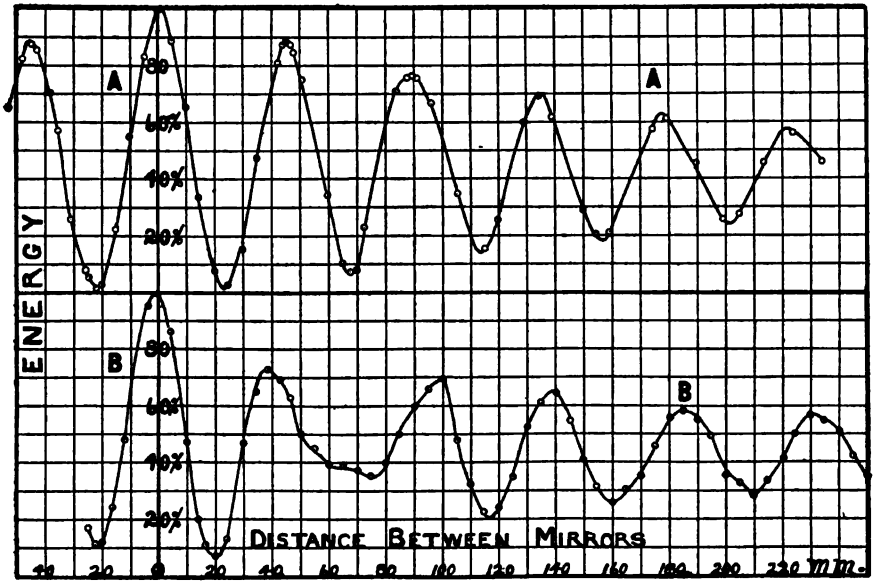
from the receiver wings. The results are shown in Fig. 8. Curve *A* shows the tuning of the receiver to the energy of the oscillator and curve *B* the tuning to the energy received from two reflections of resonator screens. The energy for the tuned length is in each case taken as 100 per cent. and the other values referred to this, and the percentages plotted as ordinates with the corresponding total receiver length in millimeters as abscissæ. These curves as is usual for receiver tuning curves show the overtone at approximately three times the fundamental length. The first overtone cannot appear in the receiver as it causes a node of current at the center where the thermo-junction is located. That the first overtone does exist in a resonator was shown in the preceding section.

The average value of the ratio of wave-length of resonators to the fundamental resonance length of receiver was found to be 2.46 (2.39, 2.43, 2.48, 2.50, 2.50). It must be stated in this connection that for mechanical reasons the base of the receiver used for this tuning work could not be made exactly the same as that used in the non-selective receiver for wave-length determinations. This fact may have something to do with the exact value of ratio obtained above.

Of this ratio more will be said in a following paper.

The subject of overtones will be further mentioned in a paper in this issue on the investigations of the effect of the dimensions of the resonators, particularly the width, on their resonance qualities. A comparison of Fig. 5 with Fig. 9 will, for the present, serve to show this effect. In each figure *A* is the vibrator curve, *B* is the curve from one screen and *C* is the curve from two screens of resonators. In Fig. 5 the resonators are linear while in Fig. 9 they are square. The vibrator curve is the same in the two cases and the *C* curves are seen to be of the same wave-length, but the *B* curves in the two cases are quite different. *B* in Fig. 9 shows a very marked tendency to break down into an overtone, or at least gives a complex radiation due to the fact that resonators and oscillator are not in close enough tune. It is important to note that the second screen makes use of this rather complex source and is caused to oscillate with its own natural frequency.

Fig. 11, *B*, is a further illustration of the effect of vibrator and resonators not being in close enough tune. The maxima and minima are not evenly spaced, nevertheless the oscillation is maintained for about fifteen wave-lengths as can be seen in Fig. 10. Fig. 11, *A*, shows a typical wave-length curve when screens and oscillator are in tune with each other.



Interference curves.

Fig. 11.

If this form of apparatus is to be used as a source of short, undamped electric waves, then it is evident, the closer the tuning between oscillator and resonator screen the better. If two reflections can be had then the exciting wave may differ from the natural wave-length of the resonators by as much as 20 or 30 per cent., while if only one reflection can be had this difference should not be more than 10 or 15 per cent.

SUMMARY.

1. The results of multiple reflection of electric waves from selective resonator screens are very similar to those of multiple or selective reflection in the infra-red.

2. Multiple reflection affords a practicable and easy method of obtaining short electric waves having a very definite period, less damped than the oscillator and free from "mirror action" and other disturbing influences.

3. The damping coefficient for resonators, whose width was negligible compared to their length, was found to be about 0.15, which agrees with the theoretical value, whereas the experimentally determined coefficient is about 1.4 for the Righi vibrator.

4. Resonators can be made to oscillate in their first overtone, *i. e.*, with a frequency twice that of the fundamental.

In a following paper is discussed an application of this multiple reflection method to a systematic study of the resonators themselves and the distribution required for any desired wave-length.

The authors take pleasure in expressing here their thanks to President E. F. Nichols, at whose suggestion these investigations were undertaken and they wish especially to express their appreciation for the many helpful criticisms and valuable suggestions of Dr. H. W. Webb throughout the work.

PHOENIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY,
March, 1913.

A SYSTEMATIC STUDY OF LINEAR AND NON-LINEAR RESONATORS FOR SHORT ELECTRIC WAVES.

By W. S. Nelms and W. L. Severinghaus.

Introduction.

IN the preceding paper the so-called "multiple reflection method" for investigating the radiation from screens of resonators has been discussed. The results and advantages of this method have been given there. It was shown that multiple reflection gave a radiation which was little damped and contains a single wave-length. With this kind of radiation we have determined the relation of the dimensions and the distribution of the resonators to the wave-length, damping, and amount of energy radiated from a screen. The question as to how wide the resonator may be and still be considered linear has also been investigated, in as much as the term "linear resonator" has been used rather loosely by other investigators.

The apparatus described in the first paper was used again in this investigation. There was a primary oscillating system (*source*), a reflecting system, and a receiving system. The oscillating system, or primary source of energy, consisted of a Righi vibrator placed at the focus of a spherical parabolic mirror. The reflecting system was the screens of tin-foil resonators to be studied together with a Boltzmann mirror for measuring the wave-lengths. The receiving system had a non-selective receiver of the Klemenčič type at the focus of a parabolic mirror similar to that used in the oscillating system. There was also a non-selective check receiver, the use of which was explained in the previous paper. The dimensions and details of the apparatus, as well as the arrangement of the parts, are given at length in the same place. Throughout the present investigation the apparatus was kept as nearly as possibly the same. It was found best to keep the period of the primary source as nearly as possible the same as the natural period of the resonators under investigation. This was accomplished by changing the vibrator from time to time.

A change in the dimensions of the resonators must be considered as due to a variation of the length, width or thickness: and by a change in the distribution is to be understood a change in the distance between consecutive resonators measured parallel to the electric component or a change in the distance measured perpendicular to this. The thickness of all the resonators was kept small and constant. In the discussion which follows, the dimension of a resonator measured parallel to the electric component of the oscillation in the incident energy is called the length, the dimension perpendicular to this is called the width. In general, the length was greater than the width but in some cases the width was several times as great as the length. Fig. 1 will make clear the meaning of the terms to be used.

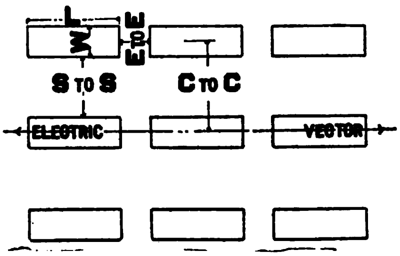


Fig. 1.

C to *C*, center-to-center distance; *S* to *S*, side-to-side distance; *E* to *E*, end-to-end distance; *W*, width; *L*, length.

The length of a resonator is the most important factor in determining its natural frequency. It seems well established that the wave-length reradiated is directly proportional to the length of the linear resonator, if the separation be kept in a constant ratio to the length of the resonator. The relation may be expressed by an equation of the form $\lambda = KL$, where K is a constant whose value depends on other factors than the length. The value of this constant given by theoretical and experimental investigators[1] seems to lie between 2 and 2.6. The theoretical values apply to the case in which there is but one resonator in the field. The primary object of this investigation was to find the value of K for any distribution of resonators for which the corresponding energy is sufficiently large to permit of the application of the present apparatus and method.

The results of this investigation are presented under three headings, viz, the effect on the period of oscillation of the resonators brought about by a change in

I. The distance between the resonators measured *parallel* to the electric component in the energy,

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[1] MacDonald, Electric Waves, p. 111; Poincare, Les Oscillations Electric; M. Abraham, Wied. Ann., 1898, 66, p. 435; Blake and Fountain, Phys. Rev., 1906, Vol. XXIII., p. 276; Webb and Woodman, Phys. Rev., 1909, Vol. XXIX., p. 131; Ives, Phys. Rev., 1910, Vol. XXX., p. 199.

II. The distance between the resonators measured *perpendicular* to the electric component in the energy,

III. The *width* of the resonator.

I. THE EFFECT OF A CHANGE IN THE DISTANCE BETWEEN THE RESONATORS MEASURED PARALLEL TO THE ELECTRIC OSCILLATION.

It was not expected that the distance between the ends of the resonators (E to E, Fig. 1) would affect their period of oscillation. This proved to be true for linear resonators. There is but little mutual capacity effect for such small surfaces, and there is no interlinking of the field of one with the circuit of the other, hence no mutual induction effect. Even with very wide resonators when the distance between the ends of the resonators was changed experiments showed that there was no effect on the period of oscillation of the resonator.

For the present "linear resonators" shall be considered as those having a width not greater than *one tenth* of the length. Later it will appear that a "linear resonator" may have a width as great as one fourth of the length. Table I. shows the results due to end-to-end variations. Using linear resonators, this end to end distance between resonators was varied from 4 mm. to 22 mm. The length of the resonators was changed from 23 mm. to 55 mm. The changes in wave-lengths noted were those due to changes in length of the resonators and changes in the distance between the resonators in the direction perpendicular to the electric oscillation. The fourth column of the table (Measured Wave-lengths) gives the wave-lengths measured by the Boltzmann mirror for the different screens of resonators; the fifth column gives the wave-length calculated for screens of resonators having the same dimensions and center-to-center distance, but having an end-to-end distance of 10 mm. How these calculated wave-lengths were obtained will appear in the next section, which relates to the effect of varying the center-to-center distance. The method of measuring the wave-lengths is accurate to about two per cent.; this is about the maximum variation of any measured wave-length from that calculated for the similar screen with a different end-to-end distance. The results in Table I. must be considered for each screen separately, for the separation center-to-center was not the same for the different cases, and screens having one separation cannot be compared with those having some other separation. With the resonators 226 and 590 mm. wide only one row of resonators was put on the screens, hence the different screens may be compared directly.

The fact that there was no end-to-end effect for the linear resonators

TABLE I.

Width, Mm.	Length, Mm.	End-to-end Distance, Mm.	Wave-length.	
			Measured, Mm.	Calculated for *E* to *E* 10 Mm., Mm.
2	23	22	58	57
2	36	10	95	96
2	36	15	97	96
2	50	4	113	113
2	55	5	110	113
3	36	10	83.5	85
3	36	15	95	97
18	36	10	91	91
18	36	20	81	81
18	36	15	100	97
226	36	10	72.3	
226	36	56	72	
590	50	43	110	
590	50	15	112	
590	50	20	110	

does not require that there be no such effect for the wide resonators. It was, however, demonstrated that there was no effect of this kind even for the very wide resonators. As appears in Table I., resonators varying in width from 18 mm. to 590 mm. (that is, from one half to twelve times the length) gave little end-to-end effect. With resonators 18 mm. wide there was no change in the wave-length when this end-to-end distance was changed from 10 mm. to 20 mm.; resonators 226 mm. wide showed no change in wave-length for a variation of this distance from 10 mm. to 56 mm. The end-to-end distance was changed from 15 to 43 mm. with the very wide resonators (590 mm. wide), the change in wave-length was less than two per cent. .

This work on the effect of a change in the distance between the resonators measured parallel to the electric oscillation confirms the work of Blake and Fountain[1] and extends it to apply to the case of non-linear resonators. We find then that there is no effect on the wave-length of the radiation from a screen of resonators when the resonators are moved apart in the direction of the electric component of the oscillation.

II. THE EFFECT OF VARYING THE DISTANCE BETWEEN THE RESONATORS MEASURED PERPENDICULAR TO THE ELECTRIC COMPONENT OF THE OSCILLATION IN THE RESONATORS.

A change in the distance between adjoining resonators measured perpendicular to the electric oscillation produces a change in the wave-length of the energy radiated by a set of resonators of given length. Table II. and Fig. 2, *A*, give the results of this investigation on the effect of this change of distance for the linear resonators (resonators 36 mm. long). The curve represents the relation between the wave-length and the center-to-center separation for these linear resonators. There is a minimum value for the wave-length when the separation has the value of 9 mm., *i. e.*, the distance between the resonators is 0.4 of the length of the resonators. From this point the curve is practically a straight line, the wave-length increasing as the separation is increased. This straight line relation cannot hold indefinitely, but apperas to hold past the point for which the separation has the value of 90 mm. or 2.5 times the length of the resonators. The separation represented by this value of 90 mm. is the greatest for which the multiple reflection methods could be used. For separations greater than this the energy became very small because of the few resonators in the field, and the method was then unreliable.

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[1] Blake and Fountain, PHYS. REV., 1906, Vol. XXIII., p. 268.

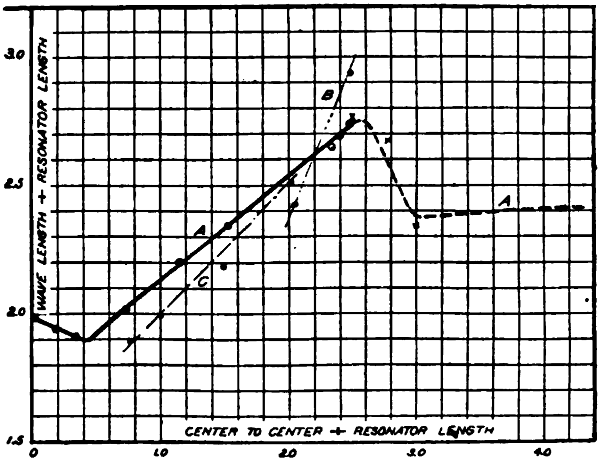

Fig. 2.

⊙, points taken by "multiple reflection"; ×, points taken by tuning method; ●, non-linear resonators.

The abscissæ of the curve as it is plotted in Fig. 2, *A*, were taken as the ratios of the distance between the resonators to the length of the resonator, and the ordinates as the ratios of the wave-length to the resonator length. These coördinates were chosen in order to make the curve more general in its application. With these coördinates the curve can be applied to any resonator system regardless of the length of the resonators. For with any two sets of resonators both variables are divided by the same factor, and the shape of the curve is maintained.

TABLE II.

C to C Distance, Mm.	Wave-length Measured, Mm.	$\frac{\text{Wave-length}}{\text{Reso. Length}}$	$\frac{C \text{ to } C}{\text{Reso. Length}}$	Wave-length of Source, Mm.
7	69	1.94	0.194	76
12	68	1.91	0.333	76
26	73	2.02	0.723	76
42	79.5	2.20	1.166	76
56	84.6	2.35	1.555	76
82	95	2.63	2.380	76
86.5	99	2.69	2.400	87
90	97	2.72	2.500	87
	All resonators 2 mm. wide and 36 mm. long.			

The data contained in Table II. were obtained by using the same linear resonators throughout, keeping the same distance between the ends of the resonators, and varying only the distance between the resonators in the direction perpendicular to the electric oscillation. As will be seen from the table, this distance was changed by successive small amounts from 7 mm. to 90 mm. In all cases two reflections were used and the source was kept fairly well tuned to the screens. Under these conditions, as shown in the preceding paper, the character of the resulting radiation depends almost entirely upon the resonators and is practically independent of the source.

For separations greater than 90 mm. or 2.5 times the resonator length, a method of tuning the receiver to the radiation from the screens had to be introduced. This method (described in the preceding paper) is open to the same objections[1] as the tuning methods used by other investigators. In any tuning method of wave-length measurement, the maximum energy may not and very probably is not received when the tuning is exact, *i. e.*, when the system has the same period as the incident energy. There is in all cases a decrease in the energy due to the decrease in the size of the resonator or receiver, as the case may be. This will tend to shift the length taken as the tuning length toward the large values and give an incorrect value for the wave-length. This objection may be brought against that part of the curve in Fig. 2, *A*, which lies beyond the abscissa 2.5. It may also be brought against the work of all investigators who have used a tuning method for studying resonators of this type.

Although the method of study is open to the above objections, it was adopted in the present investigations since it was very desirable to carry the curve beyond the value of 2.5 for the abscissa. From the part of the curve beyond this point it is possible to estimate the value of the ordinate of the curve when the resonators are separated to such a distance that they no longer influence the period of one another. The value of the constant *K* connecting the resonator length and the wave-length when there is only one resonator in the field would thus be found. The exact shape of the curve beyond the point for which the abscissa is 2.5 has not been accurately determined. However, it appears to turn at about this point and to approach the ordinate having the value of about 2.4 as its value when the resonators no longer affect each other. This then would indicate the ratio of the wave-length to the resonator length when there is but one resonator in the field. The exact value of *K* is still in doubt, but the most probable value does not agree with the theoretical value given by Abraham.[1] The value does agree very well with that obtained by other investigators. For a radiating system the ratio of the wave-length to the resonator length seems to be greater than two.[2]

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[1] Webb and Woodman, PHYS. REV., Aug., 1909, Vol. XXIX.

Table III.

C to *C* Dist., Mm.	Wave-length Measured, Mm.	Wave-length Calculated, Mm.	$\frac{C \text{ to } C}{\text{Reso. Length}}$.	Length of Resonator, Mm.	*E* to *E* Dist., Mm.	Wave-length of Source, Mm.
42	58	57	1.825	23	22	72
42	58.5	57	1.825	23	22	76
42	91	90	1.000	42	10	87
42	94	90	1.000	42	10	118
67	113	113	1.340	50	4	87
42	111	113	0.722	55	5	76
84	116	120	1.400	60	10	118
42	119	122	0.700	60	10	118

As a test of the accuracy of the curve showing the relation of the separation to the wave-length emitted, a number of screens were made and the corresponding wave-lengths measured and compared with those calculated from the curve. The results are shown in Table III. In one column are given the measured values of the wave-lengths and in a parallel column the calculated values. Screens of resonators varying in length from 23 mm. to 60 mm. and having a *C* to *C* separation varying from 40 to 84 mm. were tested. The distance between the ends of the resonators are also given, but as shown in the last section this factor does not affect the wave-length. Linear resonators only were used, the width being 2 mm. in all cases. A comparison of the two columns mentioned above shows that in no case is the variation of the calculated value for the wave-length from the measured value greater than about 3 per cent. over a range of wave-lengths varying from 58 mm. to 119 mm.

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[1] M. Abraham, Wied. Ann. d. Phys., 1898, 66, p. 435.

[2] Pierce's Principles of Wireless Telegraphy, p. 116.

Not only does this curve represent the results for the apparatus used in the present case, but the results of other investigators using other forms of apparatus agree very well with this curve. Aschkinass and Schaefer,[1] in 1901, published the results of some work done to determine the dielectric constants of certain liquids. Their investigation was made by studying the effect of the liquid on the period of resonators of the same type as ours when immersed in the liquid. The resonators had the dimensions and distribution given below:

Length of resonator	45 mm.
Separation perpendicular to length	27 mm.
Width of resonators	"linear"
Wave-length in tune with resonators *in air*	90 mm.

For a screen of this description the curve Fig. 2, *A*, calls for a wave-length of 90 mm.

Woodman and Webb[2] found that resonators 29 mm. long, with a separation of 30 mm., were in tune with a source having a "wave-length of 62 to 64 mm." The curve shows that a screen with resonators of these dimensions should have a natural frequency corresponding to a wave-length of 62.5 mm. The wave-lengths given by the various observers mentioned above were measured by different methods, and differing from that used in obtaining the curve, so that the agreement between their results and the calculated values cannot in any way be due to a similarity of apparatus or methods. Hence it is evident that the curve given in Fig. 2, *A*, which represents the effect of the separation on the wave-length expresses very closely the correct relation between these two quantities.

Blake and Fountain[3] in a paper relating to linear resonators, give the following data for three of their screens:

No.	Resonator Length, Mm.	Separation, Mm.
1	53	30
2	49	60
3	44	100

The curve obtained in the present investigation gives the following values for the wave-lengths from these screens:

No.	Wave-length.
1	104
2	109
3	115

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[1] Aschkinass and Schaefer, Ann. d. Phys. 1901, 5, p. 489.

[2] Woodman and Webb, PHYS. REV., Vol. XXX., 1910, p. 575.

[3] Blake and Fountain, PHYS. REV., Oct., 1906, Vol. XXIII.

The investigators state that these screens were in tune with a vibrator having a wave-length of 99 mm.[1] Our study of their interference curve for the wave-length of their source seems to indicate that the wave-length may have been about 108 mm. Webb and Woodman[2] have shown that the tuning method is not very reliable for wave-length measurements and that the tuned receiver is unsatisfactory for this kind of work. The results given for these screens do not agree as well with the calculated results from the curve as those of the other investigators mentioned. But the errors to be expected from the tuning method are more than sufficient to account for the discrepancy.

In the same paper (page 268) Blake and Fountain have given a curve representing the relation of the separation to the wave-length for the screen. Their curve does not seem to fit the facts as found in the present investigation. The discrepancy is one of amount and not of kind.

The curve, Fig. 2, *A*, applies to linear resonators only. If the multiple reflection method is to be of value, we must be able to determine beforehand what the frequency of the resonators will be on any given screen. Hence the effect of the separation on the wave-length for the *non-linear* resonators must also be known. The effects for square resonators (36 mm. long and 36 mm. wide) and for resonators 36 mm. long and 18 mm. wide are shown in Fig. 2, *B*, and 2, *C*. It is seen that the curves are similar to that for the linear resonators but have a greater slope. Hence for wide resonators the wave-length increases by a greater amount for the same increase in separation than for the linear resonators. For the intermediate widths the slope of the curve is also intermediate.

When the wide resonators are used a question arises as to how this separation is to be measured. Shall it be taken as the distance between the adjacent edges of the resonators or shall it be taken as the distance between corresponding points in the resonators, as for example between the centers (*C* to *C*)? A study of this point shows that the results are more simply correlated if the separation be taken as the distance between the centers of the resonators. The reason for this conclusion will be taken up more fully in the next section.

The curves in Fig. 2 were used for determining the length and distribution of resonators on screens to give desired wave-lengths. The length chosen depended on the energy required, for the energy reradiated decreased with a decrease in the length of the resonator as well as with the separation. A very satisfactory condition was found when the length of the resonator was approximately one half of the wave-length desired.

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[1] Blake and Fountain, Phys. Rev., 1906, Vol. XXIII., pp. 263–4.

[2] Webb and Woodman, l. c., p. 123.

Having chosen the length of the resonator the exact distribution to be used was determined from the curve. By taking the ratio of the length of the wave to the length of the resonator as the ordinate, the ratio of the separation to the length of the resonator was the abscissa of the point on the curve. The distance between the ends of the resonators (*E* to *E*) does not affect the wave-length, hence this distance was made 10 or 15 mm. for all the screens. A number of screens were made up in this way and in all cases the measured and the calculated wave-lengths were in close agreement.

III. THE EFFECT OF A CHANGE IN THE WIDTH OF THE RESONATORS ON THE WAVE-LENGTH OF THE ENERGY RERADIATED BY THEM.

It was now possible to take up the question of the effect of the width of the resonator on the wave-length. In this investigation it was found possible to determine the limiting dimensions of what may be called a "linear resonator." It was found that a change in the width changes the wave-length but slightly. The damping, however, and the amount of energy radiated by a set of resonators depend very largely upon the width of the resonators.

The effect of the width on the wave-length was first investigated by a method differing essentially from the multiple reflection method. The method was one of tuning, in which the primary source of the energy was tuned to the resonator screens. The energy which was transmitted, *i. e.*, passed through the screen, fell upon a receiver (non-selective): the tuning was judged from the relative amounts of energy absorbed when sources of different wave-length were used. This method was however unsatisfactory and had to be abandoned, as the effect on the wave-length of the changes in width is too small to be determined in this way. The method of multiple reflection was then employed.

Resonators 24 mm. wide and 36 mm. long were placed on screens with a distance between the ends of 10 mm. and a distance of 66 mm. between adjoining sides, *i. e.*, 90 mm. between centers of the resonators measured perpendicular to the electric oscillation. An interference curve was taken with the Boltzmann mirrors, with the energy successively reflected from two of these screens. The resonators were then made narrower by cutting equal amounts of foil from the sides of each resonator. The distance between the sides was in this way changed by the amount of foil taken from a resonator, the distance between the centers remaining the same. An interference curve was taken for the altered system of resonators. The resonators were then narrowed still more and the interference curve again taken. This process was continued until the resonators were too narrow to allow of further cutting from the sides. This set of screens will be referred to as "Set 1."

Table IV.

Width, Mm.	Wave-length Measured, Mm.	Wave-length Calculated, Mm.	Side-to-side Distance, Mm.
24	98	97	66
18	100	97	71
12	100	97	78
6	98.5	97	84
4	96.5	97	86
3	95	97	87
2	96	97	88

End-to-end distance	15 mm.
Center-to-center distance	90 mm.
Length of resonators	36 mm.
Wave-length of source	76 mm.

Two reflections used.

Table IV. gives the dimensions and distributions of the resonators for Set 1, and the observed wave-lengths taken from the interference curves. It will be seen from column I. that the width varied from 24 mm. to 2 mm., and from column II. that the wave-length varied from 95 mm. to 100 mm. only. The small variations in the wave-length do not follow the changes in the width. For example, with resonators having a width of 18 mm. the wave-length is 100 mm. while a width of 24 mm. gives a wave-length of 98 mm., a width of 3 mm. a wave-length of 95 mm. and a width of 2 mm. a wave-length of 96 mm. It is probable that this random apparent variation in the wave-length is due to errors of observation and does not represent a true effect. The greatest deviation of any value for the wave-length from the mean value is about 2.5 per cent., the average value of the wave-length being 98 mm. The greatest deviation between any two measured wave-lengths is only about 5 per cent. for a variation in the width of some 1,200 per cent. This small and random variation in the measured wave-lengths suggested to the authors that there was no very great effect on the wave-length caused by the change in width and further that the distance between the centers of the resonators, rather than the distance between the sides, was the important factor in determining the effect of separation perpendicular to the electric oscillation. This conclusion seems to be borne out by the results obtained from the screens of a second set, called Set 2.

The screens of Set 2 were made with resonators of the same width as those used in Set 1. With these screens the distance between the sides of the resonators was kept constant, necessitating that the distance between the centers of the resonators be varied. It was necessary with these screens to shift the resonators toward the center row each time the width was changed, by an amount equal to the change in the width of the resonators. The distance between the sides of the resonators was constant and equal to 54 mm. Table V. gives the results for the screens of Set 2.

TABLE V.

I.	II.		III.	IV.	V.
	Wave-length				
Width, Mm.	*A* **Measured, Mm.**	*B* **Calculated,[1] Mm.**	**Center-to-center, Mm.**	**Wave-length of Source, Mm.**	**End-to-end, Mm.**
0.3	84.6	83.9	54.3	76	10
2.0	84.6	84.6	56	76	10
3.0	83.5	85.0	57	76	10
5.0	85.0	85.0	59	76	10
9.0	86.5	87.5	63	76	10
18.0	91.0	91.0	72	76	10

[1] Calculated for "linear resonators" with same distribution as center-to-center distance of the wide ones.

Side-to-side distance constant and equal to 54 mm.

The assumption that the width does not greatly affect the wave-length and that the center-to-center distance is the important factor in determining the wave-length explains the results from the screens of Set 2 as well as those from Set 1. In the case of Set 1 this view requires that the wave-length be very nearly a constant, since the center-to-center distance was constant. And as was shown in Table IV. this was very nearly true, though the width was varied by several hundred per cent. Again this assumption requires for the screens of Set 2 that the wave-length correspond very nearly to the changes in the center-to-center distance between the resonators. This is also shown to be true, in Table V. (cf. column II., *A*, and III.). Other assumptions were made to explain these results and it was found that an assumption involving the width as an important factor and the distance between the sides as the important separation factor, necessitated the assumption of a relation between the wave-length, width and separation which was not so simple. On the other hand, the relation between the wave-length and separation on the assumption here made appears to be practically linear, a small correction being required only in case the width was large compared with the length of the resonator. From these results it appears that the width has but a small effect on the wave-length of the reradiated energy, and that the important factor to be considered is the distance between the centers of the resonators rather than the distance between the sides.

If the width does not greatly affect the wave-length, then the wave-length for a screen of non-linear (width comparable to the length) resonators should correspond rather closely to that for a screen of linear resonators for the same distribution. The separation distance for the linear resonators must be taken the same as the distance between the centers of the wide resonators. Calculating the wave-length which should be given by a screen of linear resonators arranged to have a *C* to *C* separation of 90 mm., the center-to-center distance for all screens of Set 1, it is found that the curve Fig. 2, *A*, requires a wave-length of 97 mm. for these linear resonators. The average value of the measured wave-length for these screens was 98 mm. In Table V. a column headed "calculated wave-length" (col. II., *B*) gives the wave-lengths for the linear resonators having the *C* to *C* separation corresponding to the center-to-center distance for the wide resonators. It is seen that these calculated values agree very well with the wave-length measured for the screen. Although the width was made to vary from 0.3 mm. to 18 mm. and the center-to-center distance from 54 mm. to 72 mm., the maximum value of the deviation of any one measured value from the corresponding calculated value is about 3 per cent. The actual variation in the wave-length is some 16 per cent.

All the results tend to show that the wave-length, or the natural period of the resonators, is very nearly independent of the width. That there is a slight variation due to the change in the width is shown by the fact that the curve for the effect of the separation in the case of the wide resonators is steeper than that for the linear resonators (see Fig. 2). But the variation of the wave-length for the square resonators from that for the linear ones having the same separation (*C* to *C*) is small. In the case when the separation is as great as can be tested by the multiple reflection method, the variation was only about 10 per cent. The change in width for this case was some 1,200 per cent.

Damping.—A change in the width of the resonators exerts a marked influence on the damping of the oscillation. This damping is evident from (1) changes in the amoumt of energy reradiated, (2) changes in the number of maxima and minima that may be taken, (3) changes in the purity of the radiation, or the "smoothness" of the interference curves. Table VI. shows the variation in the damping constant and amount of energy radiated as the width is changed. The damping constant[1] given in the last column was calculated as explained in the previous paper. This constant has a value of .15 for the narrow resonators and increases with the width up to a value of .31 for the square resonators. The damping constant of the radiation from the source has a value of 1.36.

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[1] Klemenčič u. Czermak, Ann. d. Phys., 1893, Vol. 50, p. 179.

TABLE VI.

Width, Mm.	Energy, One Reflection.	Energy, Two Reflections.	Damping Constant.
0.3	308 divisions	112 divisions	0.15
2	458	243	0.19
3	518	279	0.19
5	565	357	0.19
9	616	413	0.22
18	522	355	0.29
36	291	110	0.31
Strips	110	23	——
Source	——	——	1.36

With this highly damped source and total reflecting surfaces in the place of the resonator screens, it was possible to get only two or three maxima and minima in the interference curve. Using the narrow, linear resonators it was possible to take as many as fifteen or twenty such maxima and minima. That there is an effect on the purity of the radiation resulting from a change in the width is quite evident from a glance at interference curves for resonators of different widths and the same length. The curves become more and more irregular as the width is increased and secondary maxima and minima become more evident especially when the source is not well tuned to the resonators. With the wider resonators there seems to be a greater tendency of the source to force its own period on the resonators.

The energy increased with the width of the resonators until the width became about one fourth of the length, 9 mm. for the resonators used. If the width was increased beyond this value the energy decreased and when the width was equal to the length (square resonators) the energy was about the same as for the very narrow resonators. The increase in the width seemed to act as an increase in the number of resonators in the field. At the same time this increase in the number of resonators increased the damping of the oscillations in them. It appears that those factors which increased the damping also decreased the amount of energy reradiated. When the width has become as great as one fourth of the length the losses just balance the gain from the increased foil.

A rather extensive study was made of the resonators having their width equal to their length, *i. e.*, 36 mm. long and 36 mm. wide. It was found that if two reflections be used and the C to C separation be not greater than 2.5 times the length of the resonator, the interference curves were similar to those for the linear resonator; the damping however was greater. If the separation be greater than this the curves are very irregular and it is difficult to determine the wave-length from them. These irregularities in the curves become more pronounced as the separation is increased, and there seems to be a tendency for the resonator to oscillate with the period of the source. In order to assure good results and reliable interference curves, it is necessary that the incident energy have a wave-length of not less than 0.8 of the wave-length of the resonators, provided two reflections be used. With one reflection the interference curves are not very reliable, unless the source be very closely tuned to the period of the resonators. If the source have a wave-length of less than 0.7 of the wave-length of the resonators the radiation from one reflecting screen will probably correspond to that of the first overtone of the resonator.

It is to be expected that with resonators having as great a surface as the square ones there will be a certain amount of energy regularly reflected, as from a metal sheet. In order to obtain information on this point, a set of screens was made in such a way that this regularly reflected energy did not arrive at the receiving system. This was accomplished by rotating each resonator on the screens through an angle of about thirty degrees about a line through its axis parallel to the electric component of the oscillation. In this way the corresponding points in all the resonators were held in the same plane, so that the Huygens wave-front of the radiated energy was still a plane and the reradiated energy had the same path as before. The angle taken was more than enough to guarantee that this energy did not arrive at the receiving system. It was found that the interference curves were somewhat smoother, and that the energy was reduced by about 25 per cent.

The results from the square resonators therefore show that they are not very satisfactory as resonators. The damping is high, the energy is small and they show rather marked tendency to reflect the irregularities present in the incident energy. They also show a tendency to oscillate in their overtone when the tuning is not close.

In Table VII. is given the results of some work on resonators for which the width was greater than the length. The widths were varied from one to twenty times the length of the resonator. It will be seen that the wave-length measured for such resonators is in general very near to that for the source. It is evident that when the width is greater than the length there remains but little resonance effect, and there is no definite period for the resonators. The damping is high and the energy is small.

TABLE VII.

Width, Mm.	Length, Mm.	Wave-length of Source, Mm.	Center-to-center, Mm.	End-to-end, Mm.	Reflections.	Wave-length, Mm.
27	36	76	200	10	1	73
36	36	76	72	10	2	87
54	36	76	90	10	2	71
72	36	76	108	10	2	72
72	36	76	108	10	1	73
104	36	76	108	10	2	70
108	36	76	—	10	1	70
166	36	76	—	10	1	69
226	36	76	—	10	1	72.3
226	36	76	—	56	1	72
226	36	76	—	10	2	69
320	36	76	—	10	1	70
320	36	76	—	10	2	70
328	36	76	—	10	2	68.5
550	40	87	—	10	2	86
590	36	120	—	10	2	109
590	50	120	—	43	2	110
590	50	120	—	15	2	112
590	55	120	—	20	2	112
695	36	76	—	10	2	70.5

If a "linear resonator" means one whose wave-length does not change with a decrease in the width, then the results of this investigation show that a resonator may be considered "linear" so long as the width is less than one fourth of the length of the resonator. If the additional condition be imposed that the damping must also be independent of the width, the ratio of the width of the resonator to its length must be smaller than this value.

SUMMARY.

The object of the present investigation has been to determine the relation of the dimensions and distribution of the resonators to the resulting wave-length. The present study is an application of the "*Multiple Reflection Method*" described in the preceding paper by the same authors.

A summary of this investigation follows:

1. A change in the distance between the resonators measured along the direction of the electric oscillation has very little effect on the period of oscillation of the resonator. This applies to both linear and non-linear resonators.

2. For a screen of resonators the ratio of the wave-length of the energy radiated to the length of the resonators depends very largely upon the separation of the resonators measured perpendicularly to the electric

oscillation. The value of this ratio (K) is shown by the curves in Fig. 2 These curves may be used to determine the proper distribution of resonators to give any desired wave-length.

3. The effect on the wave-length of the distance between the resonators measured perpendicular to the electric oscillation is more simply expressed in terms of the center-to-center distance between the resonators than in terms of the side-to-side distance.

4. An increase in the width of the resonators of a given length has a very small effect on the wave-length reradiated.

5. An increase in the width of the resonators increases the damping of the oscillation. As a result the interference curves are irregular and the oscillation shows a more marked tendency than in the case of the linear resonators to correspond to that of the first overtone. For widths greater than one fourth of the length of the resonator, the energy decreases with an increase in the width.

6. If the width of the resonators be greater than 1.5 times the length, there is very little effect of resonance. The wave-length of the energy from the screen is determined almost entirely by the wave-length of the incident energy.

7. The ratio of the wave-length to the resonator length for a resonator not under the influence of other resonators has been shown to be greater than 2. The value estimated from this investigation is about 2.4.

8. It appears from the results of this investigation that any resonator having a width less than one fourth of its length may be taken as a "linear resonator." If the ratio be greater than this there is but a small change in the wave-length but a marked change in the character of the reradiated energy.

The authors take pleasure in expressing here their thanks to President E. F. Nichols, at whose suggestion these investigations were undertaken, and especially do they wish to express their appreciation for the many helpful criticisms and valuable suggestions of Dr. H. W. Webb throughout the work.

Phœnix Physical Laboratory,
Columbia University,
New York City,
March, 1913.

Note.—Since the foregoing papers were sent to the publishers an article by K. F. Lindman has appeared in the Ann. d. Phys., April, 1913, p. 992. His results are, in general, in agreement with the results of the present work, although the methods are quite different.

SPECIFIC INDUCTIVE CAPACITY AND ATOMIC CHARGES.

BY FERNANDO SANFORD.

IN a paper published in 1911[1] an attempt was made to find the explanation of the Volta effect in the different specific inductive capacities of the metals, though at that time it was quite generally held, especially by chemists, that the Volta effect is due to some kind of electrolytic charging of the opposed metallic surfaces by air or other gases. Since that time I have been able to show[2] that the Volta effect is independent of the character of the opposed surfaces, and that the potential difference between two metals when separated and in metallic contact with the earth must accordingly be due to some characteristic property of the metals themselves.

In several recent papers I have undertaken to show that the atoms of different elements have characteristic charges which are proportional to the products of the atomic mass into the mobility of the atomic ion in electrolysis, and that it is these charges which determine the position of the metal in the voltaic series. It is the purpose of this paper to call attention to the apparent relation of these atomic charges to the specific inductive capacities of their respective elements.

Unfortunately, these charges have been calculated for only three elements whose specific inductive capacities are known in the solid or liquid state. These elements are chlorine, bromine and iodine. For these elements, the specific inductive capacity in the liquid or solid state varies as the square root of the atomic charge of the respective atom. This is shown in Table I., where e' represents the atomic charge and k the specific inductive capacity as given in Landolt & Boernstein's Tables.

TABLE I.

Element.	e'	k	e'/k^2
Cl	35.2	2	8.8
Br	82.8	3.1	8.6
I	129	4	8.06

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[1] A Physical Theory of Electrification, Leland Stanford, Jr., University Publications.

[2] On the Nature of the Volta Effect, PHYS. REV., XXXV., 484, December, 1912.

Since the atomic volumes of Cl, Br and I are nearly equal, their electrical densities will be proportional to their atomic charges, hence the relations shown in Table I. hold as well for the electrical densities of the atoms as for their atomic charges.

Since the specific inductive capacity of metals has not been measured, differences in its magnitude can only be inferred from other properties which are known to vary with specific inductive capacity in non-metallic substances. Fortunately, the metals have a number of such properties in common with dielectric substances. One of these properties is cohesion.

The relation of specific inductive capacity to cohesion is shown for a number of gases in Fig. 1, in which the specific inductive capacity of the gases is plotted as ordinates and the corresponding values of van der Waal's constant "*a*" are plotted as abscissas. The temperatures at which the specific inductive capacities were determined are not given in the tables (Kaye & Laby's) from which the data for Fig. 1 were taken. The constant "*a*" is calculated for the critical temperature. The data are given in Table II.

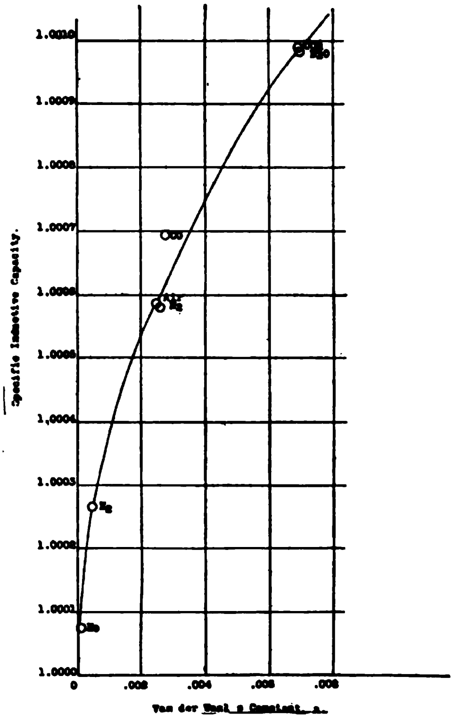

Fig. 1.

TABLE II.

Gas.	*k*	*a*
He	1.000074	.00006
H_2	1.000264	.00042
N_2	1.000581	.00259
Air	1.000586	.00257
CO	1.000695	.00275
CO_2	1.000985	.00717
N_2O	1.000990	.00710

The relation of specific inductive capacity in the solid or liquid state to cohesion may be shown with the same three elements whose specific inductive capacities are given in Table I. In a paper in Ann. d. Phys., XXXVII., 739, 1912, A. Heydweiller has calculated cohesion constants for a large number of elements. I have already shown[1] the relation of Heydweiller's cohesion constants to the characteristic charges of the atoms. Heydweiller's values for the cohesion constants of chlorine, bromine and iodine are respectively 3.02, 6.68 and 10.27. In Fig. 2 these values are plotted as abscissas and the corresponding values of the specific inductive capacities as ordinates. It will be seen from this curve that Heydweiller's values may be calculated from the specific inductive capacities by multiplying them by a constant.

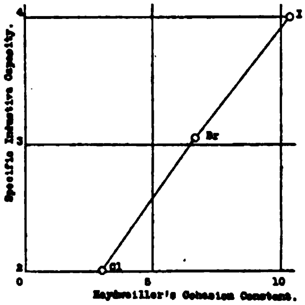

Fig. 2.

Another property of the elements which depends upon cohesion is the melting point. Both the melting point and the specific inductive capacity in the liquid or solid form are given for seven elements in Landolt and Boernstein's Tables. These are given in Table III., the melting point being given in absolute temperature.

TABLE III.

Element.	k	m.p.
Oxygen	1.465	38
Chlorine	2	171
Bromine	3+	266.7
Phosphorus	3.6	317
Iodine	4	387
Sulphur	4	387
Selenium	7.4	490

In Fig. 3 the specific inductive capacities of these elements are plotted as abscissas and their melting points as ordinates. It will be seen that with the exception of selenium the specific inductive capacities may be calculated by multiplying the melting points by a constant as closely as they may be measured by the usual methods.

The three curves already given seem to show that the cohesion between the molecules of elements vary as the specific inductive capacities of the elements. I have already shown in several papers that cohesion is an electrical force and that it varies proportionally with the characteristic charges of the molecules, hence the specific inductive capacity in the cases here considered must also vary proportionally with the molecular charges.

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[1] On The Electrical Nature of Cohesion, PHYS. REV. I., 211, March, 1913.

While the molecular charges can be calculated with a reasonable degree of probability for only a few compounds, we have in the halogen acids a group of three such compounds for which the specific inductive capacities have been determined. In a paper by Schaefer and Schlundt, Jour. Phys. Chem., XII., 669, 1909, is given a determination of the specific inductive capacities of HCl, HBr and HI in the liquid state at temperatures of from 21.7 to 27.7 degrees and also at temperatures of from – 50 to – 90 degrees. In either set of determinations the specific inductive capacity decreases at a constant rate as the melting points and the negative molecular charges (calculated as the algebraic sum of the charges of the positive and negative ion) increase. Both of these relations are shown in Fig. 4, the specific inductive capacities at the higher temperatures being used. The same relation is shown as well when the specific inductive capacities at the lower temperatures are used. In both curves given in Fig. 4 the specific inductive capacities are used as abscissas. In Curve I. the melting points and in Curve II. the molecular

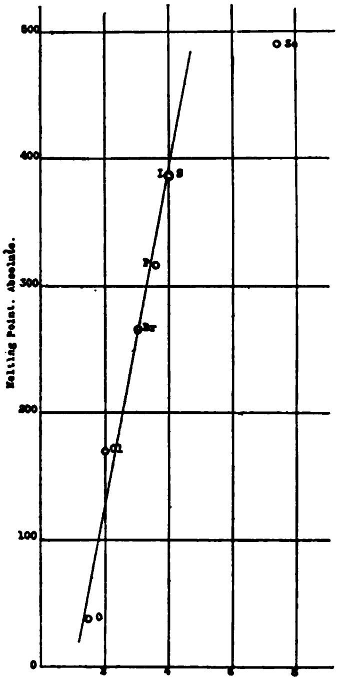

Fig. 3.

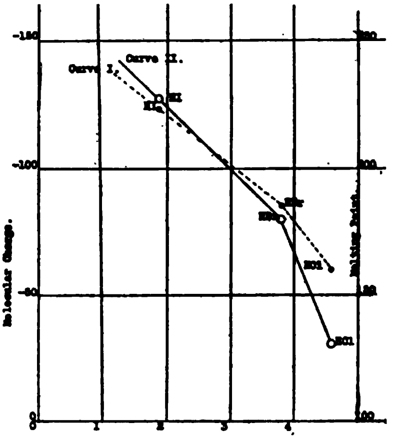

Fig. 4.

charges are taken as ordinates. The data from which the curves are plotted are given in Table IV.

TABLE IV.

Molecule.	k	Mol. Chg.	Mltg. Pt. Abs.
HCl	4.6	−31.5	161
HBr	3.82	−79.9	186
HI	2.9	−127.5	223

The same relation between specific inductive capacity and melting point is shown in the compounds of the nitrogen group with hydrogen. In Jour. Phys. Chem., XVI., 253, 1912, Schlundt and Schaefer have given the specific inductive capacities of this group of compounds. Their values for specific inductive capacity at 15 degrees are given along with the melting points of the compounds in Table V.

TABLE V.

Compound.	k	Melting Point.
NH_3	15.9	63
PH_3	2.88	317
AsH_3	2.05	631
SbH_3	1.81	998

The data given in the preceding tables seem to show an unquestionable relation between specific inductive capacity and cohesion for groups of similar elements or compounds, and accordingly that in these cases a similar relation must exist between specific inductive capacity and atomic or molecular charges.

Another property which is generally understood to vary with specific inductive capacity is the refractive index or the refraction constant as calculated from the Gladstone or Lorentz formula. This refraction constant has been calculated by Gladstone for a large number of the elements, and was found to vary roughly with the square root of the atomic weight, a relation which also holds for the atomic charges. That it also, like specific inductive capacity, varies with cohesion is shown by the following considerations.

In Zeitsch. f. phys. Chem., VII., 140, 1891, J. W. Brühl gives a new determination of the atomic refraction for a considerable number of elements. For five of these elements van der Waal's constant "a" has been calculated and for six of them Kleeman[1] has calculated a constant "C_a" which determines the surface tension in the liquid state of the molecules to which the atoms belong. In Table VI. are given Brühl's atomic refractions, van der Waal's constant "a" and Kleeman's values of "C_a" for a number of elements.

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[1] Phil. Mag., XIX., 784, 1910.

TABLE VI.

Element.	At. Ref.	"a"	"C_a"
Hydrogen	1.103	.00042	1
Carbon	2.365	——	5.30
Nitrogen	2.76	.00165	—.—
Oxygen	2.328	.00273	5.94
Chlorine	6.014	.01063	8.40
Bromine	8.863	.01434	10.65
Iodine	13.808	——	15.49

The relations shown in the above table are represented graphically in Fig. 5. The refraction constants are plotted as abscissas. In Curve I. van der Waal's constant "a" and in curve II. Kleeman's constant "C_a" are plotted as ordinates.

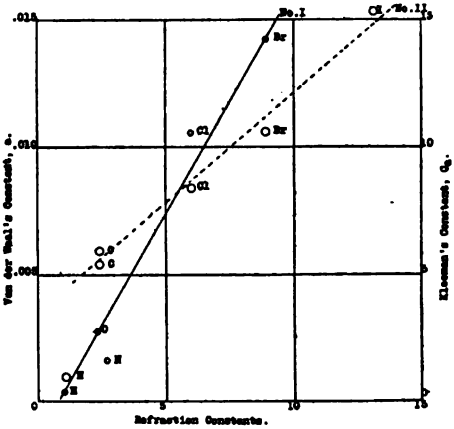

Fig. 5.

The relation of refraction constant to cohesion may be shown even in the gases of the helium group, where cohesion is weakest. In Smiles's Chemical Constitution and Physical Properties, p. 245, is given a table from Cuthbertson in which are given values of $(n - 1)10^6$ for a considerable number of elements, among them the gases of the helium group. A. O. Rankine[1] has calculated Sutherland's cohesion factor "C" for these gases from experimental data on their viscosity. The data from Cuthbertson's and Rankine's tables are given in Table VII.

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[1] Phil. Mag., XXI., 45, 1911.

TABLE VII.

Element.	$(n-1)10^6$	C
He	72	70
Ne	137	56
Ar	568	142
Kr	852	188
Xe	1380	252

These relations are shown graphically in Fig. 6.

It would be easy to multiply examples showing the relation of atomic refraction to cohesion. The sole purpose in introducing this relation here is to argue from it to the relation between atomic charges and specific inductive capacity. I have elsewhere[1] called attention to the relation between atomic refraction and atomic charges in the alkali metals. It may not be superfluous to show here the same relation for the metals of the barium group.

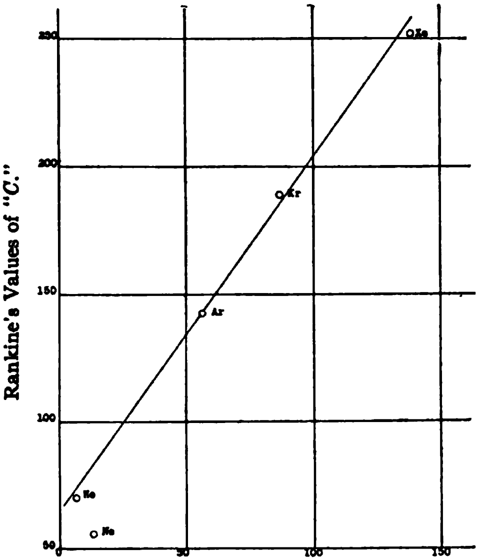

Fig. 6.

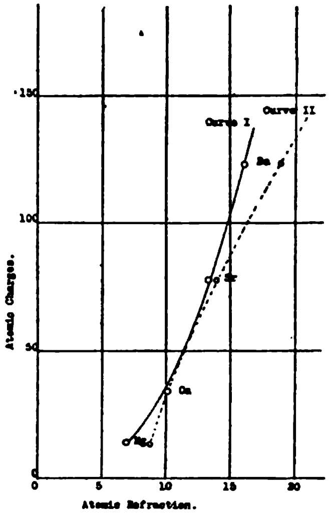

Fig. 7.

Smiles has given, besides Gladstone's atomic refraction for the metals of this group a refraction equivalent for Mg, Sr and Ba determined by Pope from other considerations. Both these values as well as the atomic charges calculated from the actual velocity of the ions in electrolysis are given in Table VIII., and their relations are shown graphically in Fig. 7.

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[1] Pressure-Shift of Spectral Lines, Astrophysical Journal, XXXV., 8, 1912.

Gladstone's atomic refractions are taken as abscissas in Curve I. and Pope's refraction equivalents in Curve II. The atomic charges are taken as ordinates in both cases.

Table VIII.

Element.	At. Ref. Pope.	At. Ref. Gladstone.	At. Chg.
Mg	8.81	6.99	13.2
Ca	—.—	10.1	34.8
Sr	13.95	13.3	77.5
Ba	18.94	16.1	123

The relation between atomic charges and refraction may also be shown after the atoms are combined into compound molecules. On page 272 of Ostwald's Solutions are given the refraction constants of a number of salts as calculated from the Gladstone formula $(n - 1)m/d$. These values and the sums of the atomic charges of the molecules are given in Table IX. It will be noticed that all but two of the salts are from monovalent acids, and that in the case of the two sulphates only one metallic ion has its charge counted. I do not know why this should be necessary to make the bivalent salts fall on the same curve as the monovalent salts, but I have observed that it is so.

Table IX.

Salt.	$\frac{n-1}{d}m$	Sum of Charges.
HCl	14.44	41.5
NaCl	15.11	52.3
KCl	18.44	76.1
HBr	20.63	90
NaBr	21.7	100.6
KBr	25.34	124.5
HI	31.17	137.5
NaI	31.59	148.2
KI	35.33	172
HNO_3	17.2	70.1
$NaNO_3$	18.7	81
KNO_3	21.8	104.7
KSO_4	30.5	143
HSO_4	22.4	108.4

The above relations are shown graphically in Fig. 8.

Another property of transparent substances which H. Becquerel has shown to be related to their refraction[1] is their magnetic rotatory power for polarized light. This property should accordingly be related to the specific inductive capacity.

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[1] Becquerel's law is $r/n^2(n^2 - 1) =$ const. where r is magnetic rotatory power and n is the refractive index.

On page 500 of Smiles's Chemical Constitution and Physical Properties is given a table of the atomic rotatory power of five metals as calculated from solutions of their salts. The calculations are by Schönrock. His values, together with Gladstone's atomic refraction and the atomic charges, are given in Table X. It will be seen that while the magnetic rotatory power and atomic refraction vary approximately together, the magnetic rotatory power may be calculated to within the probable error of its determination by multiplying the positive atomic charges by a constant. This relation is shown graphically in Fig. 9.

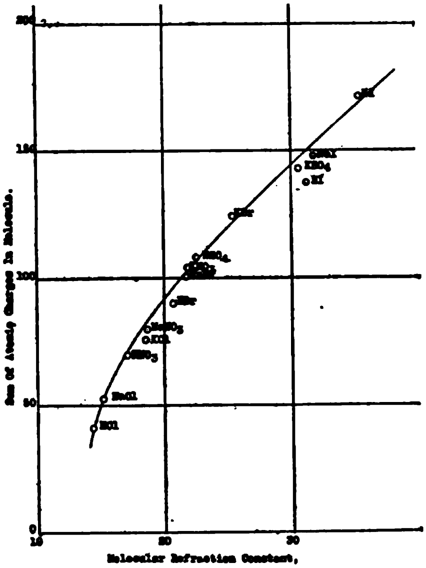

Fig. 8.

TABLE X.

Metal.	Mag. Rot.	At. Ref.	At. Chg.
Li	.398	3.6	3.6
Na	.558	4.66	15.7
Mg	.577	6.99	17.3
Ca	.691	10.1	32.5
K	.809	8	39.6

Perhaps one further relation may be presented to show that elements from the extremes of the periodic series and with opposite characteristic charges show the same relation between their charges and cohesion, viz., that cohesion diminishes with increasing positive charges. Landolt and Boernstein give the atomic heats of fusion of four monovalent elements whose charges may be calculated from the mobility of their ions. These elements are potassium, sodium, bromine and iodine. Their atomic heats of fusion and their characteristic charges are given in Table XI., and the relation between the two sets of properties is shown graphically in Fig. 10. The atomic heats of fusion are taken as abscissas and the atomic charges with their appropriate signs as ordinates.

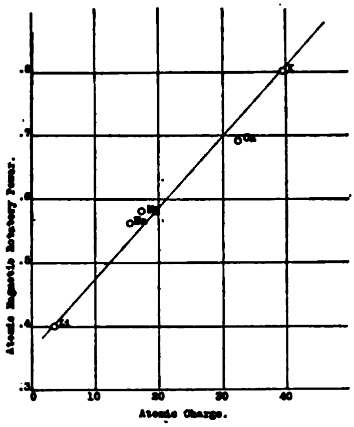

Fig. 9.

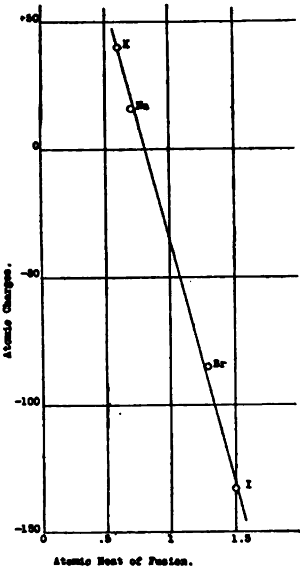

Fig. 10.

TABLE XI.

Element.	Heat of Fusion.	Atomic Charge.
K......................	.6	+39.6
Na.....................	.7	+15.75
Br.....................	1.3	−84.9
I......................	1.5	−132.5

SUMMARY.

In this paper and in preceding papers[1] I have undertaken to show that if the mobilities of ions as calculated from electrolysis be multiplied by their respective ionic masses numbers are obtained which are closely related to many of the physical and chemical properties of the ions. On the assumption that the ions in electrolysis move only short distances as free ions, and that in a water solution of given concentration the ions of different solutes are free for the same average time between dissociation and recombination, these numbers become proportional to the electric charges of the ions, and I have accordingly spoken of them as the characteristic ionic charges. Relative ionic charges which agree among themselves in the same way as those calculated from electrolysis may be calculated from the ratio e/m determined from positive ions given off by hot bodies.

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[1] The following papers dealing with this question have been published: A Physical Theory of Electrification, Leland Stanford Junior Publications, May 15, 1911; On Positive Atomic Charges, PHYSICAL REVIEW, XXXII., 512, May, 1911; Atomic Charges and Cohesion, PHYSICAL REVIEW, XXXII., 518, May, 1911; Pressure-Shift of Spectral Lines, Astrophysical Journal, XXXV., 1, Jan., 1912; On Characteristic Atomic Charges and Resultant Molecular Charges, PHYSICAL REVIEW, XXXV., 276, Oct., 1912; On Convergence Frequency in Spectral Series, Astrophysical Journal, XXXVI., 255, Oct., 1912; On the Electrical Nature of Cohesion, PHYSICAL REVIEW, 2d Series, I., 211, March, 1913.

These ionic charges have been shown to be related to all the properties of bodies which depend upon cohesion or affinity. They have been, for considerable groups of elements, related by definite laws to cohesion constants however calculated, to melting points, boiling points, critical points, expansion coefficients, solubilities, specific gravity, hardness, compressibility, heats of formation and heats of fusion, both in elementary substances and compounds. They have also been shown to be closely related within a given group to the pressure-shift of spectral lines, and the convergence frequencies of spectral series have been calculated from them.

In the present paper their relation to specific inductive capacity, refractive power and magnetic rotatory power have been shown in some cases, while a number of other properties which have previously been related to them have been shown to be related also to specific inductive capacity and to refractive power.

There can be no doubt that these atomic charges are actual physical quantities which are widely related to the physical and chemical properties of the atoms. Their relation to specific inductive capacity in the non-metallic elements is such that a high positive charge is associated with a high specific inductive capacity and *vice versa*, and the indications are very strong that a similar law holds in the case of the metals. This would seem to indicate that the electric elasticity of the ether is greater around negative electrons than around or within positive sub-atoms of the elements, and that in metals as in non-metals it is greater around some kinds of positive atoms than around others.

STANFORD UNIVERSITY,
February, 1913.

A STUDY OF THE REVERSIBLE PENDULUM.

Part III.[1] A Critique of Captain Kater's Paper of 1818.

By John C. Shedd and James A. Birchby.

CAPTAIN KATER's problem was the determination of a standard of length. To this end he proposed to use the dimensions of some form of physical pendulum vibrating seconds in the latitude of London. In speaking of his investigations he says, "I imagined that the least objectionable mode of proceeding would be to employ a rod drawn as a wire, in which, . . . the center of oscillation, as is well known, would be very nearly at the distance of two thirds of the length of the rod from the point of suspension; and I proposed by inverting the rod, and taking a mean of the results in each position to obviate any error which might arise from a want of uniformity in density or figure." This plan, he goes on to say, proved disappointing by reason of the impossibility of obtaining a rod sufficiently uniform. Abandoning his original plan he cast about for some other property of the pendulum that might lend itself to his purpose and he soon hit upon another form of interchangeability; he says: "It is known that the centers of suspension and oscillation are reciprocal."

This property was first pointed out by Huygens in 1673 but appears to have been first used experimentally by Kater in 1817.

Having recurred to this theorem of Huygens, Kater next noticed that the distance between these points depended upon the figure of the pendulum that "if the arrangement of its particles be changed, the place of the center of oscillation will also suffer a change."

His next suggestion is to have two fixed knife-edges and a movable mass the adjustment of which is to bring the coincidence of the knife-edges with the centers of suspension and oscillation respectively. Having so done the distance between the knife-edges becomes the length of the equivalent simple pendulum and the problem is solved.

In making his pendulum Kater had three adjustable masses the larger of which was supposed to remain in position throughout the work. A near adjustment was made by means of the second mass and the final adjustment secured by means of the smallest one, which he calls "the slider."

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[1] Presented at the Cleveland meeting of the Physical Society, December 30, 1912.

Kater had evidently carefully considered the position of the various masses for he says in italics: "*Whatever alteration may be made in the ARRANGEMENT of the weights, the effect on the vibrations (except in one particular instance) will be the same in both positions of the pendulum, always increasing or diminishing their number in both cases, though in different degrees: and the vibrations will be least affected by such change when the great weight is below, and will consequently be the nearest the truth in this position.*" From this he concludes that the adjustment secured by the first movable weight must be such that "the number of vibrations . . . must be left *in defect*" for "there is a point in the pendulum where the effect of the slider (the smaller of the movable weights) in increasing the number of vibrations is a maximum: and it appears from Dr. Young's investigations that this point in one position of the pendulum is different from that in the other. *Very near* either of these points, the pendulum being in its corresponding position, the motion of the slider produces scarcely any change in the number of vibrations; but the slider being then more distant from the point of maximum belonging to the other position of the pendulum, the corresponding increase of the number of vibrations arising from such motion of the slider, will, in that position be very perceptible.

"In the present instance, the point of maximum in either position of the pendulum, is about 4/10 inch below the middle, and consequently the distance of the two points from each other is about 8/10 inch."

The above may be easily understood by referring to curve C, Fig. 10. By "number of vibrations" Kater means number of vibrations done in 24 hours. This number will vary inversely with t, the period of the pendulum. The statement that a movement of the slider affects both periods in the same sense is equivalent to saying that the slope dt/dx of both curves is (in general) of the same sign for a given value of x.[1] By "in different degrees" is evidently meant that the values of dt/dx are different in the two cases.

The exception noted in the parenthesis is elaborated in the next following paragraph. By point of maximum effect on the vibration rate is meant the maximum point on the curve between d and n. Since t and n vary reciprocally this would be the minimum point on the d–t curve. Now the minimum points of the two curves do not coincide (save under one condition) and it is plain that between the minimum points the slopes of the two curves will have opposite signs. Also since the minimum point falls below the "*g*" line (for Kater adjustment) it is clear that if the slider be adjusted so as to give points above the "*g*" line then one will be outside the minimum points and hence the slopes will here have the same sign. Kater's precaution "that the number of vibrations be left in defect" amounts to saying that the period shall be in excess, *i. e.*, the point on the curve shall be above the "*g*" line and hence the slopes (dt/dx) be of the same sign.

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[1] To avoid confusion x is here put for d.

The balance of the italicized statement is also easily understood. It implies that the slopes of the two curves are unequal and also that the period on the curve having the smaller slope is nearer the correct or "*g*" period (always bearing in mind that points above the *g*-line are intended). A glance at any pair of curves in Fig. 4 shows this at once.

From the data given in Kater's paper it is possible to determine with close approximation the constants of his pendulum as he used it and to construct the working equations. The data are not in very convenient form but the following values were obtained:

Summary of Dimensions of Kater's Pendulum.—Pendulum bar, of brass, 1/8 by 1½ by 50 inches, mass 3.30 lbs. On this bar were three masses or "weights," all of brass.

First weight: Cylinder 3½ in. diam., 1½ in. thick, mass 2 lbs. 7 oz. (2.437 lbs.). This weight was placed 8.9 cm. outside of one of the knife edges (K_1).

Second weight: Rectangular in shape, mass 7½ oz. (.47 lb.). This weight was finally placed at 81.56 cm. from K_1 toward K_2.

Third weight: Mass 4 oz. (.25 lb.). This weight constituted the "slider" and was moved over a small portion of the bar midway between the knife edges, this part of the bar being graduated in divisions of 1/20 in. (.05 in.).

The mass of the whole pendulum including the knife edges but exclusive of the third weight was very approximately 9.45 lbs. The mass of the movable weight being 0.25 lb. the value of R becomes

$$R = \frac{.25}{9.45} = 0.0264.$$

The distance between knife edges is the value of L and was

$$L = 39.441'' = 100.181 \text{ cm.}$$

The other constants are b and $(\alpha^2 + R\beta^2)$. In getting these, Kater's experimental data may be used but the results are in the nature of the case approximate, some shifting being resorted to to secure results consistent with Kater's final experimental values.

First it seems fairly close to conclude that the movable weight was in final adjustment at 1 in. from the mid-way point. Using this and other data the value of b was found to be 36.5+ and the value 36.56 was settled upon. The corresponding value of $(\alpha^2 + R\beta^2)$ is 2,392± and the value used was 2,391.98. The constants thus become:

$$\begin{aligned} L &= 100.181, \\ R &= 0.0264, \\ b &= 36.56, \\ (\alpha^2 + R\beta^2) &= 2{,}391.98. \end{aligned}$$

The resulting working equations are

$$t_1 = 0.0100613\frac{d_1^2 + 141{,}235}{d_1 + 1{,}384.85},$$

$$t_2 = 0.0100613\frac{(100.181 - d_2)^2 + 243{,}920.3}{2{,}510.23 - d_2}.$$

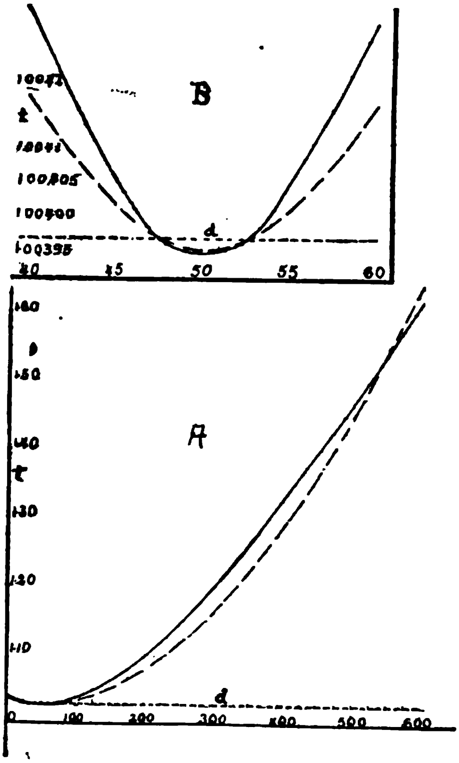

Fig. 17.

These equations are platted in Fig. 17, the lower curve showing all three intersections, while the upper curve gives the "Kater" intersections much magnified.

The following special points on the curves are of interest:

Coördinates of the three intersection points,

$$\begin{aligned} d_1 &= 47.45 \text{ cm.}, & t_1 &= 1.00396 \text{ sec.}, \\ d_2 &= 52.63 \text{ cm.}, & t_2 &= 1.00396 \text{ sec.}, \\ d_3 &= 562.5 \text{ cm.}, & t_3 &= 1.54038 \text{ sec.} \end{aligned}$$

Coördinates of minimum points,

$$\begin{aligned} \delta_1 &= 50.0875 \text{ cm.}, & \tau_1 &= 1.00393 \text{ sec.}, \\ \delta_2 &= 50.1136 \text{ cm.}, & \tau_2 &= 1.00373 \text{ sec.}, \\ \delta_1 - \delta_2 &= 0.1136 \text{ cm. or } 1/22.4 \text{ inches.} \end{aligned}$$

In a passage already quoted Kater intimates that this distance is 8/10 in. but this value is inconsistent with his other recorded data; it must therefore have referred to the pendulum before the second weight had been finally adjusted.

The angles which the curves make with the "g" line at the I. and II. intersection points are:

$$\begin{aligned} &\text{First curve,} & \varphi_1 &= 0^\circ\, 0'\, 2''.965 & \varphi_2 &= 0^\circ\, 0'\, 2''.86 \\ &\text{Second curve,} & \varphi_1' &= \underline{0^\circ\, 0'\, 2''.361} & \varphi_2' &= \underline{0^\circ\, 0'\, 2''.26} \end{aligned}$$

The angle between the

curves, *i. e.*, $\varphi_1 - \varphi_1' = 0^\circ\, 0'\, 0''.604$, $\varphi_2 - \varphi_2' = 0^\circ\, 0'\, 0''.60$.

It will at once be seen that these curves are practically tangent and that Kater was working under conditions approximating to curve E, Fig. 10, *i. e.*, he had his pendulum very near to the limiting condition for a real intersection of the curves. As a matter of fact he was working for the first intersection (where $d < \frac{1}{2}L$) and there is nothing in his paper that directly shows whether he knew of the possibility of the adjustment of the pendulum corresponding to the second intersection to say nothing of the third. The reference to Dr. Young might imply that Young could possibly have enlightened him on both these points.

It is thus clear that Captain Kater was using the pendulum under the least favorable conditions. It also seems likely that he merely happened upon this adjustment. (This is more complimentary to him than to think that he deliberately made it by choice.) A careful examination of the problem shows that a slight change in the distribution of the masses would have more nearly given curve C, Fig. 10 and thereby have greatly improved the conditions of the experiment.

For example had the first movable weight been placed on the opposite side of the first knife edge and at the same distance from it, the improvement would have followed. The constants of the pendulum now become L and R as before. $b = 41.1$, $(\alpha^2 + R\beta^2) = 2{,}410$. The working equations are,

$$t_1 = 0.0100613 \frac{d_1^2 + 155{,}273}{d_1 + 1{,}556.8},$$

$$t_2 = 0.0100613 \frac{(100.181 - d_2)^2 + 223{,}591}{2{,}338.07 - d_2}.$$

The coördinates of the intersection points now become

$$d_1 = -6.45 \text{ cm.}, \quad t_1 = 1.00396 \text{ sec.},$$
$$d_2 = 106.95 \text{ cm.}, \quad t_2 = 1.00396 \text{ sec.},$$
$$d_3 = 385 \text{ cm.}, \quad t_3 = 2.092 \text{ sec.}$$

The coördinates of the minimum points are

$$\delta_1 = 29.58 \text{ cm.}, \quad \tau_1 = 0.77153 \text{ sec.},$$
$$\delta_2 = 53.78 \text{ cm.}, \quad \tau_2 = 0.9668 \text{ sec.}$$

The angles which the curves make with the "g" line at the I. and II. intersection points are

First curve,	$\varphi_1 = 0° \, 1' \, 15''.4$	$\varphi_2 = 0° \, 1' \, 10''.6$
Second curve,	$\varphi_1' = 0° \, 0' \, 54''.9$	$\varphi_2' = 0° \, 0' \, 52''.1$
	$\varphi_1 - \varphi_1' = 10''.5$	$\varphi_2 - \varphi_2' = 8''.47$

These values of $\varphi - \varphi'$ while not large are over seventeen times as large as Kater's and constitute a great improvement over his conditions of working.

Fig. 18 shows in diagram the position of the various masses as Kater placed them (I.) and also (II.) the new position here suggested for the large weight.

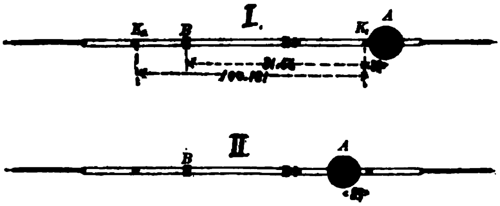

Fig. 18.

To secure a still larger value for $\varphi - \varphi'$ and thereby better working conditions for using the pendulum, would require increasing the value of R; but this would be to change the pendulum beyond what could rightly have been expected of Captain Kater and the pendulum would cease to be the one with which he worked. This therefore is beyond the scope of the present paper.

In the next paper it is proposed to inquire into the best values to be given to the various constants of the various types of reversible pendulum.

December 30, 1912.

ON A METHOD OF COMPARING INDUCTANCE AND CAPACITY.

By W. E. Forsythe.

IN comparing inductance and capacity by the ordinary bridge methods, it is often difficult to know just what resistances to choose. This is due to the fact that there are generally two conditions of balance, the first, the steady current balance, and the second, the one due to the reactance; the second condition, in general, depending upon the first. It often happens that with the capacity and inductance to be compared, the first balance will be so chosen that the second balance cannot be obtained. In the method outlined below this difficulty has been overcome, due to the fact that the two conditions of balance are independent.

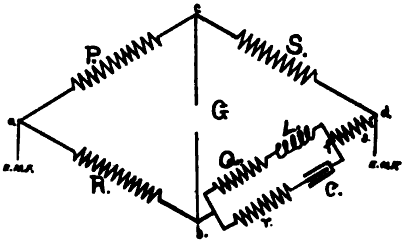

Fig. 1.

The arrangement of the circuit is given in Fig. 1, where P, R, S, Q, r, s are non-inductive resistances, L the inductance being compared with the capacity C. To obtain the conditions for balance, make P equal R, then the impedance of the arm b–d must equal that of c–d. Let Z equal the impedance of the part of the arm b–d made up of inductance and capacity, that is, b–f. We then have

$$\frac{1}{Z} = \frac{1}{Q - jX} + \frac{1}{r + jK}, \tag{1}$$

where $j = \sqrt{-1}$, $X = wL$, $K = 1/wC$, $w = 2\pi n$, n being the frequency of the alterations. As $P = R$, $(Z + s) = S$, or

$$\frac{1}{Z} = \frac{1}{S - s} \tag{2}.$$

From (1) and (2)

$$\frac{1}{Q - jX} + \frac{1}{r + jK} = \frac{1}{S - s}.$$

Or clearing

$$(S - s)(r + jK) + (S - s)(Q - jX) = (Q - jK)(r + jX). \tag{3}$$

Equating reals,

$$(S - s)r + (S - s)Q = Qr + XK. \tag{4}$$

From which

$$XK = (S - s)Q + (S - s)r - Qr. \tag{5}$$

From imaginaries

$$(S - s)K - (S - s)X = QK - rX,$$

or

$$K[(S - s) - Q] = X[(S - s) - r]. \tag{6}$$

Now it is known that for the bridge to balance, the steady current condition must hold, that is, the resistance of the arm *c–d* must equal that of *b–d*, that is,

$$S = s + Q. \tag{7}$$

From (6) and (7)

$$(S - s) = r. \tag{8}$$

From (5) and (8)

$$XK = r^2,$$

or

$$L = Cr^2.$$

Thus there are three conditions of balance, as follows:

$$S = s + Q, \tag{I.}$$

$$S = s + r, \tag{II.}$$

$$L = r^2C. \tag{III.}$$

From (I.) and (II.) it is seen that Q must equal r. Thus to balance the bridge, set $Q = r$ and then with direct current on the bridge, obtain the steady current balance, $Q + s = S$. Then with the alternating E.M.F applied to the bridge, obtain the inductive balance, with either an A.C. galvanometer or a telephone. This last balance is obtained by shifting from Q to s, keeping at the same time $r = Q$. Thus, while the bridge has three conditions of balance, the two of these due to the reactance, can be set at the same time without disturbing the steady current balance. Conditions (I.) and (II.) above might be determined directly from the bridge by noting that for the bridge to be independent of the frequency, it must hold for $w = 0$ and also for $w = \infty$. For $w = 0$ we have the condition for steady current balance, as for the steady current no current will pass through the arm containing the condenser. For $w = \infty$ no current will pass through the inductance and the capacity will have no impedance which will make $S = s + r$.

Among the advantages of the above method, the following may be mentioned:

1. The conditions of balance are practically independent.
2. It allows a rather large range of inductance to be compared with the same capacity, without any change in the steady current balance.
3. Change of resistance to bring about the inductive balance does not materially affect the amount of current flowing through the inductance; thus the inductance can be calibrated for its entire range on the same current.
4. As the resistances in the four arms of the bridge are almost equal, the effect of the distributed capacity of the resistance boxes will be reduced to a minimum, due to the fact that the distributed capacity in each arm of the bridge will be practically the same, thus its effect will be very small. The same is true for the inductance due to the resistance coils.

The above method has been used for the past year in this laboratory and has given good satisfaction for comparing small capacities of the order of a few thousandths of a microfarad with the laboratory standard of inductance.

Department of Physics,
University of Wisconsin,
March, 1913.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SIXTY-SEVENTH MEETING.

THE regular Spring Meeting of the Physical Society was held in Washington, D. C., on Friday afternoon, April 25, and Saturday, April 26, 1913. The Friday session was held at the Cosmos Club, President B. O. Peirce presiding. The Saturday sessions were held at the Bureau of Standards, Dr. S. W. Stratton presiding at the morning session and President B. O. Peirce at the afternoon session. At the Saturday morning session the chairman introduced to the Society Professor Willy Wien, who spoke informally on questions connected with radiation.

The following papers were presented:

The Production of Temperature Uniformity and Constancy in an Electric Furnace. A. W. GRAY.

The Determination of Thermal Expansivity at High Temperatures. A. W. GRAY.

A New Thermo Element for the Measurement of High Temperatures. C. C. BIDWELL.

Note on the Value of Throttling Experiments as a Basis for Computing Steam Tables. HARVEY N. DAVIS.

Preliminary Report on Throttling Experiments with Superheated Steam. HOWARD M. TRUEBLOOD.

Further Thermoelectric Methods in Calorimetry. WALTER P. WHITE.

A Redetermination of the Specific Heats of Platinum and Quartz. WALTER P. WHITE.

A Specific Heat Calorimeter for Small Temperature Intervals. D. R. HARPER.

A Precision Combustion Calorimeter. H. C. DICKINSON.

Note on a Calorimeter designed to Measure the Change in the Specific Heat of a Gas with Temperature. C. C. MURDOCK.

The Heat of Fusion of Ice. H. C. DICKINSON, D. R. HARPER, and N. S. OSBORN.

Improvements in the Resistance Thermometer Bridge. E. F. MUELLER.

The Comparison of Resistance Thermometers. E. F. MUELLER.

Calorimetric Cooling Corrections and the Richards "Adiabatic" Method. H. C. DICKINSON.

On the Construction of Calorimetric Thermoelements. Walter P. White.

A New Type of Temperature Regulator for Electric Furnaces. L. H. Adams.

Some Measurements with the Micropyrometer. G. K. Burgess and R. C. Waltenberg.

An Attempt to Measure in Radiation Units the Intensity of the Source Producing Photo-electric Effects for Various Wave-lengths. P. B. Perkins.

On the Photoelectric Potentials and Currents from Thin Cathode Films. K. T. Compton and Otto Stuhlmann.

The Theory of Metallic Conduction. (By title.) O. W. Richardson.

Observations of Sea Water Temperatures in the Vicinity of Icebergs and in other parts of the Ocean. C. W. Waidner, H. C. Dickinson, and J. J. Crowe.

On the Continued Appearance of Gases in Vacuum Tubes. George Winchester and Dorothy Dunlap.

On Magnetic Rays. Louis T. More and S. J. Mauchly.

The Electric Spark. Spectrum of the Spark. William P. Sawtelle.

Line Spectrum from Uncharged Molecules. C. D. Child.

The Spectrum of Mercury in the Schuman Region. Theodore Lyman.

The Spectrum of Magnesium Vapor. James Barnes.

The Spectrum of Active Nitrogen. (By title.) E. P. Lewis.

A Striking Instance of the Slow Transfer of Heat through a Thin Layer of a Rarefied Gas. E. L. Nichols.

14. On the Maximum Value of the Magnetization Vector in Iron. B. Osgood Peirce.

A New Method of Comparing Mutual Inductances. F. B. Silsbee.

A Vibration Electrometer. Harvey L. Curtis.

Ionization in Unstriated Discharge. C. D. Child.

Some Characteristics of Total Radiation Pyrometers. G. K. Burgess and P. D. Foote.

Final Results with the Silver Voltameter. E. B. Rosa, G. W. Vinal, and A. S. McDaniel.

A Modification of the Carey Foster Method of Comparing Resistances. Frank Wenner.

On the Radiation from Molten and from Solid Iron. C. C. Bidwell.

Preliminary Report of Methods for Measuring Electrolytic Resistance. F. Wenner and E. C. McKelvey.

Note on the Construction of Thermopiles. W. W. Coblentz.

Ernest Merritt, *Acting Secretary.*

A Modification of the Carey Foster Method of Comparing Resistances.[1]

By Frank Wenner.

The method was developed for use in the measurement of the resistances of a number of similar conductors connected in series, such for example as the coils of a low resistance potentiometer. In such measurements it is not possible to put a slide wire between the resistance being measured and the standard, so it is necessary to provide some other means of making and reading the small changes necessary to balance the bridge. This is accomplished by using a variable ratio coil set of the direct reading type instead of fixed ratio coils such as are regularly used in Carey Foster bridges. The ratio set used has a range of 1 per cent., and while designed for other work has been found to be very satisfactory for this purpose.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

The method will be understood by reference to the figure, in which X represents the conductor whose resistance is to be compared with that of the standard S, L is a connecting link interchangeable with S, x_1 and x_2 are conductors permanently connected to X, m and n, connecting leads, A and B the two sides of the ratio set, and g and g' terminals of the conductor X. In general the same characters are used to represent the value of the resistances. We shall also let the ratio of A to B be represented by $1 + r + c$, where r is the reading and c the correction to the reading of the ratio set.

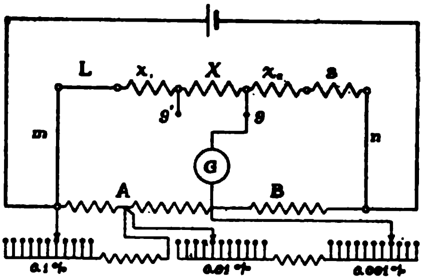

Fig. 1.

The bridge is balanced first with the arrangement shown in the figure then with the link and standard interchanged and the galvanometer connected at g'. From the two relations it follows that

$$2(X - S + L) = (r + c)(x_2 + S + n) - (r' + c)(X + x_2 + L + n) \quad (1)$$

where r' is the second reading of the ratio set. If the correction c is the same for both readings and $r + c$, $r' + c$, and L are all small in comparison with X and S then $x_2 + S + n$ is very nearly equal to $X + x_2 + L + n$ and so approximately

$$X = S - L + \frac{x_2 + S + n}{2}(r - r'). \quad (2)$$

In the case where a number of similar conductors are being compared x_1 and x_2 are of the same nominal value as X and S while m and n are small, in comparison, so equation (2) may be put in the form

$$X = N(1 + s - l + r - r') \quad (3)$$

where N is the nominal value of the resistances X and S, s the excess in proportional parts of the resistance of the standard above its nominal value, l the ratio of the resistance of the link to the resistance of the standard, and r and r' the two readings of the ratio set.

A Specific Heat Calorimeter for Small Temperature Intervals.[1]

By D. R. Harper, 3d.

THE calorimeter consists of a copper cup with thick walls into which are built an electric heating coil and a platinum resistance thermometer. A given quantity of electrical energy supplied to the cup and its contents is accurately measured by the aid of a potentiometer installation and a chronograph. The temperature change of the same system is determined with the thermometer.

Metallic specimens are machined to fit snugly into the hollow cylindrical interior of the cup; liquid specimens are poured directly into the cup and the equalization of the temperature is accelerated by immersing a suitable "cage" of radial sheet copper fins.

The calorimeter is well adapted to operation by the method due to T. W. Richards, where the outer jacket temperature is altered at the same rate as the calorimeter temperature. When so used, the time elapsing before the final equilibrium condition is attained may be quite long without serious disadvantage, so that the usefulness of the instrument is not confined to the better heat conductors.

The calorimeter is very similar to one devised and used by Nernst, although its inception was entirely independent and the instrument was constructed and in use before the publication of any description of Nernst's apparatus. Of the metals, copper is the only one upon which any work has been done to date, and the results of this are not ready for publication. Of other materials, some light and heavy coal tars, and some creosote oils have been measured. The results are of no general interest, but the work served very well indeed to test the usefulness of the calorimeter.

The operation of the first instrument is so successful that a second, embodying the same principles, is under construction for use in determining the latent heats of ammonia, carbon dioxide, and other liquids at various pressures, and the specific heat of these at a number of temperatures. For holding the liquids under the requisite pressures, the copper cup is provided with a lining of steel as thin as safety will permit, fitted with the necessary inlet tubes and valves.

Bureau of Standards,
Washington, D. C.

The Spectrum of Active Nitrogen.[1]

By E. P. Lewis.

THE writer in 1899 discovered a new form of afterglow in nitrogen produced by a condenser discharge with spark gap when the gas was freed from oxygen as far as possible. This afterglow had a characteristic spectrum containing a number of new bands. Further details were published in 1904,[2] and attention called to the remarkable fact that the vapors of mercury from the pump and from the aluminum, zinc and platinum electrodes participated in the afterglow. Strutt investigated the phenomenon on a larger scale, and gave to the glowing gas the name "active nitrogen," and Strutt and Fowler published further details on the spectrum.[1]

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[2] Ann. der Phys., 2, 249, 1900; Phys. Rev., 18, 125, 1904; Astrophys. Jl., 12, 8, 1900; 20, 49, 1904.

The writer was at first inclined to believe that small traces of oxygen were responsible for the effect, although larger quantities extinguished it; for in the earlier experiments the third group of bands, attributed by Deslandres to nitric oxide, were always present. Strutt and Fowler believed the afterglow to be an attribute of pure nitrogen, Comte favors the former view,[2] while König and Elöd[3] hold the same opinion as Strutt and Fowler.

In some recent experiments the writer failed to obtain the characteristic ultra-violet bands of the third group and those called β bands by Strutt and Fowler. The nitrogen had been for a long time in storage in the drying system. It was found that the addition of a small quantity of oxygen would cause these bands to reappear in the afterglow spectrum. They also appeared when the nitrogen was freshly prepared from sodium nitrite and ammonium sulphate, but disappeared after the gas had been for several days in contact with pyrogallic acid and the drying materials. It appears that nitric oxide is present in the nitrogen prepared in this manner, and that it is gradually absorbed. When this has happened, the third group and β bands disappear from the afterglow spectrum, and the only remaining features of the ultra-violet are the cyanogen bands, which appear only when no oxygen is present, and mercury lines. It is evident, therefore, that these bands, attributed by Strutt and Fowler to pure nitrogen, are caused by the presence of oxygen or nitric oxide. The same results were obtained with nitrogen prepared from air by passing over phosphorus.

It appears that the bands in the green, yellow and red, with heads at 5054, 5442, 5855 and 6323, which are a part of Deslandres' first group, are the only invariably characteristic features of the afterglow spectrum, all other bands so far described being due to impurities.

It seems, therefore, that Strutt and Fowler's nitrogen was contaminated with oxygen, and that their observations do not prove that the afterglow appears in perfectly pure nitrogen. This may be the case, for undoubtedly the afterglow increases in brightness as the amount of oxygen diminishes, but an amount of oxygen too small to be detected spectroscopically might possibly give rise to such an effect. It seems, therefore, still an open question whether or not "active" nitrogen is pure nitrogen.

UNIVERSITY OF CALIFORNIA,
March 10, 1913.

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[1] Proc. Roy. Soc., 85, 219, 1911; 85, 777, 1911.

[2] Phys. Zeit., 14, 74, 1913.

[3] Phys. Zeit., 14, 165, 1913.

On the Construction of Calorimetric Thermoelements.[1]

By Walter P. White.

1. We have been able to obtain, from the Electrical Alloy Company, of Morristown, New Jersey, constantan wire homogeneous enough to make, without the labor of selecting and matching special portions, thermoelements of the highest precision ordinarily needed (*i. e.*, to 1/20,000) and it is probable that wire of this quality can be regularly obtained. With this wire it is only necessary to make a rapid examination, and reject the few irregular spots that occasionally occur.

2. The insulation of the junctions of a multiple element can be secured with the greatest ease and certainty merely by distributing them through a length of from one to four centimeters, according to their number. This method has no appreciable drawbacks. In general it requires that the bare junction shall not project far from the uninjured silk, but the projection can easily be kept below a millimeter if the soldering is done by dipping in a bath of solder, and if, before the wire is cut, the silk is cemented in place by shellac.

3. Inclosure in glass tubes is quick, easy, adaptable and excellent thermally. For small elements metal tubes are nearly as satisfactory in these respects, and the thermoelement, once finished, is almost indestructible.

The final result of these methods is that thermoelements of the highest sensitiveness and precision can be constructed in a few hours, without special skill or experience.

Geophysical Laboratory.

Further Thermoelectric Methods in Calorimetry.[1]

By Walter P. White.

FOR some years calorimetric determinations of great delicacy have been made by means of twin calorimeters, between which runs a very sensitive multiple thermoelement.

1. In this and similar two-body methods the smallness of the electrical quantity to be measured gives to the temperature measurement extraordinary precision combined with extraordinary ease. These methods, accordingly, while specially advantageous for precision in determining very small heat quantities, are for nearly all cases comparable with the best in convenience and certainty.

2. If the two calorimeters are alike, the external temperature and its changes affect both alike, and hence have but a slight effect upon the thermoelement reading which measures the difference in the temperatures of the two. This has sometimes been counted another great advantage of the method. A complete jacket, however, is much more effective in this direction.

3. On the other hand, there is a cooling correction error peculiar to the twin method. It arises whenever the two calorimeters are not exactly alike,

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

and can easily be shown to be $(Ka - Kb)(\theta b - \theta j)$ where Ka and Kb are the cooling rates of the two calorimeters, θb and θj the temperatures of the comparison calorimeter and of the jacket, respectively.

4. This error is nearly eliminated if three observation periods are run (as in the usual Pfaundler Method with one calorimeter), but it can be more easily avoided by making the temperature difference, $\theta b - \theta j$, small, say, below 0.2°. Only two observation periods are then needed.

5. The trouble of the temperature adjustment is largely or wholly avoided by either of two schemes, both of which utilize the low cooling rate of a vacuum-jacketed flask, which is used for the comparison calorimeter. In one scheme, a regulator keeps the jacket temperature constant. The flask is then constant to a very high degree of precision (easily to .0001°) without any further attention whatever. This has been called the constant comparison body method.

6. The other scheme avoids the regulator. A specially adjusted (shunted) thermoelement is used in determining the cooling correction. The two unlike calorimeters can then be operated as easily and conveniently as if they were twins, but the temperature adjustment is so much less exacting as to be almost negligible. This has been called the compensated calorimeter method.

7. A combination of the two methods can be made at will, either temporarily or permanently, involves no difficulties, and reduces precautions and corrections to a very low minimum.

8. The gain in simplicity and ease of construction by using vacuum-jacketed bottles is evident.

9. The best of these methods call for a complete inclosure by the jacket, but this is almost indispensable in any calorimetry of precision, and can be easily obtained. For instance, the calorimeter chamber may be a cylindrical pot, suspended in a commercial paper tub. The cover is a square box, filled with water, and closed save for two pipes, which project downward through the surface of the water below, and so provide for a complete circulation.

10. By a suitable and rather evident arrangement of thermoelements, any of these schemes can be operated with the two calorimeters in two separate jackets. This procedure is necessary in the case of adiabatic methods.

11. The great advantage of these methods is in the ease and precision of the temperature measurement. Otherwise, they present disadvantages, which, however, are evidently slight, and far outweighed by the advantages.

GEOPHYSICAL LABORATORY.

ON THE PHOTOELECTRIC POTENTIALS AND CURRENTS FROM THIN CATHODE FILMS.[1]

BY K. T. COMPTON AND OTTO STUHLMANN, JR.

RECENT results obtained by Dike[2] and by Dike and Brown[3] for the velocities of photoelectrons from thin platinum cathode films show values which are too large to be consistent with Einstein's or Richardson's theories of the photoelectric effect. We have repeated Dike's experiments with the purpose of investigating whether these unusually high velocities are real or only apparent velocities. Dike's apparatus as shown in Fig. 2 of his paper was duplicated with the addition of a small movable arm for testing contact differences of potential. This arm could be turned back so that it rested on the bottom of the chamber when the photoelectric measurements were in progress.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[2] PHYS. REV., Vol. 34, p. 459, 1912.

[3] PHYS. REV., Series 2, Vol. 1, p. 254, 1913.

1. Films of platinum were sputtered on quartz in a separate vessel and were then introduced into the apparatus where velocities, saturation currents and contact differences of potential were measured. These films showed properties identical with those of ordinary polished platinum. The velocities, corrected for contact differences of potential, were equivalent to from 1.4 to 1.8 volts.

2. Films were sputtered on quartz in the test chamber and the above measurements were repeated. In this case the films did not come into contact with the air. The highest velocities obtained under these conditions were equivalent to 2.5 volts, which, when corrected for contact difference of potential, gave 5.5 volts. The current curves were similar to those obtained by other investigators.

3. Films sputtered on a polished platinum plate and investigated in the chamber in which they were sputtered gave maximum potentials, corrected for contact difference of potential, of 3.6 volts. The saturation current in this case increased logarithmically with the thickness, approaching a constant value much above that obtained from unsputtered platinum.

4. The above deposition of cathode films on polished platinum was repeated after a small quantity of soft wax had been placed on the cathode. The high velocities obtained by Dike were now obtained. Also the contact differences of potential ran practically parallel with the changes in the photoelectric potential. The maximum velocities thus obtained corresponded to 59 volts, with an accompanying contact difference of potential of 57 volts. The saturation current was not quite as large as in the preceding cases where no wax was present.

It is evident that the high photoelectric velocities are due to the presence of soft wax or grease in the apparatus. The fact that the apparent contact difference of potential between the emitting and receiving electrodes increases along with the photoelectric velocities shows that these high velocities are not true photoelectric velocities but are caused by the electrostatic attractions and repulsions of a charged insulating layer on the emitting electrode. Even the comparatively high velocities obtained when no wax was intentionally present in the chamber were doubtless due to small quantities of oil or grease unavoidably present in the metal apparatus or the ground glass joints.

There is therefore no evidence that thin metal films emit electrons with velocities different from those of electrons emitted from ordinary metals or that they form exceptions to Einstein's and Richardson's formula

$$(\tfrac{1}{2}mv^2)_{\max} = eV = h\nu - w_0,$$

which holds for ordinary metals.

PRINCETON UNIVERSITY,
UNIVERSITY OF PENNSYLVANIA.

AN ATTEMPT TO MEASURE IN RADIATION UNITS THE INTENSITY OF THE SOURCE PRODUCING PHOTO-ELECTRIC EFFECTS FOR VARIOUS WAVE-LENGTHS.[1]

BY P. B. PERKINS.

RECENT investigations seem to have established the independence of the intensity of the light and velocity of the photo-electrons produced. In opposition to this view is the work of Griffith[2] who, working with ultra-violet light falling on a negatively charged zinc plate, concluded that the ratio of current to light intensity is not constant, but increases with increasing intensity.

Lenard[3] has shown that the velocity of the electrons also depends on the quality of the light used as source.

Millikan[4] thinks the apparent difference in velocity of the electrons due to spark and arc sources, is due to the effect of long electro-magnetic waves sent out by the spark being superimposed on the ordinary photo-electric effect.

In any study of the relative photo-electric currents produced when different substances are used for electrode, it is necessary to keep the intensity of the source constant over comparatively long intervals of time. In spark sources this condition is especially difficult to meet. It would therefore seem of value to use some method of measuring the intensity of the source of the light and of noting any changes that take place as the investigation proceeds.

The literature of the subject reveals but few attempts to eliminate intensity changes and little or no effort to measure the intensity of the source directly. Thus far the writer has been unable to find any attempt to measure the intensity for various wave-lengths in the ultra-violet either for arc or discharge sources simultaneously with photo-electric effects.

Ladenburg,[5] in order to eliminate changes in intensity, used a null method in which the light fell through tubes sealed at 45° on electrodes of aluminum and copper sealed near the ends of a glass tube. The central conductor to which the electrons from both electrodes were projected was connected to earth. Light of different wave-lengths was obtained by means of mica screens of various thicknesses.

Linehop,[6] working on the photo-electric effects at low temperatures, used a control electrode to which a part of the light from the source was reflected from a quartz plate in the path of the rays. The constancy of the effects produced on this control was investigated from time to time.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[2] Phil. Mag., 6, 14, 1907.

[3] Ann. d. Physik., 8, 1902, p. 149.

[4] Proc. Am. Phys. Soc., November 30, 1912.

[5] Ann. d. Physik., 12, 1903, p. 558.

[6] Ann. d. Physik., 21, 1906, p. 281.

In the earlier work of Richtmeyer[1] on photo-electric effects in a sodium cell, an incandescent light was used and its intensity varied by slips of paper. The relative intensities were judged by the comparative brightness of the sheets of paper; later[2] however an acetylene flame, having a constant intensity, was used for which the energy distribution for different wave-lengths had been previously determined by Nichols and Merritt.

Hughes[3] in obtaining photo-electric currents for compounds of zinc, tin, lead, potassium and sodium used a comparison electrode at the same angle with the arc source as the electrode on which the compounds were placed.

If photo-electric currents for different wave-lengths are to be measured, these must be corrected for differences in energy distribution for these wave-lengths. This presupposes a previous knowledge of the distribution of energy in these short wave-lengths for the source used, an extrapolation by the application of some law of radiation which is known to hold for the longer wave-lengths or a separate investigation of the energy distribution. If the intensity of the light source for these short wave-lengths could be measured in radiation units at the same time as the photo-electric effects produced, any changes in intensity would be apparent and the necessary correction could be made, a comparison of the intensity of arc, spark and other sources could be at once made in radiation units and the work of different experimenters using widely differing sources and intensities could be quantitatively compared.

It is hoped that with a very powerful source of ultra-violet light, a thermopile might be used for this purpose in series with a sensitive galvanometer or other means of measuring the small changes in electromotive force produced by the incident light. This could be compared with the effect due to a standard candle at a given distance from the thermopile. The current could be measured in amperes by obtaining the deflection produced by the current from a standard cell in series with a very large known resistance. Consequently a thermopile[4] and an astatic galvanometer were constructed after the design of Coblentz, of the Bureau of Standards. The thermopile contains twenty silver-bismuth junctions and has a resistance with connections to the key of 12.03 ohms. The galvanometer is surrounded by a series of iron shields and has a resistance of 6.48 ohms. The deflections are read by a telescope and illuminated scale at a distance of two meters. The thermopile is enclosed in a small brass cylinder closed by a brass screw cap at one end and slides on a brass collar fitting into the telescope end of a Hilger monochromatic illuminator with quartz prism. At the center of the collar in the brass base is a slit through which the light passes. When the thermopile is pulled up by a weight over a small pulley, the light passes through the slit to the electrode under investigation; when the weight is released, the thermopile falls of its own weight and the light passes through an extremely thin quartz window, inserted to prevent eddy currents, to the junctions.

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[1] Phys. Rev., July, 1909, p. 71.

[2] Phys. Rev., March, 1910, p. 385.

[3] Proc. Camb. Phil. Soc., 16, 4, 1911, p. 376.

[4] Jour. Franklin Inst., Dec., 1911.

While awaiting the arrival of the illuminator, a large induction coil with eight large condensers across the secondary inclosed in a metal box has been used to obtain the positive potential gained by a platinum electrode enclosed in a bell jar of about half a liter capacity due to light from different spark sources. The carbon arc and a quartz mercury arc have also been used. The inside of the bell jar is lined with tin covered with a thick coating of carbon, obtained by burning camphor, and connected to earth. The quartz window is 2 mm. thick. Amber insulation is used. The leads pass through iron tubes connected to earth. A Dolezalek elcterometer with a scale 2.3 meters distant is used enclosed with all keys in a metal-lined box, earth connected. The glass jar is first pumped out with an oil pump, finally with a Toepler mercury pump. The pressure is read on a McLeod gauge. The system also contains a phosphorus pentoxide drying tube and a bulb containing about eighty grams of cocoanut charcoal. It is hoped with such a large amount of charcoal, by first heating, then cooling with carbon dioxide snow, to obtain sufficiently low vacuum conditions.

A tilted Wilson electroscope is being calibrated for use if it is found that the capacity of the electrometer is too large for measuring the photo-electric effects for the shorter wave-lengths.

An attempt is also being made to construct a very sensitive capillary electrometer with the hope that, should local conditions seriously impair the sensitiveness of the galvanometer necessary for measuring effects due to the shorter wave-lengths, it may be used in place of the galvanometer.

SCIENCE HALL, HOWARD UNIVERSITY,
April 5, 1913.

THE SPECTRUM OF MAGNESIUM VAPOR.[1]

BY JAMES BARNES.

THE study of the charges which are produced in the spectrum of magnesium by different forms of electrical discharges and by varying the pressure and chemical nature of the gas surrounding the electrodes has been the subject for a number of important investigations. To these the author wishes to add a few further observations.

A piece of clean magnesium rod was placed in the center of a long silica tube which was closed with water-cooled caps and quartz plates. The tube was then placed in the core of an electric furnace and was also connected with a Geryk exhaust pump. As the magnesium slowly boiled away and condensed on the sides of the tube protruding from the furnace the vapor was rendered luminous by the discharge from a large induction coil. The spectrum was photographed with a large Rowland concave grating and also at the same time with a Hilger prism spectroscope.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

The negatives show that the radiations λ 2852 and λ 4571 are the strongest lines in the spectrum of magnesium vapor excited in this manner. The strong "spark line" λ 4481 is absent. The remarkable intensity of λ 4571 and the absence of λ 4481 is noteworthy. The three bands with heads at λ 5622, λ 5211, and λ 4845 are strongly depicted on the plates. These bands have been attributed to a magnesium and hydrogen compound and their wave-lengths were found by Fowler[1] to correspond with many lines in the sun-spot spectrum. These results together with an observation made by Adams[2] that the only magnesium line which shows any marked increase in the spot over the sun is λ 4571 suggests that the physical conditions which exist in magnesium vapor rendered luminous as described above are the nearest we have yet approached in the laboratory to the conditions which exist in sun-spots.

Bryn Mawr College,
April, 1913.

PRELIMINARY REPORT ON THROTTLING EXPERIMENTS WITH SUPERHEATED STEAM.[3]

By Howard M. Trueblood.

THIS paper describes an apparatus for the measurement of the specific heat and of the coefficient of the Joule-Thomson effect in steam.

The steam is generated in an 18 in. Stanley automobile boiler, gas heated, the boiler pressure being regulated to within about 0.2 lbs. per sq. in. by means of an electrical device which controls the flow of gas to the fire. From the boiler the steam passes through a lagged pipe to the main superheater, which is composed of six 30-in. lengths of double strength 2 in. wrought-iron pipe joined together by cast steel return bends. In each of these lengths of pipe is arranged a series of sheet copper discs which act as water separators, baffle the steam, and serve also as a means of conducting heat from the walls of the pipe into its interior. This superheater is gas heated. From it the steam passes through a short, heavily lagged pipe, containing a copper-gauze strainer, into a secondary superheater, and thence into and through the porous plug. This secondary superheater and the plug casing are immersed in a rapidly stirred oil bath, electrically heated, and thermostated. The secondary superheater consists of about 40 feet of three-eighth in. copper pipe, and is arranged in a coil surrounding the plug casing. The steam ordinarily enters it at a temperature somewhat higher than is desired on the high side of the plug, and is cooled to temperature by the bath. The heat exchanging capacity of the secondary superheater is great enough so that, even with flows as large as 30 kilograms per hour, the temperature of the steam as it enters the superheater can be varied by 20° or 30° C. without affecting the reading of the high-side thermometer. The surroundings of the plug are therefore always at exactly the same temperature as the high-side steam.

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[1] Phil. Trans. Roy. Soc., 209, A447, 1909.

[2] Astrophys. Journal, 30, 105, 1909.

[3] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

The plug proper consists of a block of soapstone through which a considerable number of small holes are drilled. Through these holes is threaded a manganin heating wire for use in specific heat measurements. The plug is contained in a casing built up of a cast iron box and two pieces of seamless steel tubing. The steam, in its passage through the plug, and for some distance before entering and after leaving the plug, is lagged with soapstone to prevent conduction of heat inward from the bath.

The difference between the temperature of the steam on the high side of the plug and its temperature on the low side is directly determined by measuring the difference of resistance of two platinum thermometers. The drop of pressure experienced by the steam in passing through the plug is directly measured on a mercury manometer. The heat supplied to the steam while passing through the plug is determined by measuring the current supplied to the manganin heating wire and the drop of potential across the wire.

The steam, after leaving the plug, passes into a coil of pipe surrounded by cooling water, and is condensed and weighed.

Some results obtained at a single pressure and temperature as tests of the apparatus are presented.

JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

NOTE ON THE VALUE OF THROTTLING EXPERIMENTS AS A BASIS FOR COMPUTING STEAM TABLES.[1]

BY HARVEY N. DAVIS.

IT is the purpose of this note to show how large a part a satisfactory series of throttling experiment on superheated steam could play in increasing the accuracy of the steam tables used by engineers and others. Of the six sorts of fundamental data needed for computing a set of steam tables, two are already at hand in the literature. Of the remaining four, three could be supplied or improved, wholly or in part, more accurately by good throttling experiments than in any other way.

In the first place, the Joule-Thomson plug experiment, if carried out with a heating coil as suggested by Buckingham, would give a direct determination of c_p as well as of the Joule-Thomson coefficient μ; for a properly arranged series of experiments at each high-side pressure and temperature would enable one to plot a straight line graph whose intercepts on the axes of the figure are proportional to μc_p and to μ respectively, so that the slope of the line is c_p itself.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

Furthermore, a knowledge of μ, or of the product μc_p, over the whole superheated steam region would enable one to spread to higher pressures a single c_p curve, such as that at atmospheric pressure, which had been determined directly by other sorts of experiments. This can be done in three ways proposed originally by Dodge, by Davis, and by Grindley. These three processes are substantially equivalent. Their relative convenience depends on the nature and arrangement of the available Joule-Thomson data. This indirect attack on c_p is most valuable at high temperatures where direct measurements under pressure are difficult.

In the second place, a series of Joule-Thomson experiments, especially if conducted with a heating coil, affords by far the best way now available of determining the specific volume of superheated steam. The great advantage of this line of attack is that the efforts of the experimenter are expended on determining, not the whole quantity sought, but a relatively small correction term.

And in the third place, a series of Joule-Thomson experiments affords the best way now available of determining the total heat of saturated steam, and through this, the latent heat of evaporation and the specific volume of saturated steam. Here again the process involves evaluating, not the whole quantity in question, but an increment term that is never more than 5 per cent. of the whole.

The process just mentioned involves the previous knowledge of c_p near the saturation line, but it is not sensitive to errors in these assumed values of c_p, and curiously enough, one can, by means of Planck's equation, recompute c_p from the total heats thus obtained and come out with values that are much more accurate than the values from which the process started. This cycle affords a method of successive approximations for both total heats and specific heats that can be repeated as many times as may be necessary. It is almost the only satisfactory method of determining c_p on or very close to the saturation line.

JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

LIGHT FROM UNCHARGED MOLECULES.[1]

By C. D. CHILD.

THE vapor passing from a mercury arc into a condensing chamber is luminous giving a line spectrum which is approximately the same as that of the arc. The light given by this vapor appears to come from uncharged molecules and not from ions, as is shown by the following.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

It is not possible that the ions are set in vibration in the *arc*, for the time that it takes for an ion to pass from the arc to the region above it is much too long for the ion to be in continuous vibration. Moreover the luminosity can be destroyed by an electric field. An electric field can remove ions from a region, but it cannot destroy the vibrations of the ions. If it removes the vibrating ions, it must remove them to some point and there must be a concentration of light in the region to which they are removed. In reality there is no concentration of light at any point.

Again it is not possible for the ionization to occur in the region above the arc, first because there is no known cause for ionization in this region. A more decisive reason is that if the ionization occurred above the arc, this region could not be cleared of light by an electric field, since a field cannot affect the uncharged molecules from which ions would be formed. But there is no other place for ionization to occur. If the light is not due to ionization occurring in either of the places, it cannot be due to ionization at all.

On the other hand if light comes from recombination of ions, it can be destroyed by removing the ions, but cannot be concentrated, since both kinds of ions cannot be concentrated in the same region. This is in accordance with the observations.

With an electric field somewhat stronger than that needed to destroy the luminosity the light is intensified and changed from a yellow to a green tint. This is due to the strengthening of the green line in the spectrum and the weakening of the yellow lines. With this stronger field there is also a large increase in the conductivity of the vapor, which is no doubt due to ionization in this region. The change in the relative intensity of the lines can be explained on the assumption that it is due to the formation of a greater number of those positive ions which lack two or more electrons.

NOTE ON THE CONSTRUCTION OF THERMOPILES.[1]

BY W. W. COBLENTZ.

WITHIN the past few years rapid strides have been made in the development of sensitive thermopiles. In this development experimenters have been greatly assisted by the theoretical principles or thermopile construction enunciated by Johansen,[2] who shows that the radii of the two wires of the element must be so chosen that the ratio between the heat conductivity and the electrical resistance is the same in both. Johansen's practice, however, is poor in that he used iron wire which rusts rapidly and in that the "cold," unexposed, junctions were joined directly to the metal posts in the frame supporting the junctions. This prevents the unexposed junctions from quickly attaining the same temperature as the "hot" junctions and hence tends to cause a "drift" as was found by Moll,[1] and more recently by the writer. The "cold" junctions should be covered with receivers having the same size and emissivity as the "hot" junctions and they should be free in the air so that there will be no drift with the variation in temperature which is caused by the incident radiation. In the Hilger thermopile which embodies the Johansen principles, the "cold" junctions appear to be joined to the metal posts in the ivory frame, hence are liable to cause a "drift" of the galvanometer needle as was found by Moll and others. It is, of course, easier to construct such a thermopile but it is not giving any forethought to the victim who must use such devices. With this thought in mind, the writer constructs his thermopiles of bismuth and silver[2] (or copper) with the "cold" junctions exposed freely in the air, and having vanes symmetrical with the "hot" junctions, which is an important, if not the most important, desideratum in thermopile construction.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[2] Johansen, Ann. der Phys. (4), 33, p. 517, 1910.

The additional labor involved in constructing such a thermopile is compensated for by a greater ease of operation with a highly sensitive galvanometer, such as is required in energy measurements in the extreme ends of the spectrum.

Usually all the junctions are joined in series. However, it is just as easy to join them in series-parallel to reduce the resistance.

Flattening the bismuth wire, by pressing it between glass plates, increases the radiation sensitivity about 9 per cent.

A Vibration Electrometer.[3]

By Harvey L. Curtis.

In the comparison of very small capacities by alternating current of low frequency, using a bridge method, it is necessary to use as a detecting instrument one which will take very little current. Such an instrument can be constructed by using some of the well-known types of vibration galvanometers, but winding them of very fine wire so that the resistance will be high. However, a limit in this direction is soon reached. The vibration electrometer here described requires less current for its operation than any of the electromagnetic instruments, though the voltage which must be applied may be higher.

Greinacher[4] has recently adapted a Wulf electrometer for use as a vibration instrument. The present instrument, however, is based upon the quadrant electrometer. The vane corresponds to the needle of the quadrant electrometer, while the quadrants become small vertical plates. A horizontal section is shown in Fig. 1.

In the use of the instrument, a suitable direct current voltage is applied to the vane, while the alternating current is connected to the opposite pairs of quadrants. The vane is mounted on a bifilar suspension whose length and tension can be readily adjusted. In this way the period of the vibrating parts may be adjusted to be the same as that of the applied alternating electromotive force. The mechanical and electrical systems are then in resonance and the amplitude of vibration will depend upon the damping of the mechanical system. As the air damping is a large part of the total, the sensitiveness is greatly increased by placing the instrument in a vacuum. In fact, the vacuum must be less than 0.1 mm. of mercury before satisfactory results can be obtained, and the sensitiveness increases in the same ratio as the pressure decreases until the pressure is as low as 0.005 mm., the lowest which was used. However, as the damping is decreased, the resonance range decreases and the response of the instrument to changes in the applied voltage becomes slower. A pressure of 0.03 mm. is as low as is desirable.

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[1] Moll, Dissertation, Utrecht, 1907.

[2] Coblentz, Bull. Bur. Standards, 9, p. 7, 1912.

[3] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

[4] Phys. Zs., 13, pp. 388 and 433, 1912.

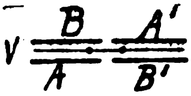

Fig. 1.

A—A' and *B—B'* are the plates to which the alternating E.M.F. is applied. *V* is the moving vane, suspended bifilarly.

The form of the instrument is such that all of the necessary constants can be determined mathematically. This is of interest for two reasons: viz., (1) some of the assumptions in the theory of the quadrant electrometer can be demonstrated for this instrument, and (2) the instrument is a type of electrostatic motor.

The instrument has been carefully tested and experiment and theory found to agree. Some of the most important facts concerning the instruments are as follows. The natural period of the instrument depends upon the voltage applied to the vane, the period decreasing as the voltage increases. If the voltage on the vane is increased and the frequency of the alternating electromotive applied to the plates adjusted until there is a maximum deflection of the mirror, then the sensitiveness increases with increasing voltage, but when the sensitiveness becomes infinite, the period becomes zero. This shows that there is electrostatic control which is opposed to the mechanical control. The sensitiveness is inversely proportional to the damping, while the resonance range is directly proportional to the damping. The current which the instrument requires may be, under working conditions, as low as 10^{-8} or 10^{-9} amperes, while the voltage may be as low as 10^{-2} volts.

RADIATION FROM SOLID AND MOLTEN IRON.[1]

BY C. C. BIDWELL.

DEVIATIONS from full radiation for surfaces of solid and molten iron for wave-length approximately .65μ have been obtained and curves plotted showing the per cent. of true temperatures which an optical pyrometer using red light gives. Knowing the true and black-body temperatures the emissivity (relative to a black-body as unity) has been computed by means of Wien's law and the relation between the relative emissivity and the absolute temperature shown. The curve for solid iron passes over to that for molten iron at about the melting-point of pure iron. The relation between emissivity and temperature for molten iron is a linear one for the temperature range studied (1175° C. to 1900° C.). The temperature coefficient of relative emissivity for molten iron was determined as .0004243. It was found possible to undercool molten iron about 300 degrees. If we except the undercooled metal, the emissivity is a minimum at about the melting-point of pure iron.

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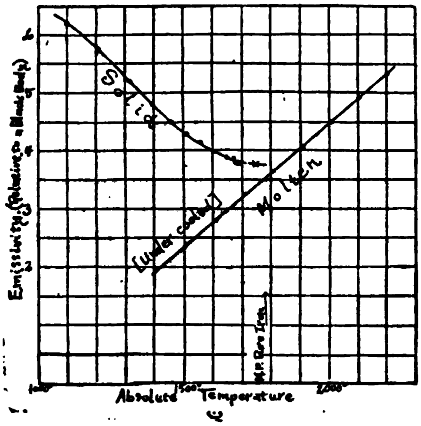
[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 to 26, 1913.



Relation between temperature and emissivity for solid and molten iron for wave-length approximately .65μ.

The true temperatures of the molten iron surface were obtained by sighting into a carbon cavity immersed in the iron and flush with the surface. Verification of the cavity reading as giving true temperatures was obtained for temperatures in the neighborhood of 1550° C. by means of a platinum, platinum-rhodium thermo-junction encased in a fine drawn quartz capillary and pressed into the surface of the iron. Solid iron was heated in a hydrogen atmosphere and in nitrogen. The surface of a large screw head was found convenient to sight upon the slit furnishing a good black-body. Thermo-junction tests showed the slit gave true temperatures.

Cornell University,
April 21, 1913.

A New Thermo-Element for High Temperature Work.[1]

By C. C. Bidwell.

A THERMAL E.M.F. between carbon and graphite has been noted and partially investigated. If the carbon and graphite is previously subjected to high heat treatment (2500° C. or more for about one minute) impurities are driven off and such samples show apparently a definite reproducible E.M.F., which varies in a definite way with the temperature. If very fine filaments are used heat conduction is reduced to a minimum. With such samples the temperature E.M.F. curve is apparently a straight line with a slope showing approximately 25.7 micro-volts per degree. A calibration has been carried to 2200° C. with no curvature apparent. There appears to be some variation with different samples of graphite and carbon. Further investigation of the junction is in progress.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

CORNELL UNIVERSITY,
April 21, 1913.

ON THE OCCURRENCE OF HYDROGEN LINES IN THE TUBE-ARC AND APPLICATION TO THE NATURE OF ENHANCED LINES.[1]

BY ARTHUR S. KING.

EXPERIMENTS have been continued on the spectra given by the arc which forms when an electric-furnace tube is forced to high incandescence and made to burn through. This arc, at high current and low voltage in a partial vacuum, gives very strongly those lines which are enhanced by the spark discharge as compared with the arc. The conditions near the center of the tube's cross-section are most favorable for these enhanced lines, this being shown by photographing the spectrum with a long slit which passed across the diameter of the tube's image and observing the distribution of intensity along the line. It is now found that the hydrogen series lines, from H_α to H_ϵ inclusive, appear in this arc and are strongest at the center of the tube. The arc lines of different elements show varying degrees of response to the conditions at the center, those of titanium and vanadium being strongest close to the arcing wall.

Tests of the electrical condition of the vapor in this " tube-arc " by means of exploring electrodes have shown, first, that very vigorous ionization takes place, and second, that the ionization is not sufficient to render the vapor nearly as good a conductor as the graphite wall of the tube. These results show that the current for the most part follows the direct path between the ends of the broken tube. The greater strength of the hydrogen and enhanced lines at the center of the tube seems to be accounted for best by a strong state of electro-luminescence in this portion, caused by the ejection of high-speed corpuscles from the intensely heated carbon. If the corpuscles shot out normally from the curved wall of the tube are most effective in rendering the vapor luminous, the number of impacts by these corpuscles will be greatest near the center. The behavior of the hydrogen lines, for which the electro-luminescence of the vacuum-tube is the most favorable of the light sources usually employed, points to such an effect of electronic impacts in the tube-arc.

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[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

The mixed condition of electro- and thermo-luminescence near the wall of the tube is most favorable for the lines characteristic of the arc.

A dissymmetry of the wider lines, becoming most pronounced near the center of the tube, has been a feature of the spectra throughout this investigation. The red side of the line is usually stronger, but some lines remain nearly symmetrical and λ 4481 of magnesium is unsymmetrical toward the violet. This effect, which corresponds in some degree with that shown by the condensed spark, may arise from a cause similar to that which gives displaced lines in certain stellar spectra.

Mount Wilson Solar Observatory,
February, 1913.

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